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Investigation of the role of the rho-meson exchange in spin-isospin strength distribution effects in a finite nucleus

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We present a unified study of the role of the ρ-exchange interaction in spin-isospin strength distribution effects in a finite nuclear system. We study both the longitudinal ($\vec{\sigma}\cdot\hat{q}\tau_\lambda$, where $\vec{q}$ is the momentum transfer to the nucleus) and the transverse ($\vec{\sigma}\times\hat{q}\tau_\lambda$) spin channels for a large range of momentum transfer ($q\sim 0$—600 MeV/c). We examine a number of effective ρ-coupling schemes used in the literature. Using the finite-nucleus formalism of Toki and Weise, we examine in detail the response function in the presence of the ρ-meson exchange term. The renormalization of matrix elements of spin-isospin sensitive probes is given for the $J^P=1^+$, $T=1$ level of ^{12}C. We analyze the results, gaining some insight into the nature of the longitudinal versus transverse channels and into approximations suggested in the past for handling finite-nucleus calculations. A comparison with local density approximation and infinite nuclear matter with a constant density results is presented for a variety of cases.

I. INTRODUCTION

The excitation of pionlike levels ($J^P=0^+$; $T=0\rightarrow J^P=0^-, 1^+, 2^-, 3^+, \ldots$; $T=1$) and the response of nuclear systems to spin-isospin sensitive probes has received considerable attention in recent years.[1,2] An excellent review of the subject is by Oset, Toki, and Weise,[2] whose general formalism we shall mainly follow in this work. Some authors[1,3] have limited their treatment of this problem to the context of infinite nuclear matter, while others present finite-nucleus treatments.[4–8] There have also been recent studies[9] of spin-isospin strength distribution effects using the local density approximation (LDA) for intermediate-energy reactions.

A major characteristic of the finite nucleus treatment (see Oset *et al.*[2] and Ref. 10) is a nonlocality in momentum space. Instead of the linear momentum conservation (characterizing infinite nuclear matter), the response function is treated with a well-defined angular momentum. This nonlocality proves to be extremely important in finite-nucleus treatments.

We focus on pionlike excitations obtained by one-particle—one-hole (1p-1h) configurations, including both nucleon-hole and Δ-isobar-hole ones, where the interaction is taken to be one-boson exchange with a repulsive term represented by the Migdal-Landau parameter g'. Although the main contribution of the meson-exchange interaction comes from the pion term, a ρ exchange is sometimes also included.[11–18,20] Some authors have found the ρ to be of considerable importance, especially for finite systems. On the one hand, most careful studies in the past have shown that[19] the ρ meson is very important in some cases and less interesting in others, regarding the N-N interaction in nuclei. On the other hand, it might be necessary to include the ρ contribution when dealing with transverse probes (proportional to $\vec{\sigma}\times\vec{q}$, where $\vec{q}$ is the momentum transfer to the nucleus), such as $M1$ or Gamow-Teller. The reason for this is that the ρ component of the particle-hole interaction is properly aligned with the transverse nature of the operator. This component may also contribute in the longitudinal channel (proportional to $\vec{\sigma}\cdot\vec{q}$, i.e., the pion source operator) in finite nuclei (for $J\geq 1$), due to surface effects. (Note that the ρ exchange does not contribute to $J^P=0^-$ pionlike excitations. In this and other respects[8] such levels are *purely* longitudinal.) These considerations result in a mixture of the π and ρ terms of the interaction even for pure pionic excitations.[11–15] By the same argument, the pion term is important in the transverse channel for finite nuclei.[6,11,13,14] This has been studied thoroughly in Ref. 8.

In this paper we present a detailed study of the role of the ρ exchange in spin-isospin strength distribution effects in a finite nucleus. We study both the longitudinal ($\vec{\sigma}\cdot\vec{q}$) and the transverse ($\vec{\sigma}\times\vec{q}$) channels for a large range of momentum transfer ($q\sim 0$—600 MeV/c). We also discuss and examine a number of effective ρ-coupling schemes used in the literature, and compare our results to the LDA and infinite-nuclear-matter treatments. We adopt a finite nucleus formalism given by Toki and Weise[2,4,13] and follow exactly the formal comments in our earlier papers[7,8] in constructing the meson self-energy tensor (Fig. 1). In Secs. II and III we examine in detail the response function

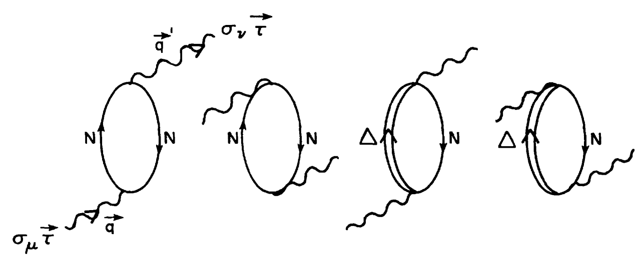

FIG. 1. Nucleon-hole and Δ(1236)-hole excitations ring-diagram contribution to the meson self-energy.

when the ρ-meson exchange is taken into account for the 1^+ level of ^{12}C. The application of the formalism to the renormalization of matrix elements of spin-isospin sensitive probes in ^{12}C is presented in Sec. IV. A comparison with LDA and nuclear matter with a constant density results is given in Sec. V.

We find the ρ meson to be of limited importance in both the longitudinal and the transverse channel. We then analyze this behavior and present a number of checks to clarify the origins of these results. This enables us to gain some insight into the nature of the transverse versus longitudinal channels and into the approximations suggested in the past for handling approximate finite-nucleus calculations.

The LDA and infinite nuclear matter calculations are found to be completely unsatisfactory even for semiquantitative estimates of the pertinent renormalization effects and of the role of the ρ meson in this scheme. This is the case because Fermi gas calculations involve translational invariance and do not allow for momentum space nonlocality. This nonlocality turns out to be very important for the ρ-exchange interaction and the transverse channel as well as for the one-pion exchange (OPE) interaction and the longitudinal channel.

We shall also discuss (at the end of Sec. II) some of the effects of the quark model and SU_6 symmetry on the physical input used (such as coupling constants, cutoffs, etc.) and on the results presented in this work.

II. THE FINITE-NUCLEUS RESPONSE FUNCTION: FORMALISM

A. The ρ-exchange interaction

The formalism pertinent to the response function is presented in Refs. 5, 7, and 8. We are interested in the iterated self-energy of Fig. 1. [For infinite nuclear matter this iteration results in the diamesic function renormalization, which represents the many body random phase approximation (RPA) renormalization of the Fermi-gas self-energy.] Both nucleon-hole and Δ-isobar-hole excitations are included. The momentum space nonlocalities result in an integral equation for the response function R. For the static ($\omega=0$) case this equation is

$$\langle \vec{q}\,' \mid R \mid \vec{q} \rangle = \langle \vec{q}\,' \mid \Pi^0 \mid \vec{q} \rangle + \int \frac{d^3k}{(2\pi)^3} \langle \vec{q}\,' \mid \Pi^0 \mid \vec{k} \rangle D(k) \langle \vec{k} \mid R \mid \vec{q} \rangle \,, \tag{1}$$

where $D(k)$ is the particle-hole interaction including an OPE potential, a ρ-exchange term, and a term for short-range effects, represented by the spin-isospin parameter g', as in Refs. 11–15. The latter references differ in the manner in which the ρ-exchange potential is incorporated in the particle-hole interaction. Delorme *et al.*[11] treat this part as a sharp, well-defined ρ exchange with mass $m_\rho=770$ MeV. The ρNN coupling constant f_ρ is linked to the πNN one (f) at zero momentum and to the appropriate mesonic masses by

$$c_\rho = (f_\rho^2/m_\rho^2)/(f^2/m_\pi^2) \,, \tag{2}$$

where c_ρ is not well determined, and ranges between 0.8 and 2.18.[11] At nonzero momenta, appropriate form factors are introduced.[11,13,16–18] While Cenni *et al.*[16] treat the nuclear matter case using $c_\rho=1.0$–2.2, most other authors use only high values of c_ρ. These tend to enhance, and perhaps overestimate, the importance of this part of the particle-hole interaction, which is otherwise known to be relatively unimportant.[19] Another important effect is incorporated by Toki and Weise,[13] who include the 2π continuum, or the iterated OPE part of two-pion exchange (TPE) in addition to the resonant (ρ exchange) part. This kind of interaction was later tested also by Alberico *et al.*[18] in an infinite-nuclear-matter study of the quasielastic peak. However, Speth *et al.*[12,15] indicate that this full 2π exchange can be simulated by a one-ρ-meson exchange potential using an effective ρ-coupling constant which is about 30% larger than the actual one; this implies a c_ρ, which can be as high as 2.90. These authors also multiply the central part of the ρ-exchange contribution by 0.4 to include effects of very short range repulsive correlations of the ω-exchange potential. These modifications, which were not taken into account *as a whole* by Toki and Weise,[13] nor by Delorme *et al.*,[11] seem, according to Ref. 15, to explain some of the discrepancies between theory and experiment in Refs. 13 and 11. Later studies have incorporated the above modification to a satisfactory extent,[18,20] but not for the present approach.

B. The ρ exchange in finite nuclei

As already indicated in Sec. I, one may expect a contribution from the ρ-meson exchange in both the transverse and longitudinal channels when dealing with finite nuclei, because of surface effects. This is incorporated in the 2×2 partial wave matrix $\hat{D}_J$ of the interaction, as in Ref. 13. Using different cutoffs in the (monopole) π and ρ form factors and recalling Eq. (2) we have for the static case

$$[\hat{D}_J(k)]_{LL'} = a_{JL}\frac{-1}{k^2+m_\pi^2}a_{JL'} + \frac{g'}{k^2}\delta_{LL'} + b_{JL}c_\rho\left[\frac{\Lambda_\rho^2-m_\rho^2}{\Lambda_\rho^2+k^2}\Bigg/\frac{\Lambda_\pi^2-m_\pi^2}{\Lambda_\pi^2+k^2}\right]^2 \times\frac{-1}{k^2+m_\rho^2}b_{JL'} \,. \tag{3}$$

In Eq. (3) the matrix elements have orbital angular momentum indices $L,L'=J\pm1$, and Λ_ρ and Λ_π are the cutoffs associated with the pion and the ρ meson, respectively. We have used the values $\Lambda_\rho=2$ GeV and $\Lambda_\pi=1$ GeV throughout. The quantities a_{JL} (associated with the longitudinal channel and the pion coupling) and b_{JL} (pertinent to the transverse channel and ρ-nucleon coupling) are given by

$$a_{JL}=\langle J0\ 10\,|\,L0\rangle \tag{4a}$$

and

$$b_{JL}=\begin{cases}\left[\dfrac{J+1}{2J+1}\right]^{1/2}, & L=J-1\\[2ex] \left[\dfrac{J}{2J+1}\right]^{1/2}, & L=J+1\ .\end{cases} \tag{4b}$$

The matrix $\hat{D}_J(k)$ then enters in the integral matrix equation for the response function as follows:

$$[\hat{R}_J(q',q)]_{LL'}=[\hat{\Pi}_J(q',q)]_{LL'}+\int_0^\infty \frac{k^2dk}{(2\pi)^3}\times\sum_{\lambda\lambda'}[\hat{\Pi}_J(q',k)]_{L\lambda}[\hat{D}_J(k)]_{\lambda\lambda'}\times[\hat{R}_J(k,q')]_{\lambda'L'}\ , \tag{5}$$

where λ,λ' and $L,L'=J\pm1$, and $\hat{\Pi}_J$ is the self-energy matrix partial wave in the notation of Refs. 5, 7, and 8. The response function and the self-energy are highly nonlocal in momentum space for light nuclei.

In constructing the self-energy (see Fig. 1) we have used the πNN and πNΔ vertex operators with coupling constants (including form factors) $f(q^2)$ and $f^*(q^2)$. The ratio $f^*(q^2)/f(q^2)$ was taken to be 2 (see the discussion in Sec. II E). The difference in coupling constants between the π and the ρ is taken into account in Eq. (3). The ratio corresponding to f^*/f for the ρ case is similar to that of the pion case, since the N-Δ interaction via 2π exchange can be assumed to satisfy the scaling relation[2]

$$V_{2\pi}(\mathrm{N}\Delta\to\mathrm{N}\Delta)=\alpha V_{2\pi}(\mathrm{N}\Delta\to\mathrm{NN})=\alpha^2V_{2\pi}(\mathrm{NN}\to\mathrm{NN})$$

(used here just for extracting the vertex coupling). The value $\alpha\approx1.7$–2.0, similar to that for one pion exchange, is used in Oset, Toki, and Weise.[2] (See also the discussion in Sec. II E.)

C. The modification of the central ρ-exchange contribution

Interaction (3) can be presented in two interesting alternative forms. Separating the tensor and central spin-spin (Yukawa) parts of the OPE and ρ-exchange terms, and using the appropriate partial wave expansion, one finds[13]

$$[\hat{D}_J(k)]_{LL'}=\left[\frac{1}{3}\frac{-1}{k^2+m_\pi^2}+\tfrac{2}{3}V_\rho(k)+\frac{g'}{k^2}\right]\delta_{LL'}+\left[\frac{1}{3}\frac{-1}{k^2+m_\pi^2}-\tfrac{1}{3}V_\rho(k)\right]\times(3a_{JL}a_{JL'}-\delta_{LL'})\ , \tag{6}$$

where

$$V_\rho(k)\equiv C_\rho\left[\frac{\Lambda_\rho^2-m_\rho^2}{\Lambda_\rho^2+k^2}\Big/\frac{\Lambda_\pi^2-m_\pi^2}{\Lambda_\pi^2+k^2}\right]^2\frac{-1}{k^2+m_\rho^2}\ .$$

While this decomposition is not particularly needed for the OPE part, its inclusion in the ρ case is useful for incorporating the modification of the central part contribution,[12,15] discussed in Sec. II A. This will be done here by multiplying V_ρ in the first (central) part by the factor 0.4, as suggested by Speth *et al.*[12,15] [this factor does not affect the tensor part in Eq. (6)].

D. A comment on the ρ meson condensation

A second interesting form for Eq. (3) can be achieved by noting that

$$\delta_{LL'}=a_{JL}a_{JL'}+b_{JL}b_{JL'}$$

[which is equivalent to

$$\vec{\sigma}_1\cdot\vec{\sigma}_2=(\vec{\sigma}_1\cdot\hat{q})(\vec{\sigma}_2\cdot\hat{q})+(\vec{\sigma}_1\times\hat{q})\cdot(\vec{\sigma}_2\times\hat{q})]\ ;$$

thus,

$$[\hat{D}_J(k)]_{LL'}=a_{JL}\left[\frac{-1}{k^2+m_\pi^2}+\frac{g'}{k^2}\right]a_{JL'}+b_{JL}\left[V_\rho(k)+\frac{g'}{k^2}\right]b_{JL'}\ . \tag{7}$$

If the longitudinal and the transverse channel are almost separated, and do not interfere, we can predict on the basis of Eq. (7) precritical effects in the ρ (transverse) channel, in analogy with those already known in the pion channel. This will happen at much higher momenta and nuclear densities since the mass of the ρ meson is much larger than that of the pion. A study of the proximity to the ρ-meson condensation threshold in nuclei was given by Toki and Comfort;[21] as we shall see, our numerical results predict relatively unimportant ρ contributions and large effects of OPE in the ρ channel. This is true for q in the range 0–4 m_π. In this region our study will not predict a strong ρ-induced precritical behavior. We shall indicate a possible strong effect of the ρ meson in the transverse channel for $q\gtrsim m_\rho$. This effect, however, disappears when the modified ρ-exchange interaction due to Speth *et al.* (see Secs. II A, II C, and III B) is introduced, at least for $q\sim4$–$5\,m_\pi$.

E. Quark-model considerations

In concluding this section, we briefly discuss some of the effects the quark model and SU_6 symmetry have in the present context. (I am grateful to H. J. Weber for pointing these out to me.) We use the cutoff values $\Lambda_\pi=1$ GeV and $\Lambda_\rho=2$ GeV [see Eq. (3)]. These values are extracted from phenomenological fits (of the nucleon form factor) and are widely used in the pertinent literature. Quark models, however, predict smaller values ($\Lambda_\pi\approx0.6$–0.75 GeV) and often $\Lambda_\rho\approx\Lambda_\pi$. For such values, the renormalization effects considered here are reduced, but the general features and all our subsequent conclusions are left completely unchanged. The quark models also predict that $f^*/f=\sqrt{72/25}$. Our value[27] (Sec. II B) is an intermediate one between this prediction and the one derived from first order perturbation theory (namely, $f^{*2}/4\pi=0.37$). This is also the Chew-Low value. We use this same ratio for the ρNΔ vertex coupling, i.e., $\alpha=f^*/f$

(see the end of Sec. II B). This identity is also supported by quark models.

III. THE FINITE NUCLEUS RESPONSE FUNCTION: CALCULATIONAL RESULTS

A. The effect of ρ exchange

We start with a straightforward application of Eqs. (3)—(5). We sum over high-lying excitations up to $12\hbar\omega$ in the Q subspace[7,8] using the harmonic oscillator complete set of states, and then use numerical methods to solve the matrix integral equation (5). Special care must be taken when handling the high-lying ph states. Calculations for $[\hat{R}_J(k,q)]_{LL'}$ were performed for the $J^P=1^+$, $T=1$ level in ^{12}C. [Note that a 0^- level is unaffected by the presence of the ρ; this is formally seen from Eq. (4b). Since only the OPE term is present for $J^P=0^-$, our earlier results[7] for ^{16}O remain unchanged by the present modification of the interaction.[22]]

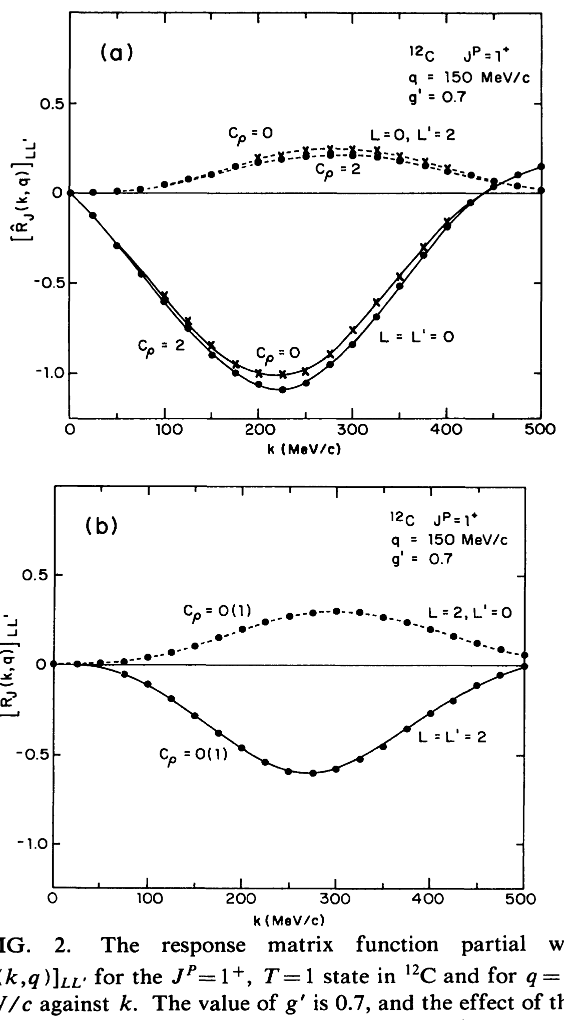

FIG. 2. The response matrix function partial wave $[\hat{R}_J(k,q)]_{LL'}$ for the $J^P=1^+$, $T=1$ state in ^{12}C and for $q=150$ MeV/c against k. The value of g' is 0.7, and the effect of the ρ meson in the interaction is examined. (a) Results for $c_\rho=2$ (full circles) and $c_\rho=0$ (no ρ exchange, OPE only, lines marked by $\times$'s) for $L=L'=0$ (solid lines) and for $L=0,L'=2$ (dashed lines). (b) Results for $c_\rho=0$ and 1 (indistinguishable on this scale) for $L=L'=0$ (full line) and for $L=2,L'=0$ (broken line).

We found that the response function is modified by the ρ term to an extent of $\sim 20\%$ or less in the range $m_\pi \lesssim q < 4m_\pi$. As wel shall see later, this does not immediately imply a similar renormalization of the pertinent matrix elements. Results for $[\hat{R}_J(k,q)]_{LL'}$ for $J^P=1^+$ and $L,L'=0,2$ in ^{12}C are shown in Figs. 2 and 3 for $q=150$ MeV/c and in Fig. 4 for $q=400$ MeV/c as a function of k. The general features of these graphs (such as the nonlocality in momentum space, the rate of convergence with the number of ph excitations included in the summation, etc.) have already been described in detail in Ref. 7. The novel point in the present paper is the study of the consequences of the inclusion of the ρ meson in the interaction. For $q \approx m_\pi$ we see that only the strong coupling gives some modification of the response function; this is readily explained by the large mass of the ρ meson. The parameters $c_\rho=2.90$ and $g'=0.4$ give a larger contribution of the ρ meson (Fig. 3), but are probably unrealistic (this value of g' is used, however, in Ref. 11). For higher momenta the ρ term affects the response function even for high g'.

B. The modified ρ exchange

We proceed now to study the effects of the modifications of the central ρ-exchange interaction,[12,15] discussed

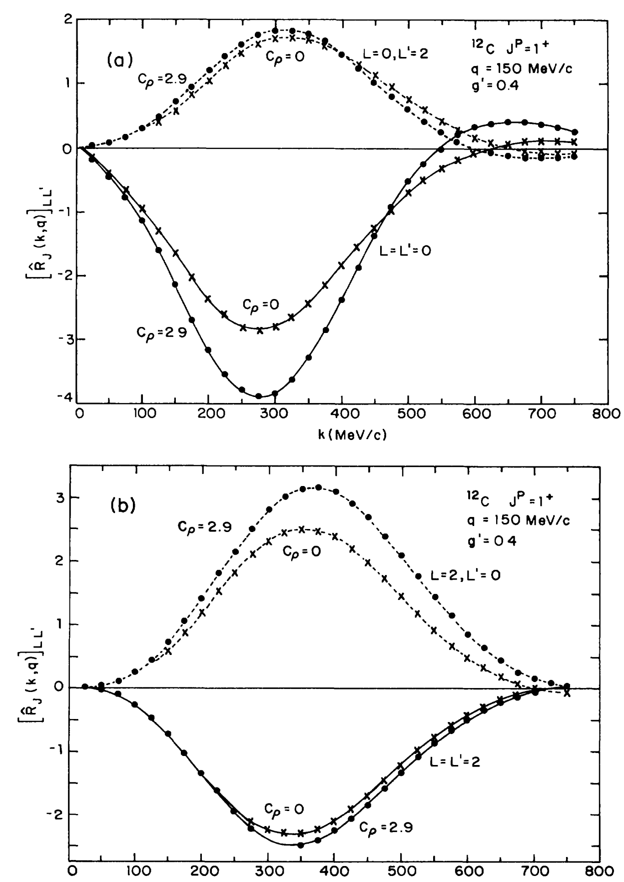

FIG. 3. As in Fig. 2, but for $g'=0.4$ and for $c_\rho=2.9$ and 0. The effect of the ρ-meson exchange is largest for $L=L'=0$. The value of g' (used in Ref. 11) is not realistic.

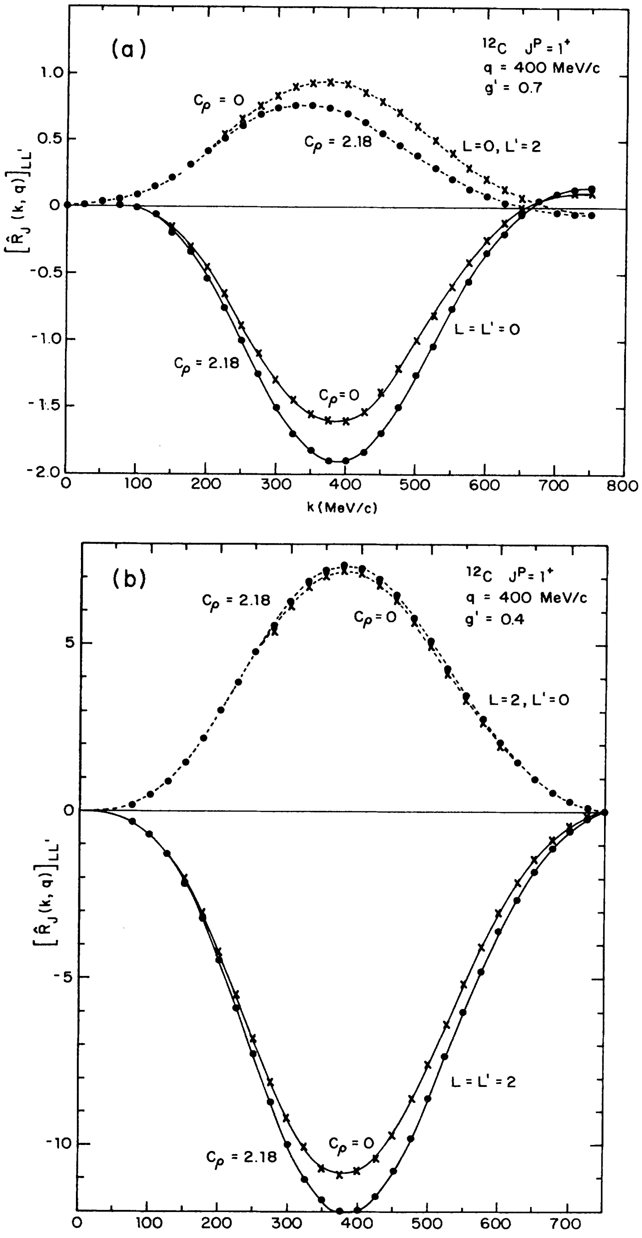

FIG. 4. The response matrix function partial wave $[\hat{R}_J(k,q)]_{LL'}$ for the state $J^P=1^+$, $T=1$ in ^{12}C for $q=400$ MeV/c against k. The ρ meson is included in the interaction. The results are shown for $c_\rho=2.18$ (full circles) and $c_\rho=0$ (OPE only, lines marked by $\times$'s). (a) Results for $g'=0.7$ and for $L=L'=0$ (solid lines) and $L=0,L'=2$ (dashed lines). (b) Results for $g'=0.4$ and for $L=L'=2$ (full lines) and for $L=2,L'=0$ (broken lines).

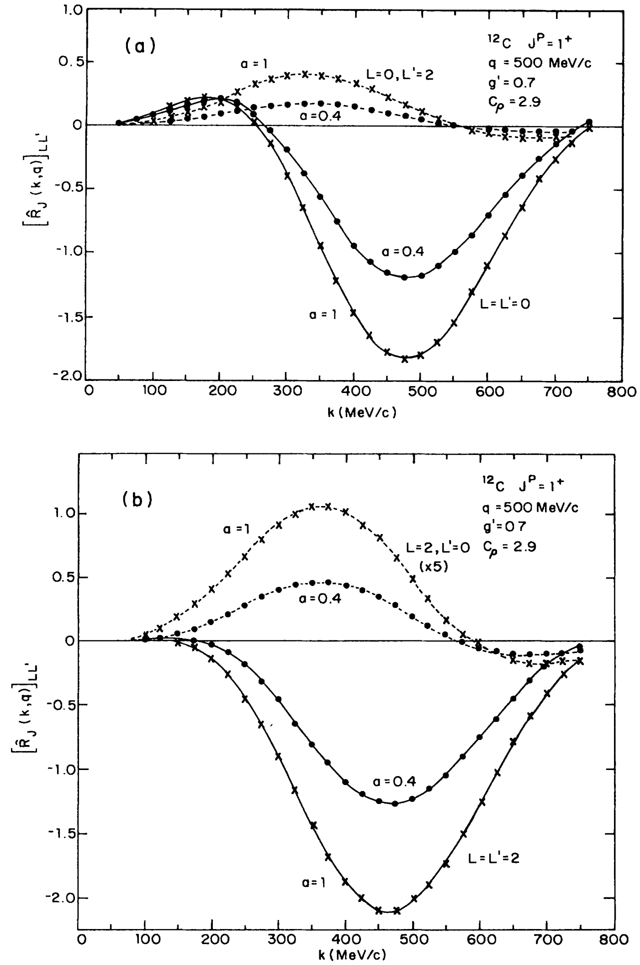

FIG. 5. The response matrix function partial wave $[\hat{R}_J(k,q)]_{LL'}$ for the state $J^P=1^+$, $T=1$ in ^{12}C for $q=500$ MeV/c as a function of k. The value of g' is 0.7, and $c_\rho=2.9$. The results are presented for $a=0.4$ (full circles) and $a=1$ (the $\times$'s lines). (a) Results for $L=L'=0$ (solid lines) and for $L=0,L'=2$ (dashed lines). (b) Results for $L=L'=2$ (full lines) and for $L=2,L'=0$ multiplied by 5 (broken lines). The corresponding graphs for $c_\rho=0$ would mainly be close to the $a=0.4$ one for $L=L'=0$, or between the two curves shown for $L=L'=2$, and are higher than the $L\neq L'$ ones in this figure.

in Sec. II C. For ^{12}C ($J^P=1^+$) this interaction has the form

$$\begin{aligned} &\tfrac{2}{3}aV_\rho(k), \quad L=L'=0\,, \\ &\frac{\sqrt{2}}{3}V_\rho(k), \quad L\neq L'\,, \qquad (8) \\ &\frac{2a-1}{3}V_\rho(k), \quad L=L'=2\,, \end{aligned}$$

where $a=0.4$ according to Speth *et al.*[12,15]

Figure 5 demonstrates the large effects of this modification on $\hat{R}_J$. It tends to lower appreciably the results for $a=0.4$ relative to those for $a=1$. For $c_\rho=0$ (no ρ exchange) the graphs are generally close to the $a=0.4$ one for $L=L'=0$, lie between the two curves shown for $L=L'=2$, and are higher than the $L\neq L'$ cases of Fig. 5.

IV. RENORMALIZED MATRIX ELEMENTS OF SPIN-ISOSPIN SENSITIVE PROBES

A. Formalism

We consider the renormalized matrix elements of the longitudinal ($\vec{\sigma}\cdot\hat{q}\tau_\lambda$) and the transverse ($\vec{\sigma}\times\hat{q}\tau_\lambda$) spin-isospin operators in the Q subspace for the 1^+ level treated above. A discussion of the Q vs $P+Q$ spaces was given in Refs. 7 and 8. The present formal comments [Eqs. (9)—(12)] are identical to material[5] in the literature, but are required here to make the subsequent results intelligible and self-contained.

The renormalized matrix element of a longitudinal, pionlike operator is

$$\mathscr{L}_{fi}^{(\mathrm{ren})}(\vec{q})=\langle \mathrm{ph}(J^PM;TM_T)\,|\,\vec{\sigma}\cdot\hat{q}e^{i\vec{q}\cdot\vec{r}}\tau_\lambda\,|\,0\rangle_{\mathrm{ren}}$$

$$=\delta_{T1}\delta_{M_T\lambda}\sum_{L,L'=J\pm1}\int_0^\infty \frac{k^2dk}{(2\pi)^3}F_{\mathrm{ph}}^{*JL}(k)[\hat{\epsilon}_J^{-1}(k,q)]_{LL'}Y_{JM}^*(\hat{q})a_{JL'}\ , \tag{9}$$

where $F_{\mathrm{ph}}^{JL}(k)$ is given by

$$F_{\mathrm{ph}}^{JL}(q)=\eta_{\mathrm{ph}}[(2L+1)(2S+1)(2j_{\mathrm{p}}+1)(2j_{\mathrm{h}}+1)]^{1/2}\begin{Bmatrix} l_{\mathrm{p}} & s_{\mathrm{p}} & j_{\mathrm{p}} \\ l_{\mathrm{h}} & \frac{1}{2} & j_{\mathrm{h}} \\ L & S & J \end{Bmatrix}$$

$$\times(-i)^L[4\pi(2l_{\mathrm{h}}+1)]^{1/2}\langle L0l_{\mathrm{h}}0\,|\,l_{\mathrm{p}}0\rangle \int_0^\infty dr\, r^2 j_L(qr)R_{\mathrm{p}}(r)R_{\mathrm{h}}(r)\ , \tag{10}$$

with $s_{\mathrm{N}}=\frac{1}{2},\eta_{\mathrm{Nh}}=2$ and $s_\Delta=\frac{3}{2},\eta_{\Delta\mathrm{h}}=\frac{4}{3}$ (see Oset *et al.*[2]).

In Eq. (10) $S=1$ is implied, and $R_{\mathrm{p(h)}}(r)$ are the radial wave functions (here taken in a harmonic oscillator basis). In Eq. (9), the diamesic matrix function is obtained as

$$[\epsilon_J(q,k)]_{LL'}=\frac{(2\pi)^3}{k^2}\delta(k-q)\delta_{LL'}-[\hat{\Pi}_J(q,k)\hat{D}_J(k)]_{LL'}\ , \tag{11}$$

where $\hat{D}_J(k)$ can be taken from Eqs. (3), (6), and (7), or else modified as in (8), and a_{JL} has been introduced in Eq. (4a).

For the transverse case the renormalized matrix element (valid for $J^P=1^+,2^-,\ldots;\ T=1$) is

$$\mathscr{T}_{fi_\mu}^{(\mathrm{ren})}(\vec{q})=\langle \mathrm{ph}(J^PM;TM_T)\,|\,(\vec{\sigma}\times\hat{q})_\mu e^{i\vec{q}\cdot\vec{r}}\tau_\lambda\,|\,0\rangle_{(\mathrm{ren})}$$

$$=\delta_{T1}\delta_{M_T\lambda}(-i)\sum_{LL'}\int_0^\infty \frac{k^2dk}{(2\pi)^3}F_{\mathrm{ph}}^{*JL}(k)[\epsilon_J^{-1}(k,q)]_{LL'}b_{JL'}\langle JM-\mu 1\mu\,|\,JM\rangle Y_{JM-\mu}^*(\hat{q})\ , \tag{12}$$

where $F_{\mathrm{ph}}^{JL}(k)$ is given in Eq. (10) and b_{JL} is from (4b).

For the 1^+ case we present results for the radial (in momentum space) parts of the renormalized matrix elements. For the longitudinal case this part is denoted by $L_J(q)$, where

$$\mathscr{L}_{fi}^{(\mathrm{ren})}(\vec{q})=L_J^{(\mathrm{ren})}(q)Y_{JM}^*(\hat{q})\delta_{T1}\delta_{M_T\lambda}\ . \tag{13}$$

For the transverse case we define the reduced element $T_J(q)$ by

$$\mathscr{T}_{fi_\mu}^{(\mathrm{ren})}(\vec{q})=\delta_{T1}\delta_{M_T\lambda}T_J^{(\mathrm{ren})}(q)\langle JM-\mu\, 1\mu\,|\,JM\rangle \times Y_{JM-\mu}^*(\hat{q})\ , \tag{14}$$

which can be expressed[7,8] as a sum of the P-subspace contribution (the subspace of configurations treated directly—no renormalization effects) and a second part, generated by the renormalization effects.

B. Results for the longitudinal case and discussion

In Table I we present results for the longitudinal renormalized matrix element with ρ exchange for strong ($c_\rho\geq 2$) and weak ($c_\rho\approx 1$) couplings. This is compared with the case where no ρ exchange is taken into account ($c_\rho=0$). The *unimportance* of the ρ in this channel is evident, even for the extreme case of $g'=0.4$, $c_\rho=2.9$. This unimportance is in agreement with Delorme *et al.*[11]

Our main goal in this section is to find out the origins of this unimportance. There are two physical points which may explain this behavior. The first one is the large mass of the ρ meson compared to the pionic one ($m_\rho/m_\pi\simeq 5.5$) and to the range of the pertinent momentum transfer; the second is the nature of the couplings (i.e., longitudinal versus transverse), as discussed in Sec. I. If the latter effect is the dominant one, one should expect the ρ to play an important role in the transverse ($\vec{\sigma}\times\hat{q}$) case, while the former fact (large m_ρ) would leave the ρ as an unimportant meson in both channels. If it is the combination of these two effects that makes the ρ unimportant in the longitudinal case, then one would expect this meson to be moderately important in the transverse case.

In order to check these points we first added *by hand* a term $-b_{JL'}b_{JL}/(k^2+m_\pi^2)$ to the interaction $\hat{D}_J(k)$ [Eq. (3)], making the pion term diagonal ($[-1/(k^2+m_\pi^2)]\delta_{LL'}$). The additional term can be thought of as a "transverse pion exchange." This should have a limited effect on the longitudinal matrix element if the reason for the very small ρ contribution in this channel is its transverse coupling; in the other case it should have a larger effect (because the pion mass is very low). For $q=150$

TABLE I. Numerical results for the longitudinal renormalized matrix element with ρ exchange (and without it for comparison). The superscript (0) means no renormalization effects (P subspace nonrenormalized results).

q (MeV/c)	$L_J^{(0)}(q)$	g'	Nature of ρNN coupling	c_ρ	$L_J^{(\text{ren})}(q)$
150	5.50	0.7	OPE only (no ρ)	0	3.34
		0.7	weak	1.00	3.36
		0.7	strong	2.00	3.39
		0.4	strong	2.90	3.43
400	−1.48	0.7	OPE only (no ρ)	0	−2.42[a]
		0.7	strong	2.18	−2.46
		0.4	OPE only (no ρ)	0	−11.79[a]
		0.4	strong	2.18	−12.38

[a]These numbers correct a numerical mistake in Ref. 7.

MeV/c, $g'=0.7$, and $c_\rho=2$ we found a large effect of this modification on the response function,[23] along with a moderate effect on the renormalized matrix element: 4.24 instead of 3.39. This renormalized result lies between the nonrenormalized one and the renormalized with correct pion term case. We can thus conclude from this first test that a transverse coupling combined with a small meson mass does have a moderate effect in the longitudinal channel for a finite system.

As a further check, we now added, again *by hand,* a hypothetical longitudinal ρ term, yielding a total diagonal ($\propto \delta_{LL'}$) ρ term. The additional term is aligned with the probe $\vec{\sigma}\cdot\hat{q}e^{i\vec{q}\cdot\vec{r}}$. If the dominant reason for the ρ unimportance is its large mass, this term would not have a large contribution. On the other hand, if the coupling is the reason, this term should make a relatively large contribution to the renormalization.

The results, for the same parameters as in the previous check, again show an appreciable effect on the response function,[24] which is induced by the ρ. These changes in $\hat{R}_J(k,q)$, of the same order as the former ones, bring about a large renormalization of the matrix element; the renormalized value drops to 2.33 MeV^{-1}, *despite the large value of* m_ρ.

We thus conclude that the nature of the meson-nucleon coupling is very important, and that the *combination* of orthogonal couplings and large-mass effects reduce the contribution from the ρ exchange in the longitudinal channel. This conclusion leaves room for moderate ρ effects in the transverse channel where the large mass is balanced by the preferred coupling; these are presented in Sec. IV C.

The last discussion offers some insight into the nature of the approximations of Toki and Weise.[4,5] These were found in our earlier studies[7,8] to be unsuccessful, and we can now understand the reason for this. Although the Bessel function distribution may *look* rather similar to the actual exact shape of $[\hat{\Pi}_J(q,q')]_{LL'}$, it does not give correct results for the renormalized matrix elements. This seems to be the case because a careless approximation to the self-energy partial wave brings about spurious components which are aligned with the driving interaction. On the other hand, a careful and correct approximation would not cause any serious errors even when it differs appreciably from the actual distribution in momentum space, provided that the differences are orthogonal to the dominant channel.

C. Results for the transverse case and discussion

Results for $q=150$ MeV/c do not show any appreciable effect of the meson on the renormalized matrix element of the transverse-channel probe (Table II): for $g'=0.7$ the ρ has an effect of 0.7%, while for $g'=0.4$ we find a 2.1% effect. These results are in agreement with Delorme *et al.*[11] and with Toki and Weise.[13]

In order to understand these results better, we now turn to an analysis similar to the one presented in Sec. IV B. We modified the interaction partial wave $\hat{D}_J(k)$ in three different ways: (a) By adding a term

$$b_{JL'}\frac{-1}{k^2+m_\pi^2}b_{JL} ,$$

thus making the pion exchange part diagonal (namely,

$$\frac{-1}{k^2+m_\pi^2}\delta_{LL'}) .$$

(b) By making the ρ-meson exchange term diagonal. (c) By considering only the OPE and one-rho exchange

TABLE II. Results for the transverse renormalized matrix element with and without ($c_\rho=0$) ρ exchange for $q=150$ MeV/c. A superscript (0) means no renormalization effects as in Table I.

g'	c_ρ	$iT_J^{(0)}$	$iT_J^{(\text{ren})}$
0.7	0	9.74	6.53
0.7	2	9.74	6.59
0.4	0	9.74	7.02
0.4	2.9	9.74	6.87

(ORE) terms in the interaction (no g'). The corresponding results for $iT_J^{(\mathrm{ren})}$ are (a) 8.15, (b) 6.42, and (c) 11.69, respectively. The large effect of (a) is expected, since the added term has the "right" coupling (i.e., is aligned *with* the $\vec{\sigma}\times\hat{q}$ probe), and also has a low mass. On the other hand, the very small change induced by (b) results from the "wrong" coupling (orthogonal to the probe channel) in combination with the large meson mass. We may recall that the same check in the longitudinal case (Sec. IV B) resulted in a larger effect, because in that case the original ORE term had the wrong coupling, while the added part was in the right channel. In case (c) we find an enhancement, since the interaction is mostly attractive.

Before further discussing these features, we would like to present some results for intermediate and large momenta in the transverse channel. The renormalized transverse matrix elements presented in Table III show a relatively low sensitivity to g' (compared with the longitudinal channel) as already found in Ref. 8, and an effect of 20—30 % of the ρ-exchange interaction in the renormalization process. This is also the extent of modification of the response function, as found in Sec. III A. This effect is much larger than the one for the corresponding longitudinal-channel results, but not as large as one could expect for the right coupling.

We are now able to understand in a better way some features of the longitudinal versus transverse probes. Both exhibit insensitivity to the ρ term at $q \lesssim m_\pi$. This is mainly due to the large m_ρ, and, in the longitudinal case, also due to the ρ-nucleon coupling. In the latter case one encounters a subtle interplay between the OPE and g' terms, especially near $q \sim 2$—$3\, m_\pi$ (the critical momentum region). This may give rise to large enhancement phenomena, or may, at least, result in appreciable renormalizations. The ρ-nucleus coupling is orthogonal to such a probe, but a hypothetic "longitudinal ρ," or a pion with mass m_ρ, does contribute to this balance and thus has a considerable effect. For the transverse probe at these values of q there is no such cancellation, and therefore the contribution of the ρ is less than what one might expect (even though it has the right coupling, i.e., aligned with the probe). These observations combine with the fact that the sensitivity to g' in the transverse case is much weaker than in the longitudinal channel.

We note in passing that a large effect of the ρ has been calculated for the transverse channel at $q=600$ MeV/c. This might be the signature of the ρ condensation[12] (see Sec. II D). As we shall see in the next section, this effect is largely reduced as a result of the attenuated ORE interaction discussed in Secs. II C and III B. We shall not pursue this point further here, because it is outside the scope of the present work, and also because such high momenta require a very high running time on a computer. We note, however, that for such large momentum transfers a fully relativistic treatment of the nuclear system is probably needed.

TABLE III. Calculational results for the transverse renormalized matrix element with ρ exchange and without it ($c_\rho=0$). We use the superscript (0) as in Table I.

q (MeV/c)	$iT_J^{(0)}(q)$	g'	c_ρ	$iT_J^{(\mathrm{ren})}(q)$
350	0.23	0.7	0	−1.07
		0.7	2.90	−1.27
400	−0.29	0.7	0	−0.59
		0.7	2.18	−0.71
		0.7	2.90	−0.77
		0.4	0	−0.9
		0.4	2.90	−1.08
500	−0.34	0.7	0	0.23
		0.7	2.90	0.27
600	−0.13	0.7	0	0.25
		0.7	2.90	0.42

D. Results for the modified ρ exchange

We now use the modified ρ-exchange interaction,[12,15] already discussed in Secs. II C and III B. Using Eq. (8) we find this to be an important modification only for the transverse case. Results for $a=0.4$, to be compared with the corresponding ones for $a=1$ or $c_\rho=0$ (Table III), are given in Table IV. The results show that the present modification makes the renormalization correction lower, but can sometimes change its sign, relative to the $a=1$ case. For $q=600$ MeV/c, the large contribution of the ρ for $a=1$ disappears when the value $a=0.4$ is used.

V. COMPARISON OF NUMERICAL RESULTS WITH TRANSLATIONAL-INVARIANT CALCULATIONS

In this section we compare the results of our treatment, especially those of Sec. IV, with some other numerical results based on the use of the LDA and on infinite nuclear matter with a constant density $\bar{\rho}$ (not necessarily the central one ρ_o).

We present an overall comparison of the longitudinal enhancement factor

$$\mathscr{R}_J^{(L)}(q) \equiv L_J^{(\mathrm{ren})}(q)/L_J^{(0)}(q)$$

and the corresponding transverse-channel ratio

TABLE IV. Theoretical results for the transverse renormalized matrix element, with the modified ρ exchange ($a=0.4$). The value of g is taken to be 0.7. The results are to be compared with those of Table III.

q (MeV/c)	c_ρ	$iT_J^{(\mathrm{ren})}$
350	2.18	−1.11
400	2.18	−0.65
	2.90	−0.67
500	2.90	0.21
600	2.90	0.27

$$\mathscr{R}_J^{(T)}(q) \equiv T_J^{(\mathrm{ren})}(q)/T_J^{(0)}(q)$$

for the $J^P = 1^+$, $T=1$ level of ^{12}C studied in this work.

A. The local density approximation

1. Formalism

The use of the LDA is based on introducing a density with radial dependence, taken here in the Fermi form and normalized to unity

$$\rho(r) = \frac{3}{4\pi c^3}\left\{\left[1+\frac{\pi^2 t^2}{c^2}\right]\left[1+\exp\left[\frac{r-c}{t}\right]\right]\right\}^{-1}, \tag{15}$$

where c denotes the nuclear half-density radius, and $a = 4.40t$ is the nuclear surface thickness. This density determines a local Fermi momentum

$$p_F(\rho) = [\tfrac{3}{2}\pi^2 A\rho(r)]^{1/3}, \tag{16}$$

which then appears in the Lindhard function[25] used for spin-isospin operator renormalization in the Fermi-gas model.[3] This assumes that the nuclear density is slowly varying so that it is meaningful at each point to assign a local Fermi momentum, determined from the nuclear density at that point, and then to calculate in the corresponding Fermi gas. Naturally, this is not necessarily a very good approximation, and we wish here to study its validity by comparing it to the treatment appropriate to a finite system.

Using the LDA in the Fermi-gas model for the operator renormalization[3] we have

$$(\vec{\sigma}\cdot\hat{q}e^{i\vec{q}\cdot\vec{r}}\tau_\lambda)_{\mathrm{ren}} = \epsilon^{-1}(q,\omega)\vec{\sigma}\cdot\hat{q}e^{i\vec{q}\cdot\vec{r}}\tau_\lambda \tag{17}$$

for the longitudinal probe, and

$$[(\vec{\sigma}\times\hat{q})_j e^{i\vec{q}\cdot\vec{r}}\tau_\lambda]_{\mathrm{ren}} = \epsilon'^{-1}(q,\omega)(\vec{\sigma}\times\hat{q})_j e^{i\vec{q}\cdot\vec{r}}\tau_\lambda \tag{18}$$

for the transverse one, where ϵ^{-1} and ϵ'^{-1} are given in Ref. 3 (see also Refs. 7 and 8 and the following discussion).

With these renormalization schemes the ratios $\mathscr{R}_J^{(L,T)}$ are given by

$$\mathscr{R}_J^{(L)}(q) = \left[\sum_L F_{\mathrm{ph}}^{*JL}(\epsilon,q)a_{JL}\right]\Big/\left[\sum_L F_{\mathrm{ph}}^{*JL}(q)a_{JL}\right] \tag{19}$$

and

$$\mathscr{R}_J^{(T)}(q) = \left[\sum_L F_{\mathrm{ph}}^{*JL}(\epsilon',q)b_{JL}\right]\Big/\left[\sum_L F_{\mathrm{ph}}^{*JL}(q)b_{JL}\right]. \tag{20}$$

The quantities a_{JL} and b_{JL} are defined in Eqs. (4a) and (4b). The quantity $F_{\mathrm{ph}}^{JL}(\mathcal{X},q)$ is obtained from $F_{\mathrm{ph}}^{JL}(q)$ [the latter is given by Eq. (10)] upon a replacement of the nonrenormalized radial integral in Eq. (10) by the renormalized one

$$\int_0^\infty dr\, r^2 \mathcal{X}^{-1}[\rho(r)]j_L(qr)R_p^*(r)R_h(r)\ (\mathcal{X}=\epsilon \text{ or } \epsilon').$$

In order to incorporate the ρ-meson exchange interaction in the diamesic renormalization of Eqs. (17) and (18) we note that for the present case of a Fermi gas, where translational invariance exists, the 1p-1h interaction in the transverse channel is given by

$$V_{\mathrm{ph}}^{(T)} = -\frac{f_\rho^2(q^2)}{m_\rho^2}\frac{q^2}{q^2+m_\rho^2} + \frac{f^2(q^2)}{m_\pi^2}g' \tag{21}$$

using the physical quantities introduced in Sec. II B. In this straightforward introduction of the ρ meson into the 1p-1h interaction the longitudinal channel is unaffected and has only the OPE and g' terms. On the other hand, when a q dependence of g' due to ρ-meson exchange is introduced[26] through the relation

$$g'(q) = g_0' + 0.4c_\rho\frac{q^2}{q^2+m_\rho^2}, \tag{22}$$

(where $c_\rho = 2.18$ and $g_0' = 0.55$ are used[26]), both channels are affected by the presence of the ρ.

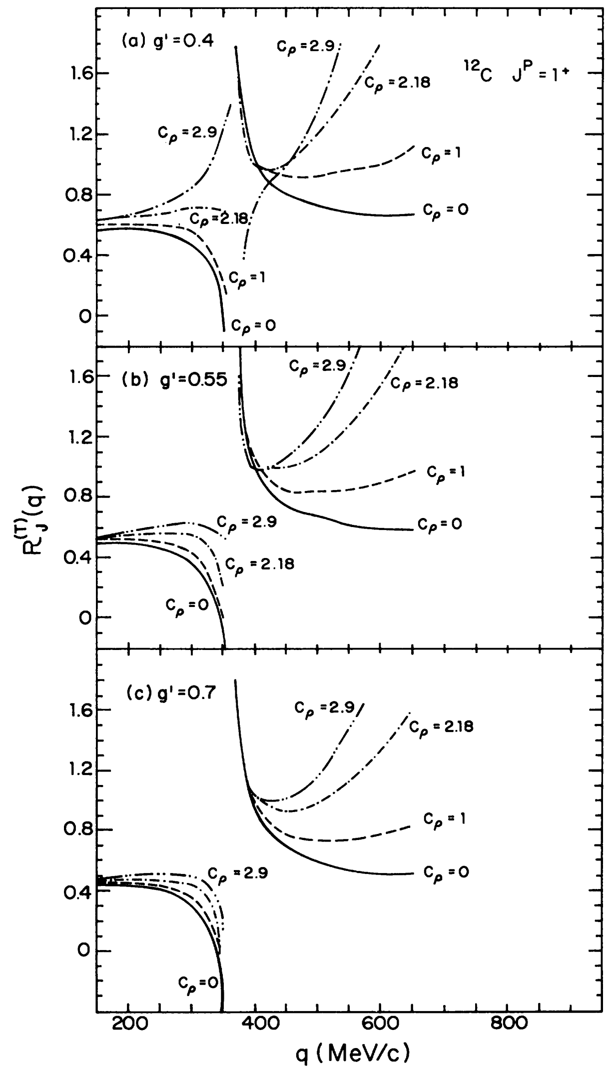

FIG. 6. Results for the transverse-channel ratio $\mathscr{R}_J^{(T)}(q)$ for (a) $g'=0.40$, (b) $g'=0.55$, and (c) $g'=0.70$. The results are shown for $c_\rho = 2.9$, 2.18, 1, and 0 for momentum transfers $q \geq 150$ MeV/c. The parameters for the LDA were taken as $a = 2.29$ fm and $c = 2.36$ fm.

2. *Numerical results and discussion*

We start with the simpler formalism, where the full ORE interaction is introduced, in the transverse channel only. The longitudinal channel is then totally unaffected by the ρ field. This is also approximately the case for the exact finite nucleus treatment when $q \lesssim m_\rho$. LDA and nuclear matter results for the longitudinal case can thus be found in Ref. 7.

We present results for the transverse channel for $g'=0.40$, 0.55, and 0.70, worked out for $c_\rho=2.9$, 2.18, and 1, and compared in each case with the corresponding results for $c_\rho=0$ (i.e., no ρ exchange at all). The LDA results are summarized in Fig. 6, and a comparison with the finite nucleus treatment (FN) is given in Table V. Since appreciable modifications due to the ρ exchange were found to exist only for $q \gtrsim 200$ MeV/c, we do not show any results for $q < 150$ MeV/c; these were already presented in our earlier studies.[7,8] We note, however, that the Fermi gas results predict at $q \to 0$ a quenching of matrix elements of the kind considered here, which, for ^{12}C, is of the order of 0.8. This is not found in the finite nucleus treatment,[8] at least for light nuclei such as ^{12}C.

Figure 6 demonstrates that the attractive ρ-exchange interaction has a large effect on the renormalized matrix element (in the LDA) for high values of q, where it overcomes the repulsive g' term. We note that the LDA gives large precritical effects in the ρ channel for $c_\rho > 0$, which become very pronounced for high values of the parameter c_ρ. In Fig. 6, the low-g' graphs help to demonstrate the interplay between the two parts of the interaction. However, as Table V shows, the LDA renormalization is completely unsatisfactory. Even the effect of the ρ meson itself is accounted for only semiquantitatively by the LDA. Exact finite nucleus calculations are clearly necessary for a reliable estimate of the renormalization effects, since they take into account the essential feature of nonlocality in momentum space.

Turning to the possible q dependence of g' [Eq. (22)], we find an effect of the ρ exchange for both (the longitudinal and the transverse) channels. As expected, this ef-

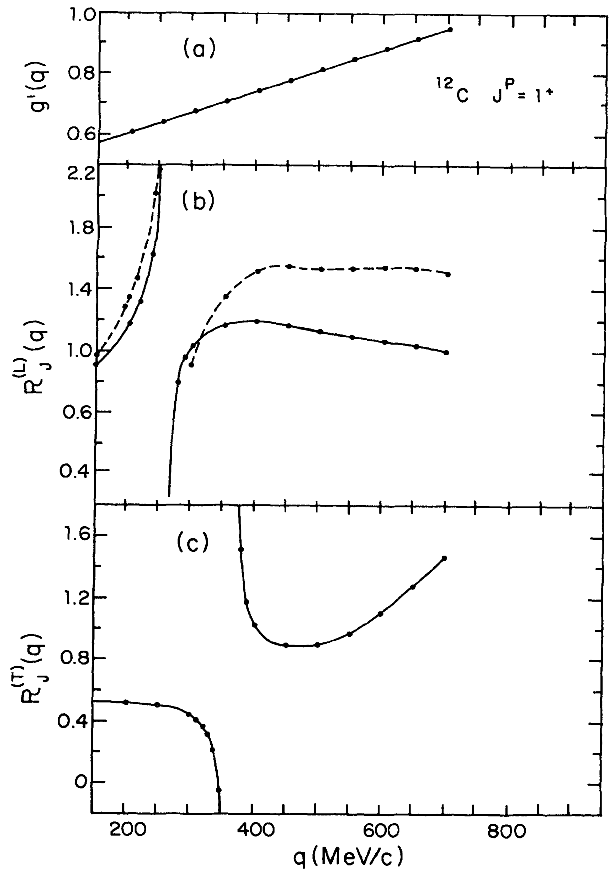

FIG. 7. Results for the LDA calculation involving $g'(q)$, Eq. (22). (a) The q dependence of g'. (b) The q dependence of $\mathscr{R}_J^{(L)}(q)$ where $g'(q)$ is used. (c) The momentum dependence of $\mathscr{R}_J^{(T)}(q)$ where both the ρ-exchange interaction and $g'(q)$ are incorporated. In (b), the dashed line represents the results for $g'(q)=g'(0)=0.55$, shown for comparison.

TABLE V. Numerical results for the ratio $\mathscr{R}_J^{(T)}(q)$ calculated for the $J^P=1^+$; $T=1$ level of ^{12}C. The results are for a finite nucleus (FN) and for an LDA calculation. The relative importance of the ρ meson in the two cases is also shown through the ratio of $\mathscr{R}_J^{(T)}(q)$ for $c_\rho>0$ to that at $c_\rho=0$. The parameters for the LDA case were taken as $a=2.29$ fm and $c=2.36$ fm.

q (MeV/c)	g'	c_ρ	$\mathscr{R}_J^{(T)}$ FN	$\mathscr{R}_J^{(T)}$ LDA	$\mathscr{R}_J^{(T)}(c_\rho)/\mathscr{R}_J^{(T)}(c_\rho=0)$ FN	$\mathscr{R}_J^{(T)}(c_\rho)/\mathscr{R}_J^{(T)}(c_\rho=0)$ LDA
150	0.7	2.0	0.68	0.47	1.01	1.07
	0.4	2.9	0.71	0.64	0.98	1.10
350	0.7	2.9	−5.52	0.13	1.19	−0.37
400	0.7	2.18	2.45	1.04	1.20	1.12
	0.7	2.9	2.66	1.04	1.31	1.12
	0.4	2.9	3.72	0.76	1.20	0.75
500	0.7	2.9	−0.79	1.16	1.17	1.90
600	0.7	2.9	−3.23	1.90	1.68	3.65

fect is equivalent to using high values of g' for high momenta, and thus only attenuates the renormalization effects there (Fig. 7). The momentum dependence of g' does not, therefore, improve the poor agreement between the exact FN and LDA results.

B. Nuclear matter with constant density

Closely related to the LDA is the use of a constant nuclear density, $\bar{\rho}$, as the simplest (and crudest) approximation to a finite nucleus. The density $\bar{\rho}$ may be different from the central density ρ_o, since it is well known that light nuclei have a relatively low density on the average.

In Tables VI—VIII we show results for the transverse-channel ratio $\mathscr{R}_J^{(T)}(q)$ when the full ρ-exchange interaction is included (and is not included in the longitudinal channel renormalization). The results are presented for $\bar{\rho}=0.17$, 0.12, and 0.08 fm^{-3}, $g'=0.70$, 0.55, and 0.40, and $c_\rho=2.9$, 2.18, 1, and 0. We show our results for $q \geq 200$ MeV/c, since for lower momenta the ρ meson is too massive to contribute to the renormalization.

The overall features are quite similar in the LDA and the constant density calculations. The attractive ρ-exchange interaction turns quenching into enhancement for high enough momenta (see the underlined numbers in the tables). The momentum value at which this happens depends on both c_ρ, g', and $\bar{\rho}$. In general, $\mathscr{R}_J^{(T)}(q)$ becomes larger than one earlier (i.e., for lower q and c_ρ) the smaller the value of g' and the higher the value of $\bar{\rho}$, since these mean less repulsion and larger renormalization effects, respectively. Also, the lower densities give less quenching as well as less enhancement, since for $\bar{\rho}\to 0$ all renormalization effects vanish. On the other hand, high values of g' give rise to more quenching but less enhancement. The agreement with the exact FN results is even poorer in this last approximation than it is for the LDA one.

We also used Eq. (22) (where g' acquires a momentum dependence), which affects both the $\vec{\sigma}\times\hat{q}$ and the $\vec{\sigma}\cdot\hat{q}$

TABLE VI. Results for the ratio $\mathscr{R}_J^{(T)}(q)$ calculated for the $J^P=1^+$; $T=1$ level of ^{12}C. The results are given for an infinite nuclear matter with a constant density $\bar{\rho}=0.17\ \mathrm{fm}^{-3}$. The underlined numbers indicate the momenta for which quenching turns into enhancement as a result of the ρ exchange attractive interaction.

q	$g'=0.40$				$g'=0.55$				$g'=0.70$			
(MeV/c)	$c_\rho=0$	$c_\rho=1$	$c_\rho=2.18$	$c_\rho=2.90$	$c_\rho=0$	$c_\rho=1$	$c_\rho=2.18$	$c_\rho=2.90$	$c_\rho=0$	$c_\rho=1$	$c_\rho=2.18$	$c_\rho=2.90$
200	0.51	0.54	0.58	0.62	0.43	0.45	0.48	0.50	0.37	0.39	0.41	0.42
300	0.54	0.62	0.75	0.87	0.46	0.52	0.61	0.68	0.40	0.45	0.51	0.56
400	0.59	0.74	$\underline{1.06}$	$\underline{1.44}$	0.51	0.62	0.83	$\underline{1.05}$	0.45	0.54	0.69	0.83
500	0.66	0.90	1.53	2.70	0.59	0.77	$\underline{1.19}$	1.79	0.53	0.67	0.97	$\underline{1.33}$
600	0.77	$\underline{1.03}$	1.76	3.08	0.70	0.93	1.47	2.28	0.65	0.84	$\underline{1.26}$	1.81

TABLE VII. Results for the ratio $\mathscr{R}_J^{(T)}(q)$ calculated for the $J^P=1^+$; $T=1$ level of ^{12}C. The results are given for an infinite nuclear matter with a constant density $\bar{\rho}=0.12\ \mathrm{fm}^{-3}$. The underlined numbers indicate the momenta for which quenching turns into enhancement as a result of the ρ exchange attractive interaction.

q	$g'=0.40$				$g'=0.55$				$g'=0.70$			
(MeV/c)	$c_\rho=0$	$c_\rho=1$	$c_\rho=2.18$	$c_\rho=2.90$	$c_\rho=0$	$c_\rho=1$	$c_\rho=2.18$	$c_\rho=2.90$	$c_\rho=0$	$c_\rho=1$	$c_\rho=2.18$	$c_\rho=2.90$
200	0.55	0.58	0.63	0.66	0.47	0.50	0.53	0.55	0.41	0.43	0.45	0.47
300	0.59	0.67	0.79	0.89	0.51	0.57	0.66	0.72	0.45	0.50	0.56	0.61
400	0.65	0.79	$\underline{1.05}$	$\underline{1.31}$	0.58	0.68	0.87	$\underline{1.04}$	0.52	0.60	0.74	0.86
500	0.76	0.93	1.28	1.67	0.69	0.84	$\underline{1.11}$	1.39	0.64	0.76	0.98	$\underline{1.19}$
600	0.83	$\underline{1.02}$	1.42	1.85	0.78	0.95	1.28	1.62	0.73	0.88	$\underline{1.16}$	1.44

TABLE VIII. Results for the ratio $\mathscr{R}_J^{(T)}(q)$ calculated for the $J^P=1^+$; $T=1$ level of ^{12}C. The results are given for an infinite nuclear matter with a constant density $\bar{\rho}=0.08\ \mathrm{fm}^{-3}$. The underlined numbers indicate the momenta for which quenching turns into enhancement as a result of the ρ exchange attractive interaction.

q	$g'=0.40$				$g'=0.55$				$g'=0.70$			
(MeV/c)	$c_\rho=0$	$c_\rho=1$	$c_\rho=2.18$	$c_\rho=2.90$	$c_\rho=0$	$c_\rho=1$	$c_\rho=2.18$	$c_\rho=2.90$	$c_\rho=0$	$c_\rho=1$	$c_\rho=2.18$	$c_\rho=2.90$
200	0.60	0.63	0.68	0.70	0.52	0.55	0.58	0.60	0.46	0.48	0.51	0.52
300	0.65	0.72	0.83	0.91	0.57	0.63	0.71	0.77	0.51	0.56	0.62	0.67
400	0.73	0.84	$\underline{1.03}$	$\underline{1.20}$	0.66	0.75	0.90	$\underline{1.03}$	0.60	0.68	0.80	0.90
500	0.83	0.96	1.16	1.33	0.78	0.89	$\underline{1.07}$	1.21	0.74	0.84	0.99	$\underline{1.11}$
600	0.88	$\underline{1.02}$	1.24	1.43	0.84	0.97	1.16	1.33	0.81	0.92	$\underline{1.10}$	1.25

TABLE IX. Results for the nuclear matter calculation involving $g'(q)$, Eq. (22). We show the ratio of $\mathscr{R}_J^{(L,T)}(q)$ to the corresponding ratio for $c_\rho=0$ [where $g'(q)=g'_0=0.55$]. For the transverse case the ρ-meson exchange interaction term is also included, as in Tables VI—VIII.

q	$\bar{\rho}=0.17$ fm^{-3}		$\bar{\rho}=0.12$ fm^{-3}		$\bar{\rho}=0.08$	
(MeV/c)	Longitudinal	Transverse	Longitudinal	Transverse	Longitudinal	Transverse
200	1.05	0.83	1.06	0.87	1.06	0.90
300	1.15	0.64	1.14	0.72	1.12	0.80
400	1.29	0.57	1.22	0.68	1.18	0.80
500	1.46	0.62	1.30	0.77	1.21	0.86
600	1.53	0.74	1.35	0.83	1.23	0.88

channels. The results for $\mathscr{R}_J^{(L,T)}(q)$ relative to those for $c_\rho=0$ (to be found in Refs. 7 and 8) are given in Table IX. For the transverse channel, the ρ-exchange term is also included in the calculation. These results show once again that the infinite nuclear matter approximation is inadequate for our purposes, and that the exact, nonlocal finite nucleus treatment is very important for quantitative calculations in this field. The results of this work are intended to serve as a guide for further study along these lines.

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[23]For this test (modified) case $[\hat{R}_J(k,q)]_{00}$ peaks at $q=275$ MeV/c with a value of -3.69 MeV^{-1}, compared with corresponding values of 225 MeV/c and -1.09 MeV^{-1} for the correct case. For $[\hat{R}_J(k,q)]_{02}$ we find a peak value of -0.56 MeV^{-1} at $q=300$ MeV/c for the test case, compared with $+0.22$ MeV^{-1}, 225 MeV/c for the correct one. Also, $[\hat{R}_J(k,q)]_{20}$ has its peak value of -0.79 MeV^{-1} at 275 MeV/c (test case) instead of $+0.29$ MeV^{-1} at 300 MeV/c (correct case). Finally, the values for the $L=L'=2$ are -0.90 MeV^{-1} at 300 MeV/c against -0.60 MeV^{-1} at 225 MeV/c, respectively.
[24]For this second test case the modified response function peaks at 250, 300, 325, and 300 MeV/c with the peak values -1.50, $+0.68$, $+0.88$, and -1.12 MeV^{-1} for $L,L'=0,2$, respectively. The correct (unmodified) values are given in Ref. 23.
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