Neutral and charged current inclusive neutrino reactions in ^{12}C and the pion-electroproduction cross section

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Total muon-capture rates and inclusive pion-electroproduction cross sections in ^{12}C are used to obtain inclusive neutral and charged current neutrino cross sections in ^{12}C. These results are obtained for 150 MeV $\leq E_\nu \leq 250$ MeV for the charged current case and 50 MeV $\leq E_\nu \leq 150$ MeV for the neutral current case. Comparison is made with other calculations.

I. INTRODUCTION

The reaction $\nu_\mu + {}^{12}\text{C} \rightarrow \mu^- + X$ has been proposed as a possible test for neutrino oscillations[1] and preliminary measurements have been performed on it. These measurements have yielded no evidence for ν_e-ν_μ oscillation but have left open the question of ν_μ-ν_τ oscillation. A number of calculations have been undertaken for this reaction based on a summing of individual final state contributions[2] and on a Fermi gas model.[3] Recently a calculation making use of closure and the nonrelativistic impulse approximation has been performed[4] in which the total cross section was given as a function of the average momentum transfer squared, $\langle \vec{q}^{\,2} \rangle$. To supplement this work we make use of an elementary particle model related result[4] and experimental values for the total muon-capture rate $\mu^- + {}^{12}\text{C} \rightarrow \nu_\mu + X$ and the pion electroproduction cross section $e + {}^{12}\text{C} \rightarrow e' + \pi^+ + {}^{12}\text{C}$ to obtain the charged and neutral current neutrino reaction cross sections in ^{12}C, namely, $\nu_\mu + {}^{12}\text{C} \rightarrow \mu^- + X$ and $\nu_\mu + {}^{12}\text{C} \rightarrow \nu'_\mu + X$, respectively.

In order to perform the actual calculations, it will be necessary to make a number of approximations which we list at the outset. These approximations will necessarily make the calculation a rough one but the results are nonetheless interesting. First, we shall assume that cross terms, particularly the vector current-axial vector current interference terms, are small. For individual transitions this is not true; however, the situation may be somewhat better for the inclusive case being considered here. This is because there will be a large number of states with axial and vector current form factors of random sign contributing to the interference term which should lead to some cancellation. We note that in the allowed (nonrelativistic) approximation[4,5] these cross terms do in fact vanish. It has been suggested that the sum of the inclusive neutrino and antineutrino cross sections should be free of this difficulty since these interference terms will cancel. We therefore obtain these sums in Sec. III of this paper.

Second, we will have to determine two parameters to fully determine σ_c and σ_N. These are obtained by making use of experimental data for the total muon-capture rate, Γ, for $\mu^- + {}^{12}\text{C} \rightarrow \nu_\mu + X$ and for the differential inclusive pion electroproduction cross section $d^2\sigma/d\Omega\, dE_\pi$, for $e + {}^{12}\text{C} \rightarrow e' + \pi^+ + X$. The treatment of the total muon-capture rate is straightforward but the accuracy of the extraction of the axial current form factors from pion electroproduction data for $e' + {}^{12}\text{C} \rightarrow e' + \pi^+ + X$ is unfortunately quite limited. However, as is remarked in Sec. IV of this paper, the total inclusive neutrino cross section, in the low and middle ranges of E_ν, is dominated by the value of a function, D, at values of $\langle q^2 \rangle$ near that of the total muon-capture rate and so is not highly sensitive to the inclusive pion photoproduction cross section. Other quantities involving these parameters will, in general, be more affected by this problem.

Finally, at the higher end of the E_ν range used here the giant dipole resonance will no longer dominate the inclusive neutrino reaction cross section and so we expect our calculation to begin to fail here. An accurate experiment might be very useful for determining the behavior[6] of D in this region. Thus the calculation presented here is a very approximate one limited by the accuracy of the assumptions as noted above. Nonetheless it is an interesting one readily extended to other nuclei. In Sec. II of this paper we calculate the matrix elements needed for the above mentioned processes. In Sec. III we obtain values for $\sigma_c(\nu_\mu + {}^{12}\text{C} \rightarrow \mu^- + X)$ and $\sigma_N(\nu_\mu + {}^{12}\text{C} \rightarrow \nu'_\mu + X)$ and in Sec. IV we discuss our results.

II. CALCULATION OF MATRIX ELEMENTS

The matrix elements for the process $\nu_\mu + {}^{12}\text{C} \rightarrow \mu^- + X$ may be written to the lowest order in G $(=1.02 \times 10^{-5}/m_p^2)$ as

$$\langle \mu^- X \mid H_w \mid \nu^{12}\text{C} \rangle = \frac{G}{\sqrt{2}} \cos\theta_C \bar{u}_\mu \gamma^\lambda (1-\gamma_5) u_\nu \langle X \mid J_\lambda^\dagger(0) \mid {}^{12}\text{C} \rangle \; . \tag{1}$$

We are not interested in specifying the final state so that σ_c the charged current cross section is given by

$$\sigma_c = \sum_k \frac{1}{2M} \frac{m_\nu}{E_\nu} \int \mid M_{ki} \mid^2 \frac{d^3 P_\mu m_\mu}{E\mu (2\pi)^3} \frac{d^3 P_k}{2E_k (2\pi)^3} (2\pi)^4 \delta^4 [P_\mu + P_\mu - (P_\nu + P_i)] \tag{2}$$

with

$$|M_{ki}|^2=\frac{G^2\cos^2\theta_C}{m_\mu m_\nu}(P_\mu^\sigma P_\nu^\lambda-P_\nu\cdot P_\mu g^{\sigma\lambda}+P_\nu^\sigma P_\mu^\lambda-i\epsilon^{\alpha\sigma\beta\lambda}P_{\mu\alpha}P_{\nu\beta})\langle k|J_\sigma^\dagger(0)|{}^{12}\mathrm{C}\rangle\langle k|J_\lambda^\dagger(0)|{}^{12}\mathrm{C}\rangle^*$$

$$\equiv\frac{G^2\cos^2\theta_C}{m_\mu m_\nu}\mathscr{L}^{\sigma\lambda}\langle k|J_\sigma^\dagger(0)|{}^{12}\mathrm{C}\rangle\langle k|J_\lambda^\dagger(0)|{}^{12}\mathrm{C}\rangle^*\ , \qquad (3)$$

where $J_\mu^\dagger=V_\mu^\dagger-A_\mu^\dagger$ is the hadronic part of the weak charge raising current, θ_C is the Cabbibo angle, M is the ^{12}C mass, the subscripts μ and ν refer to the muon and neutrino, respectively, and the sum is over the final nuclear states. We restrict our attention to excited nuclear states so that the final state is always a two-particle state. In the energy range of interest this should be the dominant mode.[4]

We integrate Eq. (2) over the four-momentum of the final nuclear state and obtain

$$\sigma_c=\sum_k\frac{G^2\cos^2\theta}{2ME_\nu}\int\mathscr{L}^{\sigma\lambda}\langle k|J_\sigma^\dagger(0)|{}^{12}\mathrm{C}\rangle\langle{}^{12}\mathrm{C}|J_\lambda(0)|k\rangle\frac{d^3P_\mu}{E_\mu(2\pi)^2}\delta(P_k^2-M_k^2)\theta(P_{k\sigma})\ . \qquad (4)$$

We wish to apply the closure approximation and so assume an average nuclear excitation δ such that

$$M_x-M=\delta\ . \qquad (5)$$

We obtain, therefore, an average E_μ and $|\vec{\mathrm{P}}_\mu|$

$$\langle E_\mu\rangle\cong E_\nu-\delta\ , \qquad (6a)$$

$$\langle|\vec{\mathrm{P}}_\mu|\simeq[(E_\nu-\delta)^2]^{1/2}-m_\mu^2\ . \qquad (6b)$$

Using Eqs. (6a), (6b), and (4) we perform the integration over E_μ and find

$$\sigma_c=\frac{G^2\cos^2\theta_C}{2ME_\nu}\int d\Omega_\mu\left[\sum_k\langle{}^{12}\mathrm{C}|J_\lambda(0)|k\rangle\langle k|J_\sigma^\dagger(0)|{}^{12}\mathrm{C}\rangle\mathscr{L}^{\sigma\lambda}\frac{\langle|\vec{\mathrm{P}}_\mu|\rangle}{\left|2\mu-2\nu+2\nu\cos\theta_\mu\frac{\langle E_\mu\rangle}{\langle|\vec{\mathrm{P}}_\mu|\rangle}\right|}\right]\ . \qquad (7)$$

Setting

$$P_k^\mu=P_i^\mu+P_\nu^\mu-\langle P_\mu\rangle\equiv P_i^\mu+\langle q^\mu\rangle\ , \qquad (8)$$

we then write

$$\sum_k\langle{}^{12}\mathrm{C}|J_\lambda(0)|k\rangle\langle k|J_\sigma^\dagger(0)|{}^{12}\mathrm{C}\rangle=\sum_{|P_f\alpha\rangle}\langle{}^{12}\mathrm{C}|J_\lambda(0)|P_f\alpha\rangle\langle P_f\alpha|J_\alpha^\dagger(0)|{}^{12}\mathrm{C}\rangle\delta^4[P_f^\mu-(P_i^\mu+\langle q^\mu\rangle)]$$

$$=\int d^4x\,e^{-i\langle q\rangle\cdot x}\sum_{|P_f\alpha\rangle}\langle{}^{12}\mathrm{C}|J_\lambda(0)|P_f\alpha\rangle\langle P_f\alpha|J_\sigma^\dagger(0)|{}^{12}\mathrm{C}\rangle$$

$$=\int d^4x\,e^{-i\langle q\rangle\cdot x}\langle{}^{12}\mathrm{C}|J_\lambda(0)J_\sigma^\dagger(x)|{}^{12}\mathrm{C}\rangle\equiv Q_{\lambda\sigma}(P_i,\langle q\rangle)\ , \qquad (9)$$

where $|P_f\alpha\rangle$ is a complete set of states which we presume is saturated by the lower lying excited states.[4]

The problem of calculating $\sigma_c(\nu+{}^{12}\mathrm{C}\rightarrow\mu^-+x)$ thus reduces to determining $Q_{\lambda\sigma}(P_i,\langle q\rangle)$. We may write the general form of $Q_{\lambda\sigma}(P_i,\langle q\rangle)$ as

$$Q_{\lambda\sigma}(P_i,\langle q\rangle)=\alpha g_{\lambda\sigma}+\frac{\beta}{M^2}P_{i\lambda\sigma}P_{i\sigma}+\frac{\gamma}{M^2}P_{i\lambda}\langle q_\sigma\rangle+\frac{\delta}{M^2}\langle q_\lambda\rangle P_{i\sigma}+\frac{\rho}{M^2}\langle q_\lambda\rangle\langle q_\sigma\rangle+\frac{\eta}{M^2}\epsilon_{\mu\nu\lambda\sigma}P_i^\mu\langle q^\nu\rangle\ . \qquad (10)$$

The η term in the sum is clearly the vector-axial vector cross term. Furthermore, if we explicitly look at the contribution[7] to the coefficients γ, δ, and η from spin 0, 1, and 2 excited states we find that they are of two kinds: (i) cross terms coming from the squared axial current matrix element or the vector current matrix element or (ii) terms proportional to the absolute value squared of a form factor, $|F_{ik}|^2$, for which there exists a corresponding larger term appearing either in α or β of Eq. (10). For a number of reasons we expect the cross terms γ, δ, and η to be small. First, the cross terms contain at most one factor of the largest term in a matrix element. In addition the cross term part of the contributions to γ, δ, and η are of the form $\sum_{i=1}^N F_{ik}F_{il}^*$. Because the number of contributing states is large and because the signs of the individual contributions to the sum should be random we expect that there would tend to be substantial cancellation. These ideas are borne out in the calculation[4] referred to previously which we shall refer to as the KM paper, where only terms proportional to g_V^2 and g_A^2 survive. We find also that contributions to the ρ term will generally be small and so we take

$$Q_{\lambda\sigma}(P_i,\langle q\rangle)\simeq\alpha g_{\lambda\sigma}+\frac{\beta}{M^2}P_{i\lambda}\cdot P_{i\sigma}\ . \tag{11}$$

Because neutrino reaction cross sections to individual states tend to be peaked in the forward direction,[4] we take $\langle\vec{q}\rangle\cong|\vec{P}_\nu|-\langle|\vec{P}_\mu|\rangle$. Under this assumption, Eqs. (11) and (7) imply

$$\sigma_c=\frac{G^2\cos^2\theta_C}{4\pi}\frac{\langle|\vec{P}_\mu|\rangle\langle E_\mu\rangle}{M(M+E_\nu)}D\ , \tag{12}$$

where

$$D=\beta-2\alpha \tag{13}$$

and is a function of $\langle q^2\rangle$ to be determined. In fact, a direct impulse approximation calculation[4] leads to a result of the form

$$D=a_0+b_0q^2\ . \tag{14}$$

We shall assume this form and attempt to determine a_0 and b_0.

We are also interested in the neutral current neutrino reaction $\nu+{}^{12}C\to\nu'+X$. Calculations similar to those leading to Eq. (7) yield a result

$$\sigma=\frac{G^2}{2ME_\nu}\int d\Omega'_\nu\left[\sum_k\langle{}^{12}C|J_\lambda^N(0)|k\rangle\langle k|J_\sigma^N(0)|{}^{12}C\rangle\mathscr{L}_N^{\sigma\lambda}\langle E'_\nu\rangle\frac{1}{|2m-2\nu+2\nu\cos\theta_\nu|}\right]\ , \tag{15}$$

where

$$J_\lambda^N(0)=J_\lambda^{(3)}-2\sin^2\theta_w J_\lambda^{\rm em} \tag{16a}$$

and

$$L^{\sigma\lambda}=(\nu^\sigma\nu'^{\lambda}-g^{\sigma\lambda}\nu\cdot\nu'+\nu'^{\sigma}\nu^\lambda-i\epsilon^{\alpha\sigma\beta\lambda}\nu_\alpha\nu'_\beta)\ . \tag{16b}$$

The sum over excited states in Eq. (15) may be written

$$\sum_k\langle{}^{12}C|J_\lambda^N(0)|k\rangle\langle k|J_\sigma^N(0)|{}^{12}C\rangle=A+B+C+D\ , \tag{17a}$$

where

$$A=\sum_k\langle{}^{12}C|J_\lambda^{(3)}(0)|k\rangle\langle k|J_\sigma^{(3)}(0)|{}^{12}C\rangle\ , \tag{17b}$$

$$B=-2\sin^2\theta_w\langle{}^{12}C|J_\lambda^{(3)}(0)|k\rangle\langle k|J_\sigma^{\rm em}|{}^{12}C\rangle\ , \tag{17c}$$

$$C=-2\sin^2\theta_w\langle{}^{12}C|J_\lambda^{\rm em}(0)|k\rangle\langle k|J_\sigma^{(3)}(0)|{}^{12}C\rangle\ , \tag{17d}$$

$$D=+4\sin^4\theta_w\langle{}^{12}C|J_\lambda^{\rm em}(0)|k\rangle\langle k|J_\sigma^{\rm em}(0)|{}^{12}C\rangle\ , \tag{17e}$$

with $J_\lambda^{(3)}=v_\lambda^{(3)}-A_\lambda^{(3)}$ the neutral member of the isotriplet $J_\lambda^{(1)}$, $J_\lambda^{(2)}$, and $J_\lambda^{(3)}$. Because $4\sin^4\theta_w$ is of the order of 0.2 we ignore term D and examine A, B, and C. Making use of

$$[I_+,J_\mu]=2J_\mu^{(3)}\ ,\qquad [I_-,J_\mu^\dagger]=-2J_\mu^{(3)}\ , \tag{18}$$

and that the fact that ${}^{12}C$ is an $I=0$ state we find

$$A\equiv\sum_k\langle{}^{12}C|J_\lambda^{(3)}(0)|k\rangle\langle k|J_\sigma^{(3)}(0)|{}^{12}C\rangle=\tfrac{1}{2}\sum_{k'}\langle{}^{12}C|J_\lambda(0)|k'|\rangle\langle k'|J_\sigma^\dagger(0)|{}^{12}C\rangle\ . \tag{19}$$

In order to obtain nonzero contributions $|k\rangle$ must be an $I=1$, $I_z=0$ state and $|k'\rangle$ must be an $I=1$, $I_z=1$ state. However, we can extend the sum over all intermediate states since others will add only a zero contribution. Thus we may define an analogy with Eq. (11)

$$Q_{\lambda\sigma}^{(3)}(P_i,\langle q\rangle)=\tfrac{1}{2}Q_{\lambda\sigma}(P_i,\langle q\rangle) \tag{20}$$

so that if $Q_{\lambda\sigma}(P_i,\langle q\rangle)$ can be determined then $Q_{\lambda\sigma}^{(3)}$ is known. We may write

$$J_\mu^{\rm em}(0)=V_\mu^{(3)}(0)+V_\mu^{(0)}(0)\ , \tag{21}$$

where $V_\mu^{(3)}(0)$ is the isovector and $V_\mu^{(0)}(0)$ is the isoscalar part of the electromagnetic current, respectively. Then

$$\begin{aligned}B&=-2\sin^2\theta_w\sum_k\langle{}^{12}C|J_\lambda^{\rm em}(0)|k\rangle\langle kJ_\sigma^{(3)}(0)|{}^{12}C\rangle\\&=-2\sin^2\theta_w\left[\sum_k\langle{}^{12}C|V_\lambda^{(3)}(0)|k\rangle\langle k|J_\sigma^{(3)}(0)|{}^{12}C\rangle+\sum_k\langle{}^{12}C|V_\lambda^{(0)}(0)|k\rangle\langle k|J_\sigma^{(3)}(0)|{}^{12}C\rangle\right]\ .\end{aligned} \tag{22}$$

The second term in Eq. (22) cannot be satisfied by any $|k\rangle$. Furthermore, we are ignoring vector-axial vector current matrix element cross terms. Under these conditions

$$\begin{aligned}B\cong C&\cong-2\sin^2\theta_w\sum_k\langle{}^{12}C|V_\lambda^3(0)|k\rangle\langle k|V_\sigma^3(0)|{}^{12}C\rangle\\&\equiv-2\sin^2\theta_w Q_{\lambda\sigma}^v(P_i,\langle q\rangle)\ ,\end{aligned} \tag{23}$$

so that from Eqs. (20), (22), and (23)

$$\sum_k\langle{}^{12}C|J_\lambda^N(0)|k\rangle\langle k|J_\sigma^N(0)|{}^{12}C\rangle\cong\tfrac{1}{2}Q_{\lambda\sigma}(P_i,\langle q\rangle)-4\sin^2\theta_w Q_{\lambda\sigma}^V(P_i,\langle q\rangle)\ , \tag{24}$$

where $Q^V_{\lambda\sigma}$ must be determined. Again ignoring cross terms we would expect

$$Q^V_{\lambda\sigma}(P_i,\langle q\rangle)\simeq\alpha^V g_{\lambda\sigma}+\frac{\beta^V}{M^2}P^i_\lambda P^i_\sigma \ . \tag{25}$$

In an impulse approximation treatment of the weak current matrix element,

$$\langle f\,|\,J_\mu(0)\,|\,i\,\rangle=\langle\Psi_f\,|\,\sum_{j=1}^{N}\bar{u}_{fj}[(\gamma_\mu g_V+i\sigma_{\mu\nu}q^\nu g_M)-(\gamma_\mu\gamma_5 g_A+g_P\gamma_5)]u_{ij}e^{i\vec{q}\cdot\vec{r}_j}\tau_j'\,|\,\Psi_i\,\rangle \ , \tag{26}$$

where g_V, g_M, g_A, and g_P are the nucleon vector, magnetic, axial vector, and pseudoscalar form factors. We see that the large term of the vector current is the time component so that the dominant term of $Q^V_{\lambda\sigma}$ should be Q^V_{00}. Requiring the same general behavior from Eq. (25) yields

$$Q^V_{\lambda\sigma}\simeq\frac{\beta^V}{M^2}P^i_\lambda P^i_\sigma \ . \tag{27}$$

Making use of Eqs. (24), (27), and (15) we obtain

$$\sigma_N=\frac{G^2\langle E_\nu'\rangle^2}{8M^2\pi}[(\beta-2\alpha)-8\sin^2\theta_w\beta^V] \ . \tag{28}$$

A direct impulse approximation calculation[4] shows that if we take $J^N_\mu=J^3_\mu$ alone, the cross section σ_N is proportional to $(g_V^2+3g_A^2)$. Thus from Eq. (13) $\beta^{(3)}-2\alpha^{(3)}$ must be proportional to $g_V^2+3g_A^2$. Furthermore, because the $\lambda=0$, $\sigma=0$ component of $Q^{(3)}_{\lambda\sigma}$ should be dominated by the vector current contributions, whereas the diagonal space components should be dominated by the axial current,[8]

$$Q^{(3)}_{00}=\alpha^{(3)}g_{00}+\frac{\beta^{(3)}}{M^2}P_{0i}P_{0i}=\alpha^{(3)}+\beta^{(3)}\simeq\beta^V \tag{29}$$

and

$$\beta^{(3)}\simeq\beta^V-\alpha^{(3)} \ ,$$

$$\beta^{(3)}-2\alpha^{(3)}=\beta^V-3\alpha^{(3)}=k(g_V^2+3g_A^2) \ .$$

From the accepted values for g_V and g_A

$$\beta^V\simeq0.2(\beta^{(3)}-2\alpha^{(3)})=0.1(\beta-2\alpha) \tag{30}$$

so that

$$\sigma_N\simeq\frac{G^2\langle E_\nu'\rangle^2}{8M^2\pi}(0.832D)$$

and both σ_c and σ_N are proportional to D.

We next consider the muon-capture reaction $\mu^-+{}^{12}\mathrm{C}\to\nu_\mu+X$. The matrix element for this process is given by

$$M_{ki}=\frac{G}{\sqrt{2}}\cos\theta_C\langle k\,|J_\sigma(0)\,|\,{}^{12}\mathrm{C}\rangle\bar{u}_\nu\gamma^\sigma(1-\gamma_5)u_\mu \ . \tag{31}$$

By a calculation analogous to that which preceded Eq. (12) we obtain for the total capture rate

$$\Gamma_{\rm tot}=\frac{C\,|\,\Phi(0)\,|^2G^2\cos^2\theta_C\langle E_\nu\rangle^2D}{8\pi M(M+m_\mu)} \ , \tag{32}$$

where we have made use of charge symmetry to equate $\langle{}^{12}\mathrm{C}\,|\,J^{(0)}_\sigma J^\dagger_\lambda(x)\,|\,{}^{12}\mathrm{C}\rangle$ and $\langle{}^{12}\mathrm{C}\,|\,J^\dagger_\sigma(0)J_\lambda(x)\,|\,{}^{12}\mathrm{C}\rangle$ and where C is a correction factor[9] which takes into account the charge spread of the initial nucleus and $\Phi(0)$ is the momentum space ground state wave function of the muon. Thus $\Gamma_{\rm tot}$ is proportional also to D evaluated at the $\langle q^2\rangle$ appropriate to muon capture.

Finally, to obtain another process involving D at a different value of $\langle q^2\rangle$ so that a_0 and b_0 may be determined we consider the pion electroproduction reaction $\mathrm{e}+{}^{12}\mathrm{C}\to\mathrm{e}'+\pi^++X$. To determine the matrix element for this process we make use of partial conservation of axial-vector current (PCAC) (Ref. 10) and write

$$\langle\,k\,|\,\partial_\mu A^\mu+iea_\mu A^\mu\,|\,\gamma\,{}^{12}\mathrm{C}\rangle=f_\pi m_\pi^2\langle\,k\,|\,\Phi^\dagger_\pi\,|\,\gamma\,{}^{12}\mathrm{C}\rangle \ , \tag{33}$$

where f_π ($=0.96m_\pi$) is the pion decay constant and m_π is the pion mass. Thus

$$-iq_\mu\langle\,k\,|\,A^\mu\,|\,\gamma\,{}^{12}\mathrm{C}\rangle+ie\epsilon_\mu\langle\,k\,|\,A^\mu\,|\,{}^{12}\mathrm{C}\rangle=f_\pi m_\pi^2\langle\,k\,|\,\Phi^\dagger_\pi\,|\,\gamma\,{}^{12}\mathrm{C}\rangle \tag{34}$$

and taking the usual soft pion $\lim q_\mu\simeq0$, we obtain

$$ie\epsilon_\mu\langle\,k\,|\,A^\mu\,|\,{}^{12}\mathrm{C}\rangle\simeq f_\pi m_\pi^2\langle\,k\,|\,\Phi^\dagger\,|\,\gamma\,{}^{12}\mathrm{C}\rangle \ . \tag{35}$$

Making use of

$$(\Box^2+m_\pi^2)\phi_\pi^2=J_\pi^\dagger \tag{36}$$

we find

$$\langle\,k\,|\,J^\dagger_\pi(0)\,|\,\gamma\,{}^{12}\mathrm{C}\rangle\simeq\frac{ie}{f_\pi}\epsilon^\mu\langle\,k\,|\,A_\mu(0)\,|\,{}^{12}\mathrm{C}\rangle \ . \tag{37}$$

Assuming one photon exchange dominates[11] this process and substituting a virtual photon in Eq. (37) we obtain

$$M_{ki}\equiv\langle\,k\,|\,J^\dagger_\pi\,|\,\gamma\,{}^{12}\mathrm{C}\rangle\simeq\frac{ie^2}{f_\pi}\bar{u}_{\mathrm{e}'}\gamma_\mu u_\mathrm{e}\frac{\langle\,k\,|\,A^\mu\,|\,{}^{12}\mathrm{C}\rangle}{q^2} \ , \tag{38}$$

where u_e' and u_e are electron spinors. In the process of interest here the outgoing pion angle and energy are measured, but the final electron is not observed. Using Eq. (38) we may write again in analogy with previous calculations,

$$\frac{d^2\sigma}{d\Omega_\pi dE_\pi}=\frac{m_\mathrm{e}^2P_\pi}{4ME(2\pi)^5}\times\int\,|\,M\,|^2P'd\Omega'dE'\delta(P_f^2-M_f^2)\theta(E_f) \ , \tag{39}$$

with

$$|M|^2=\frac{1}{2m_e^2}\frac{(-2P\cdot P'\alpha^A+2\beta^A EE'-P\cdot P'\beta^A)}{4\left[M\left[-\epsilon+\frac{E}{M}\rho_\pi-\frac{m_\pi^2}{2M}\right]+E'(M-\rho_\pi')\right]^2}, \quad (40)$$

where m_e is the electron mass; P_π and E_π the pion momenta and energy, respectively; E and P and E' and P' the initial and final electron energy and momentum and

$$\epsilon\equiv E-E_\pi-\delta\,, \qquad \rho_\pi\equiv E_\pi-P_\pi\cos\theta\,, \quad (41)$$

and where we have performed a calculation analogous to those leading to Eqs. (9)—(11) to obtain

$$\sum_k \langle k\,|\,A_\lambda\,|\,^{12}\mathrm{C}\rangle\langle k\,|\,A_\sigma\,|\,^{12}\mathrm{C}\rangle^*\equiv Q^A_{\lambda\sigma}(P_i,\langle q\rangle)\,, \quad (42a)$$

with

$$Q^A_{\lambda\sigma}(P,\langle q\rangle)=\alpha^A g_{\lambda\sigma}+\frac{\beta^A}{M^2}P_{i\lambda}P_{i\sigma}\,. \quad (42b)$$

We make use of the "peaking" approximation[12] which comes from the strong forward peak in the pion electroproduction cross section resulting from the factor of $1/q^4$ in the denominator of Eq. (40). We therefore set the $P\cdot P'$ terms to zero in the numerator of Eq. (40) since

$$P\cdot P'\simeq PP'(1-\cos\theta)\simeq 0\,. \quad (43)$$

Integrating Eq. (39) we obtain

$$\frac{d^2\sigma}{d\Omega_\pi dE_\pi}=\frac{\left[P_\pi\alpha^2\beta^A\left[\epsilon+\frac{m_\pi^2}{2M}-\frac{E\rho_\pi}{M}\right]C_{\pi+}\right]}{32\pi^2MEf_\pi^2\rho_\pi[(E-\epsilon)\rho_\pi-\frac{1}{2}m_\pi^2]}\,, \quad (44)$$

whereas in the calculations for σ_c, σ_N, and Γ_{tot} we have assumed that β is a function of $\langle q^2\rangle$ and so is a constant with respect to the integration.

We wish to be able to write Eq. (44) as a function of $D=\beta-2\alpha$ rather than as a function of β^A. Since $Q^A_{\lambda\sigma}(P_i,\langle q\rangle)$ is the matrix element of the product of axial currents, the large terms in $Q^A_{\lambda\sigma}$ should come from the space part of the current. The Q^A_{00} term should be small. If we assume $Q^A_{00}\simeq 0$, then

$$\alpha^A+\beta^A=0 \quad (45)$$

or $\beta^A=-\alpha^A$. However, the tensor $Q_{\lambda\sigma}(P_i,\langle q\rangle)$ which represents the product of the current, $J_\mu=V_\mu-A_\mu$, with itself has a $\lambda=0$, $\sigma=0$ component

$$Q_{00}(P_i,\langle q\rangle)=\alpha+\beta\,. \quad (46)$$

This is almost entirely from the $V_\mu V_\mu^\dagger$ part of the product of currents since only the vector part of the current has a large time component. Furthermore, the diagonal space-space components of $Q_{\lambda\sigma}$ come almost entirely from the product $A_\mu A_\mu^\dagger$. But

$$Q^{jj}(P_i,\langle q\rangle)=\alpha g^{jj}+\frac{\beta P^j_{(i)}P^j_{(i)}}{M^2}\,, \quad (47)$$

where no sum is intended over j. Thus since the axial current in Eq. (42) is the axial part of the $J_\mu=V_\mu-A_\mu$ current, $|\alpha|$ must equal $|\alpha^A|$ of Eq. (42b) and

$$Q_{00}(P_i,\langle q\rangle)=\alpha^A+\beta\equiv\beta^V \quad (48)$$

yielding

$$\beta=\beta^V-\alpha^A \quad (49)$$

so that

$$D=\beta-2\alpha=\beta^V-3\alpha^A\,. \quad (50)$$

Again assuming from the KM paper that $D\alpha(g_V^2+3g_A^2)$, then $|\beta^V|3/\alpha^A|\simeq g_V^2/3g_A^2$, and from Eq. (45)

$$\beta^A=-\alpha^A\cong D/3.75 \quad (51)$$

so that

$$\frac{d^2\sigma}{d\Omega_\pi dE_\pi}\simeq\frac{\left[P_\pi\alpha^2D\left[\epsilon+\frac{m_\pi^2}{2M}-\frac{E\rho_\pi}{M}\right]C_{\pi+}\right]}{120\pi^2MEf_\pi^2\rho_\pi[(E-\epsilon)\rho_\pi-\frac{1}{2}m_\pi^2]}\,, \quad (52)$$

where α here is the fine structure constant. In the above equation $C_{\pi+}$ is a correction factor due to the Coulomb interaction of the pion with the final state nucleus. We use a factor of the form[12,13]

$$C_{\pi+}=(b/\rho_\pi)/(e^{b/\rho_\pi}-1)\,, \qquad b=2\pi\alpha Z\bar{m}\,, \quad (53)$$

where α is again the fine-structure constant, Z is the charge of the final state nucleus, ρ_π is the magnitude of the pion-space momentum, and $\bar{m}$ is the reduced mass of the pion-field state nucleus system. We are now ready to obtain D and thus determine σ_c and σ_N.

III. EVALUATION OF THE CROSS SECTIONS

We assume that the quantity $D=\beta-2\alpha$ has the form

$$D=a_0+b_0q^2 \quad (54)$$

suggested by the KM paper[4] referred to previously. To determine a_0 and b_0 we first make use of the experimental total muon-capture rate[14] for ^{12}C, $\Gamma=(3.97\pm0.01)\times10^4\ \mathrm{sec}^{-1}$ and obtain from Eq. (31)

$$D=3.76\times10^9\ \mathrm{MeV}^2\,, \quad (55)$$

where $q^2\simeq\vec{q}^{\,2}\simeq(0.75m_\mu)^2$. This provides one constraint on a_0 and b_0.

Measurements of the double differential cross section $d^2\sigma/d\Omega_\pi dE_\pi$ for the process $e+{}^{12}\mathrm{C}\rightarrow\epsilon'+\pi^++X$ with parameters such that the region of the giant dipole resonance is reached have recently been performed.[15] We make use of this data to obtain values for $d^2\sigma/d\Omega_\pi dE_\pi$ leading to the largest nuclear excitation.

Our assumption in obtaining Eqs. (12) and (31), expressions for the charged and neutral current neutrino reactions σ_c and σ_N, was that a nuclear excitation, δ ($\simeq$35 MeV), corresponding to the giant dipole resonance would be a suitable average excitation. The data in Ref. 13

reached the range of the dipole resonance rather than averages at it. Therefore because the range of available states increases rapidly[16] with decreasing E_π, we extrapolate[17] the data of Ref. 13 to $T_\pi = 2$ MeV. From this value we find

$$E_\pi = 141.5 \text{ MeV} ,\quad P_\pi = 23.7 \text{ MeV} ,\quad \epsilon = 23.5 \text{ MeV} ,\quad \rho_\pi = 124.91 \text{ MeV} , \tag{56}$$

and obtain

$$D = 4.2 \times 10^9 \text{ MeV}^2 \tag{57}$$

with $q^2 = -2.46 \times 10^4$ MeV^2. We can then calculate from Eqs. (54), (55), and (57) the result

$$D = 3.61 \times 10^9 (1 + 0.074 q^2 / m_\mu^2) . \tag{58}$$

This result is sensitive to the value for T_π (or to the average value of the excitation) as was discussed above as can be seen from Fig. 1.

Using the result, Eq. (57), for D we obtain from Eqs. (12) and (31) values for $\sigma_c(\nu_\mu + {}^{12}\text{C} \to \mu^- + X)$ and $\sigma_N(\nu_\mu + {}^{12}\text{C} \to \nu\mu' + X)$. The results are plotted in Figs. 2 and 3, respectively.

Finally we note, as mentioned in the Introduction, that the vector current-axial vector current interference terms have opposite signs for neutrino and antineutrino reactions. Thus the sum of the inclusive neutrino and antineutrino inclusive sections are given by

$$\sigma_{\nu\bar{\nu}c} \equiv \sigma_{c\nu} + \sigma_{c\bar{\nu}} = 2\sigma_c , \tag{59a}$$

$$\sigma_{\nu\bar{\nu}N} \equiv \sigma_{N\nu} + \sigma_{N\bar{\nu}} = 2\sigma_N , \tag{59b}$$

where the notation is obvious and σ_c is given by Eq. (12) and σ_N is given by Eq. (30). Clearly a measurement of $\sigma_{c\bar{\nu}}$ and $\sigma_{c\nu}$ would help test the assumptions made concerning the interference terms.

IV. DISCUSSION OF RESULTS

In Fig. 4 we have plotted the results from previous calculations[2-4] and the results obtained here for the charged current cross sections $\sigma_c(\nu + {}^{12}\text{C} \to \mu^- + X)$. We note that our results are virtually indistinguishable from those of the KM paper. This is because both calculations make use of the same relationship between the total muon capture rates and the σ_c cross section as well as the expectation that $|\vec{q}| = |\vec{P}_\nu| - |\vec{P}_\mu|$. But under these assumptions $\langle q^2 \rangle$ for muon capture and for the neutrino reac-

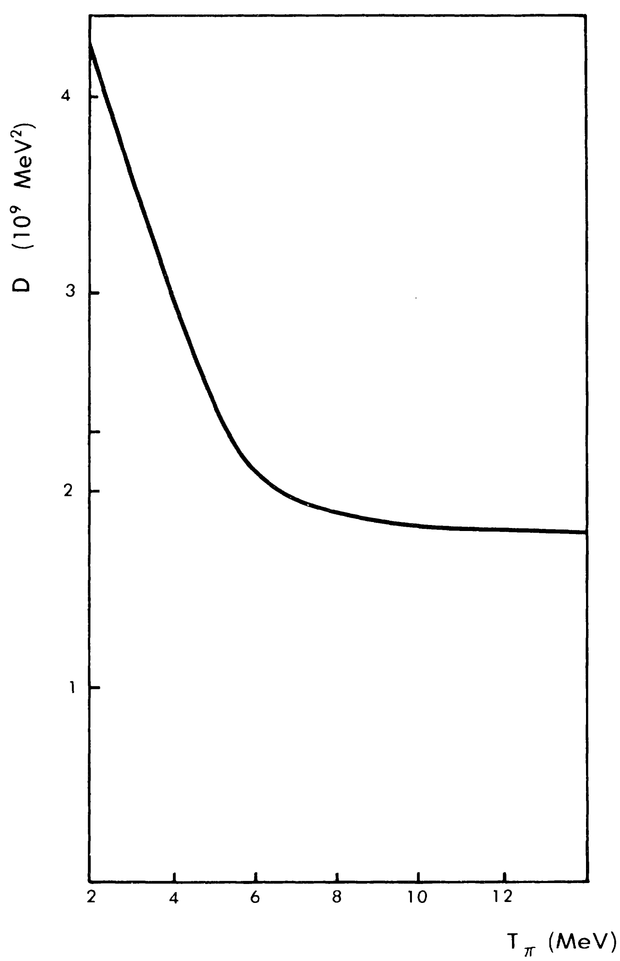

FIG. 1. Plot of D as a function of T_π, the pion kinetic energy.

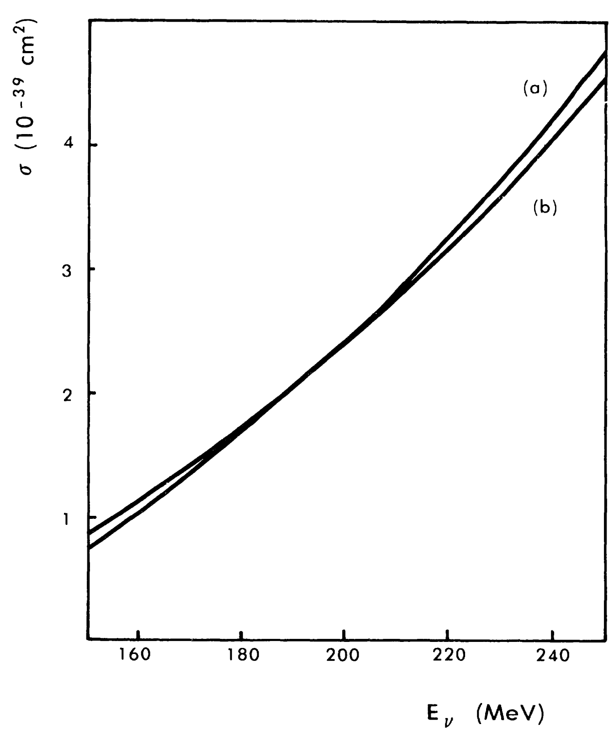

FIG. 2. Plot of the inclusive charged current neutrino cross section $\sigma_c(\nu_\mu + {}^{12}\text{C} \to u^- + X)$ as a function of E_ν, the neutrino energy. Curve (a) is the cross section for the value of D given by Eq. (58) and curve (b) is the cross section for the value of D given by Eq. (60).

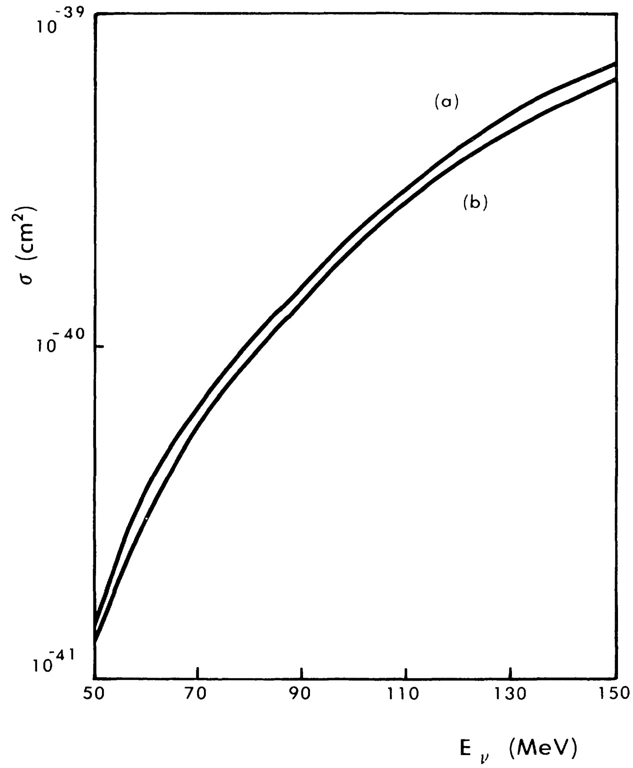

FIG. 3. Plot of the inclusive charged current neutrino cross section $\sigma_N(\nu_\mu + {}^{12}\mathrm{C} \rightarrow \nu_\mu + X)$ as a function of E_ν, the neutrino energy. Curve (a) is the cross section for the value of D given by Eq. (58) and curve (b) is the cross section for the value of D given by Eq. (60).

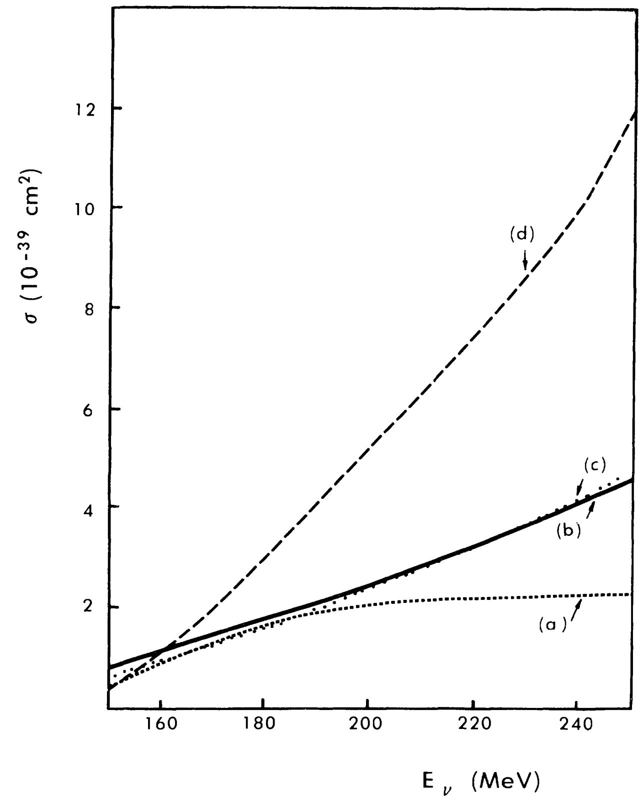

FIG. 4. Plot of the inclusive charged current neutrino cross section $\sigma_c(\nu_\mu + {}^{12}\mathrm{C} \rightarrow u^- + X)$. Curve (a) is the result obtained by making use of an impulse approximation treatment and a summation over nuclear excited states. Curve (b) is the result obtained in the KM paper using closure and a nonrelativistic impulse approximation. Curve (c) is the result obtained here and curve (d) is the result obtained using a Fermi gas model.

tions do not differ substantially, particularly over the middle of the range of E_ν used here, 150 MeV $\leq E_\nu \leq 250$ MeV. Thus the cross section is largely determined by the experimental value for Γ and kinematical factors. The relationship of the calculation presented here with the other two calculations are in line with expectations.

A more sensitive comparison of the KM calculation and the one presented here can be obtained by looking at the $\langle q^2 \rangle$ behavior of the neutrino cross section. In the KM paper

$$\sigma_c \alpha \left[1 + 0.43 \frac{\vec{q}^{\,2}}{m_\mu^2} \right], \tag{60a}$$

whereas the result of this paper is

$$\sigma_c \alpha (1 + 0.074 q^2/m_\mu^2) \,. \tag{60b}$$

This difference looks like a significant one until the sensitivity of D, Eq. (5a), to the value of T_π, the pion kinetic theory, is examined. As is shown in Fig. 1, D increases rapidly with decreasing T_π or with an increasing average nuclear excitation. Thus we expect our value of D to be somewhat of an underestimate. To obtain an estimate of this sensitivity, we note that the calculation has an error of $\pm 20 \sim 25\,\%$. If we assume D to be 20% larger than that obtained from the pion-electroproduction interaction, Eq. (57), we obtain

$$D = 3.25 \times 10^9 \left[1 + \frac{0.28}{M_\mu^2} q^2 \right] \tag{61}$$

so that a 20% change in D results in a 400% change in q^2 dependence. In Figs. 2 and 3 we present values for σ_c and σ_N, respectively, for the value of D given by Eqs. (58) and (61). We note again that the differences are not significant in view of expected experimental error in such a measurement. We would therefore conclude that the results of the KM paper and those presented here are not inconsistent although the present data available would point to a somewhat smaller q^2 dependence than that obtained in the KM paper. It would clearly be useful to have electroproduction data at higher incoming electron energies but low pion energies for obtaining a more appropriate nuclear excitations range.

Finally, we note that although the neutral current reaction would be difficult to experimentally observe, it might be interesting below $E_\nu \simeq 105$ MeV so that if done with muon neutrinos, the charged current reaction would be energetically impossible thereby eliminating background.

If measurements of the neutral current reaction cross section agree with theoretical values, but those for the charged current reaction disagree, this would be evidence for ν_μ-ν_τ oscillation. Clearly more experimental and theoretical work would be useful on this subject.

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[6]We note that the form factors describing the individual transitions have a dipole behavior which covers a wide range of q^2. Furthermore, the impulse approximation based derivation of the functional form $a_0+b_0q^2$ for D did not make use of the assumption of a particular average excitation energy. Thus even though our results here explicitly assume dominance by the giant dipole resonance, it is possible that they will have a wider range of utility. The arguments for this, however, are not compelling and careful experimental results at higher E_ν will be necessary to obtain the functional form of D.

[7]The current matrix elements for $^{12}\mathrm{C}\to N_k$, where N_k is a spin 0 and spin 1 state are given, respectively, by

$$\langle k,J=0\,|\,J_\mu(0)\,|\,^{12}\mathrm{C}\rangle=\frac{F_{1k}}{M}(P_i+P_k)_\mu+\frac{F_{2f}}{M}(P_i-P_k)_\mu$$

and

$$\langle k,J=1\,|\,J_\mu(0)\,|\,^{12}\mathrm{C}\rangle=\xi_\mu F_{3k}+(P_k-P_i)_\mu\xi\cdot(P_k-P_i)F_{4k}/M^2+\frac{F_{5k}}{M}\epsilon_{\mu\nu\rho\sigma}(P_k-P_i)_\nu(P_k+P_i)_\rho\xi_\sigma\,.$$

When these are substituted into the sum for $Q_{\sigma\lambda}(P_i,\langle q\rangle)$ and contributions for α and β are extracted,

$$\alpha=-\sum_k\left|F_{3k}\right|^2$$

and

$$\beta=2\sum_k\left|F_{1k}\right|^2+\sum_k\left|F_{3k}\right|^2\,.$$

The spin 2 case gives no large contributions to α or β. Contributions to γ and δ are of the form $\sum|F_{1k}|^2$ and $\sum F_{ik}F_{2k}^*$.

[8]We note that the diagonal space parts of $Q_{\lambda\sigma}(P_i,\langle q\rangle)$ are all $-\alpha$. From the impulse approximation the dominant space part of J_μ has coefficients g_A leading to an impulse approximation value of $Q_{\lambda\sigma}$ for the diagonal space terms proportional to g_A^2 so that to a constant of proportionality $-\alpha$ may be identified with g_A^2. In the same way $Q_{00}(P_i,\langle q\rangle)$ is of the form $\alpha+\beta$ which is proportional to g_V^2. To avoid large contributions to g_V^2 from the terms proportional to g_A^2, $\beta=\beta^V-\alpha$ so that in fact β^V is proportional to g_V^2 and alone occurs in $Q_{00}(P_i,\langle q\rangle)$. This leads to the relation

$$(\beta-2\alpha)=(\beta^V-3\alpha)=k(g_V^2+3g_A^2)$$

where k is a constant. For the $J_\mu^{(3)}(0)$ part of the neutral current an exactly analogous relation holds, $(\beta^{(3)}-2\alpha^{(3)})=(\beta^{V'}-3\alpha^{(3)})$, where $\beta^{V'}=\beta^V/2$ and $\alpha^{(3)}=\alpha/2$.

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[17]There is obviously some danger in an extrapolation of this sort but the experimental data form a smooth curve with relatively small error (±10%). Using this error the extrapolation should be accurate to 20∼25 % as noted in Sec. IV.