

Pole extrapolation in the neutron-deuteron scattering system and the deuteron asymptotic normalizations

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Differential cross sections and tensor polarizations of elastic neutron-deuteron scattering (n-d) are calculated at $E_{\text{lab}}^n < 15$ MeV with different nuclear potentials. The pole extrapolation of these model data reproduces the S -state normalization within 1% and misses the D - to S -state ratio η by about 5%. Approximate Coulomb distortion corrections to n-d tensor polarization calculations that improve the agreement with proton-deuteron (p-d) data yield smaller contributions to η than found previously within a distorted-wave Born approximation. Extrapolation of recent n-d differential cross section data and Coulomb corrected p-d tensor polarization leads to deuteron asymptotic normalizations compatible with current experimental values.

I. INTRODUCTION

Basic properties of the deuteron like, e.g., the binding energy E_D , the quadrupole moment Q , the asymptotic S -state normalization A_S , or the asymptotic ratio of the D -state to S -state normalization η have each been important for the construction of sensible N-N interaction models. It was, however, not too long ago that it became standard practice to compare, for a given model, all these deuteron quantities at one time with the values known from the experiments. To serve as real constraints for N-N potentials these experimental quantities have to be known at a high level of accuracy given the very quantitative nature of the theoretical predictions.¹ Among others a model independent relation between Q and η seems to provide very tight bounds for these quantities, thus challenging the precision of the experiments, in particular, that of the less accurately known ratio of the asymptotic normalizations. Different methods to experimentally determine η have been developed in recent years² gradually reducing the uncertainty of the measured η down to a few percent. Although there are unsettled questions within the theory, like the influence of the nucleon size,³ it seems to be mainly up to experimentalists to clarify whether or not a further improvement of their measurements is feasible.

Irrespective of this future development we address here again one of the methods that have been employed to extract the present experimental knowledge on η , namely the pole extrapolation in elastic (d,p) reactions. This question was already the subject of controversies at the "Deuteron Workshop" at the Karlsruhe conference¹ in 1983 and it seems to be worthwhile to examine once more the method of analytical extrapolation.⁴ Specifically, we treat the elastic three-nucleon scattering system, which can be described exactly by the Faddeev equations, to study the validity of the pole extrapolation procedure, i.e., to find theoretical bounds on the extrapolated values of A_S and η . For that purpose we solve the Faddeev equations with N-N potentials that differ in the deuteron asymptotic normalizations. The resulting unpolarized cross sections $[\sigma(\theta)]$ and the tensor polarizations T_{20} ,

T_{21} , and T_{22} , representing "pseudo-data," are then appropriately incorporated in functions that can be analytically continued into the unphysical region, and criteria for a reasonable extrapolation to the proton exchange pole are discussed (Sec. II). The various results obtained from extrapolating the pseudo-data are compared to the values of A_S and η inherent to the different potentials employed for the generation of the pseudo-data. From this comparison we can deduce lower bounds for the theoretical error of the extrapolated quantities (Sec. III). The experience made in extrapolating the pseudo-data is utilized in the attempt to analytically continue recent data. The Karlsruhe measurements of neutron-deuteron unpolarized cross sections are analyzed to gain information on A_S . The determination of η requires tensor polarization of the deuteron⁵ and since no such data are available for n-d scattering one has to resort to proton-deuteron data. These data, however, do contain Coulomb effects that ought to be taken into account. One way to handle such effects was proposed by Santos and Colby⁶ who introduced a correction factor for the neutron exchange pole. Here we propose another approach by applying approximate Coulomb distortion corrections⁷ in the physical region. We extrapolate n-d and Coulomb corrected n-d pseudo-data to find the Coulomb contribution to η at different energies. We then calculate η from a "n-d" polarized cross section which was generated from n-d cross section data and Coulomb corrected p-d tensor polarization data (Sec. IV).

II. PSEUDO-DATA AND POLE EXTRAPOLATION

The increasing importance of the concept of pole extrapolation in nuclear physics⁸ has been manifest also in N-d scattering. In particular, the angular extrapolation to the nucleon-exchange pole turned out to be very interesting, because the pole residues can be related to the asymptotic normalizations of the deuteron wave function. The pole strength (at the dpn vertex) obtained by extrapolating unpolarized n-d cross sections⁹ can yield information on the S -state normalization of the deuteron, whereas information on the D/S -state ratio can be gained from tensor po-

TABLE I. Potentials used in the 3S_1 - 3D_1 coupled state and some of their deuteron properties.

Potential	E_d (MeV)	A_s	η	Q	Ref.
Graz II	2.2254	0.8878	0.0274	0.281	a
Graz V212	2.2254	0.8843	0.0230	0.238	b
Reid	2.2246	0.8776	0.0262	0.280	c,d
2T7	2.2250	0.9364	0.0033		e
2T4	2.2250	0.8777	0.0260		e

^aReference 10.

^bReference 11.

^cReference 12.

^dReference 13.

^eReference 14.

larizations continued to the pole.⁵ In the latter case Coulomb corrections and the uncertainty in the data affect the extrapolation,¹ and it is not yet clear to what extent they influence the experimental value of η . To avoid both of these difficulties we have generated n-d pseudo-data with an assumed overall 1% error (the actual model error turned out to be typically smaller than 1%). The model data, namely $\sigma(\theta)$, T_{20} , T_{21} , and T_{22} , were obtained from solutions of the Faddeev equations. In detail we have used different separable potentials (Table I) with varying deuteron asymptotic normalizations as input for the three-body equations. n-d calculations using the Graz II, 2T4, and 2T7 potentials were already carried out by Koike.^{15,16} For our purpose of testing the extrapolation method via the asymptotic deuteron properties we have included only the 1S_0 and the coupled 3S_1 - 3D_1 channel. In Table I the 3S_1 - 3D_1 potentials and some deuteron properties are listed, whereas in the 1S_0 state we have used the Graz II potential throughout. For comparison we have calculated another set of model data based on the T -matrix Faddeev results of Stolk and Tjon¹⁷ for the local Reid potential. Since these pseudo-data include the effects of N-N P and D waves (at least perturbatively), they are more realistic, i.e., they better resemble the polarization data than the one obtained by the five-channel calculation.

For the purpose of angular extrapolation the polarization data should be expressed in a functional form that displays minimal complex structure.⁵ For n-d scattering we employ the formula^{18,19}

$$f_{2\nu}(z) = (-)^{\nu+1} \sigma(z) T_{2\nu}(z) \frac{k^2(z-z_p)^2}{(1-z^2)^{\nu/2}}, \quad (1)$$

where $\nu=0, 1$, and 2 , $z=\cos\theta$, k is the c.m. momentum, and z_p is the position of the proton exchange pole. Equation (1) has been used for p-d data,¹⁸⁻²² but the presence of the forward angle Coulomb singularity makes an extrapolation into the unphysical region problematic.⁴ Extrapolation of the unpolarized differential cross section is performed through the function⁹

$$h(z) = k^4(z-z_p)^2 \sigma(z). \quad (2)$$

Among the three functions involving tensor polarization primarily the function f_{22} lends itself for extrapolation purposes (Fig. 1), a finding that has been obtained previ-

ously by Amado *et al.*¹⁸ By showing $f_{2\nu}$ for one set of our pseudo-data we simply want to emphasize that for N-d scattering f_{20} and f_{21} are to be expected to yield inferior results due to their more complicated shape.

To analytically continue our pseudo-data into the unphysical region we expanded them in the interval $\cos\theta_{c.m.} = [-1, 1]$ into Legendre polynomials. Using other polynomials, like over the data orthonormalized ones or Chebyshev, led to identical representations of our data. The actual expansion was then performed in a conformally mapped variable²³ [$w\cos\theta$] to better exploit the analyticity in the angular variable. The mapping technique provides the best convergence of the polynomial expansion within the region of analyticity, but since we have only a finite number of pseudo-data with finite accuracy stability in the extrapolation for an increasing number of terms, represented by the order of the highest used polynomial, L_{\max} cannot be achieved.²⁴ Requiring the fitted curve through the data to be optimally smooth—a frequently used practice in elementary particle physics²⁵—did not improve the convergence. The polynomial expan-

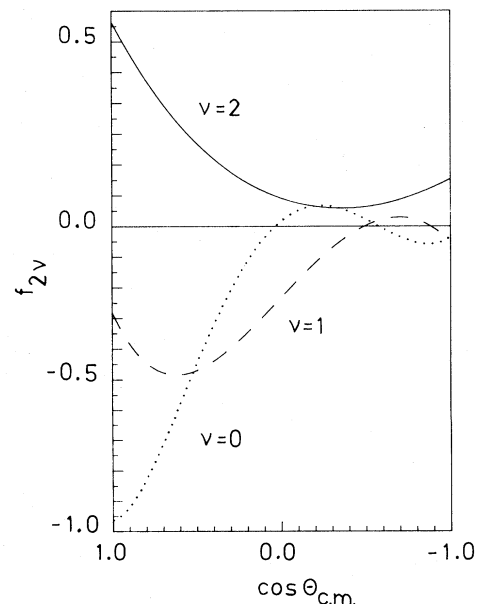


FIG. 1. Function $f_{2\nu}$ obtained from a Faddeev calculation with the Graz II potential at $E_{\text{lab}}^n = 5$ MeV.

sion of f_{22} with the pseudo-data obtained with the Graz II potential is shown in Fig. 2. An improvement of the fit in the physical region significantly affects the extrapolation to the pole. Proceeding to higher L_{\max} values of the expansion the fit does not become much better, but the extrapolation in turn deteriorates, i.e., becomes unstable (Fig. 3). In addition, with increasing L_{\max} the error in the extrapolated residue at the pole increases exponentially. Consequently, only a finite and, in fact, a small number of expansion coefficients can be taken. This truncation of the polynomial series has been used in extrapolating N-d data^{9,18-22} in the course of which the L_{\max} is determined when the $\chi^2_{\text{red}} = \chi^2 / (\text{degree of freedom})$ becomes about one. Since we are dealing with pseudo-data of unknown accuracy in the first place, we had to modify slightly the criterion for the truncation. Starting with 1% overall error in the pseudo-data we calculated the χ^2_{red} as a function of L_{\max} (Fig. 3). The series was then truncated at this value of L_{\max} where the χ^2_{red} curve displayed a tendency to flatten, i.e., where the fit did not significantly change by adding more coefficients. This happened to be at $L_{\max} = 4$ for the Graz-potential data. Typically the first plateau in the χ^2_{red} curve occurred at a value not equal to one and we therefore renormalized the χ^2_{red} to one. Thereby we obtained an estimate on the accuracy of our model data which is certainly much better than that of the N-d experiments.

The extrapolated pole residue and hence η remains quite stable for some low values of L_{\max} , but tends to oscillate when one starts to fit the "noise" in the pseudo-data. Having established the truncation criteria it was crucial to find an estimate on the error of the extrapolated residue. It is only then that we can discuss the usefulness of the method of pole extrapolation in N-d elastic scattering processes for determining the deuteron asymptotic normalizations. We have considered two possibilities to obtain the error in η . One is to extrapolate the errors of the pseudo-data through the errors in the expansion coefficients. The other possibility is to represent each data point by a Gaussian distribution within its error bounds. This redefined set of data was then again expanded into Legendre polynomials. In both cases a very similar, exponentially increasing error curve was found (dashed-

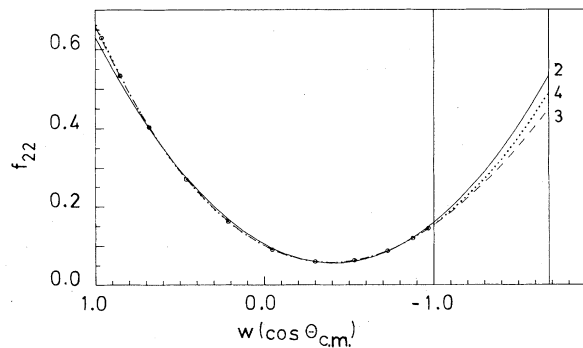


FIG. 2. Polynomial expansion of the function f_{22} of Fig. 1 plotted versus the conformally mapped variable $w(\cos\theta_{\text{c.m.}})$. The numbers at the intersection of the function and the pole position line denote the degree of the polynomial. Circles represent a sample of pseudo-data.

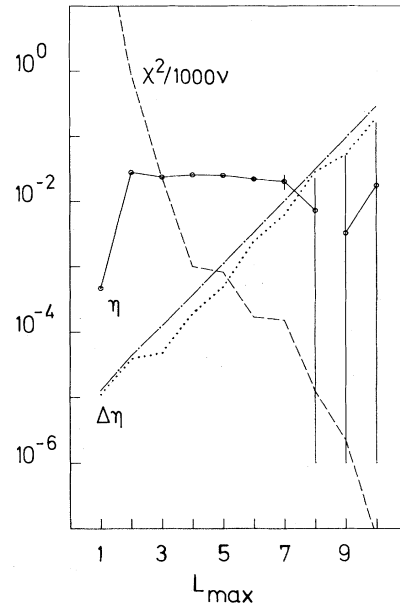


FIG. 3. η as a function of the highest used degree L_{\max} of the polynomial expansion of the Graz II potential pseudo-data at $E_{\text{lab}}^n = 5$ MeV. The line connecting η is interrupted when η changes sign. Uncertainties $\Delta\eta$ in the extrapolation are calculated on the basis of (i) the expansion coefficients (dashed-dotted line and error bars) and (ii) a Gaussian distribution (dotted line). Dashed line denotes the quality of the fit ($\chi^2/\text{degree of freedom}$).

dotted and dotted line in Fig. 3). Since this error is solely due to the statistical nature of the data and therefore neglects the error arising from truncating the polynomial expansion, the error estimate might well be too optimistic. Hence, it follows that the $\Delta\eta$ curves in Fig. 3 represent lower bounds for the uncertainty of the extrapolated value of η .

III. RESULTS WITH MODEL n-d DATA

Following the procedure outlined in the last section we have calculated the deuteron asymptotic normalizations with different sets of pseudo-data. We have chosen potentials with substantially different η to get some information on the model dependence of the extrapolation method. The calculations were performed at rather low deuteron energies, because the Coulomb effects which we will address in Sec. IV are more important there. Furthermore, the five-channel calculations become less realistic with increasing energy.

The extrapolated values for η (η^{ext}), the uncertainties in the extrapolation $\Delta\eta^{\text{ext}}$, and the deviations of η^{ext} from the true value are given in Table II. Also shown are the overall errors of the pseudo-data and the values of L_{\max} where the expansion series was truncated. The extrapolation method applied to the "separable potential data" seems to miss the true value by about 5%. The extreme case of the 2T7 potential with η being almost zero underscores in a qualitative way the applicability of the method. Taking into account that the quoted number for the extrapolation error is a lower bound, it is possible that the pole

TABLE II. η found from extrapolating various pseudo-data. $\overline{\Delta Y}$ denotes the average error of the pseudo-data, $\Delta\eta$ the extrapolation accuracy. The last column contains the deviation of η^{ext} from the values given in Table I.

Potential	E_{lab}^n (MeV)	L_{max}	$\overline{\Delta Y}$ (%)	η^{ext}	$\Delta\eta^{\text{ext}}$ (%)	$\Delta(\eta - \eta^{\text{ext}})$ (%)
Graz II	5	4	0.10	0.0258	1.05	5.25
Graz II	10	4	0.39	0.0257	1.60	5.62
Graz V212	5	4	0.10	0.0216	0.75	6.09
Graz V212	10	4	0.38	0.0218	1.70	5.14
Reid	5.5	3	0.55	0.0227	1.40	13.43
Reid	10.85	3	1.40	0.0212	1.29	19.06
Reid	14.1	3	1.90	0.0203	3.07	22.37
2T4	10	4	0.37	0.0250	2.14	3.85
2T7	10	4	0.83	0.0061	10.80	83.32

extrapolation method in the form as described by Amado *et al.*⁵ can reproduce η of the employed model. The accuracy of η^{ext} , however, would then be quite poor compared to other deuteron quantities. The results obtained with the Reid potential deviate significantly from the values given by the deuteron wave functions. If there is not a trivial reason like missing precision or errors in the used Faddeev solutions—which we could not confirm—one explanation could be a model dependence. There is, however, another possible source for said discrepancies: In the calculation of Stolk and Tjon¹⁷ only the S -wave projection of the coupled 3S_1 - 3D_1 N-N channel was treated exactly; including the D -wave part perturbatively could lead to substantial differences of the D - to S -state ratio. The overall trend of the results is to underestimate η . This trend would be even much bigger if f_{20} and f_{21} were used for extrapolation. In the first case we fail to reproduce η by 40% and in the second case by roughly 30%.

The formula used to calculate η involves the unpolarized $\sigma(\theta)$ at the exchange pole. In our attempt to find explanations for the 5% disagreement we have examined to what extent the pole approximation of $\sigma(\theta)$ (Ref. 8) could be made responsible for it. With the help of Eq. (2) we have calculated A_s with the Graz II potential data and have found (Table III) that A_s^{ext} almost reproduces the given A_s . Therefore the A_s part does not seem to explain the 5% shortcoming in η . On the other hand, it is interesting that A_s can be determined from n-d model data even more accurately than found previously.²⁶

IV. RESULTS WITH N-d EXPERIMENTS

Given the accuracy of the asymptotic normalizations obtained by extrapolating n-d model data, it is interesting to apply this procedure to recent n-d experiments. Unfortunately, there are only n-d differential cross sections

TABLE III. A_s from extrapolation of the Graz II pseudo-data. Errors as in Table II.

E_{lab}^n (MeV)	L_{max}	A_s^{ext}	ΔA_s^{ext} (%)	$\Delta(A_s - A_s^{\text{ext}})$ (%)
5	5	0.8787	0.3	1.02
10	5	0.8793	0.3	0.95

available, whereas there exist no n-d tensor polarizations to date. To extract η from measurements we have to resort to p-d tensor polarizations implying that the Coulomb force has to be dealt with. In the absence of an exact treatment, approximations have been proposed^{6,27,28} to describe the Coulomb corrections at the neutron exchange pole. Here we have handled the Coulomb contributions to the extrapolated residues somewhat differently. Employing an on-shell approximation that describes the Coulomb distortion in an effective two-body manner⁷ we have Coulomb corrected the p-d tensor polarization data. Assuming nuclear charge symmetry we have thus obtained n-d predictions that can be combined with measured cross sections for the purpose of pole extrapolation. This set of n-d data was then subjected to the procedure described in Sec. III except for the renormalization which is redundant in this case.

The usefulness of our Coulomb distortion corrections has already been demonstrated before by explaining, at least qualitatively, the measured differences of n-d and p-d differential cross sections²⁷ and nuclear analyzing powers.³⁰ As a further example we show the Coulomb corrections to the tensor polarizations as given by the Reid potential calculation¹⁷ [Figs. 4(a)–(c)]. The comparison with experiments by the Zürich group³¹ again indicates that adding the approximate Coulomb corrections make the Faddeev results more compatible with the data. This positive trend of the Coulomb corrections is not contained in more simple approximations like the one which modifies the n-d amplitudes simply by the Coulomb phases to yield Coulomb distorted amplitudes [dotted lines in Figs. 4(a)–(c)]. Sperisen *et al.*³² have already noted that this approximation fails for T_{20} and T_{21} . On the other hand, our approximate Coulomb distortion corrections would bring their Faddeev solutions much closer to the p-d data.

In Table IV we show the influence of the Coulomb distortion on the extrapolation of the Reid-potential pseudo-data. Here it should be mentioned that the function $f(z)$ certainly cannot be analytically continued into the unphysical region due to the Coulomb singularity. Alternatively one can use a modified function as proposed by Londergan *et al.*⁴ which does not contain this singularity. We obtained, however, with $f(z)$ identical results within

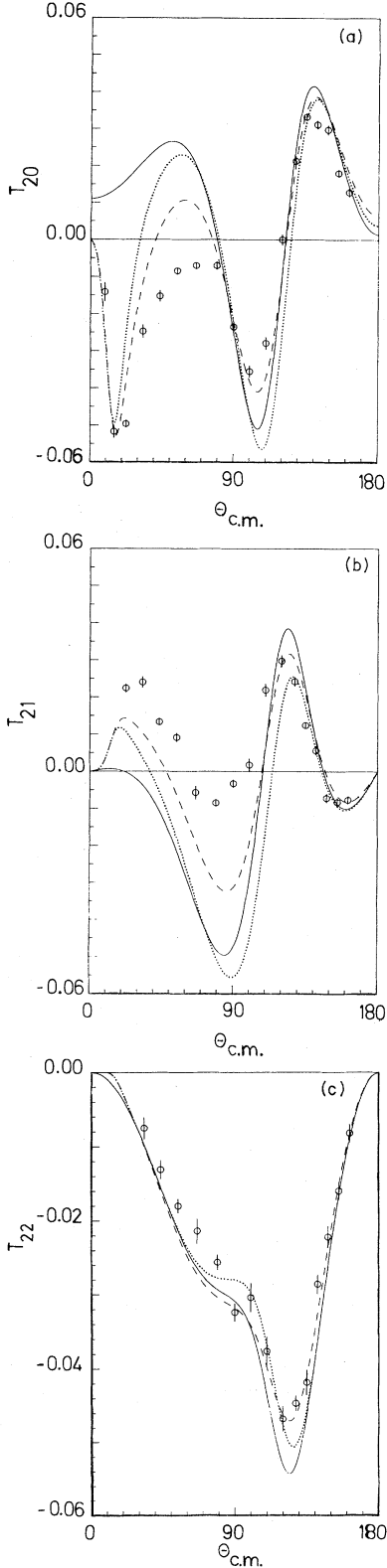


FIG. 4. (a)–(c) Tensor polarization T_{20} , T_{21} , and T_{22} at $E_{\text{lab}}^d = 11$ MeV of the Reid potential n-d calculation of Ref. 17 (solid line) with n-d results where (i) lowest order Coulomb corrections as in Ref. 32 (dotted line) and (ii) approximate Coulomb distortion effects (dashed line) are included. The p-d data at $E_{\text{lab}}^d = 10$ MeV are taken from Ref. 31.

TABLE IV. Effect of the approximate Coulomb distortion corrections on η calculated on the basis of the Reid-potential pseudo-data.

E_{lab}^n (MeV)	L_{max}	$\Delta(\eta^{\text{pd}} - \eta^{\text{nd}})$ (%)
5.5	3	19.84
10.85	3	16.50
14.1	3	9.83

the error bars when we suppressed the Coulomb peaked forward direction of the pseudo-data, an approach usually taken for extrapolating p-d data. The corrections display the typical decreasing behavior as a function of energy. The magnitude of the effect on η , however, is smaller than previously found by the DWBA-type calculation of Santos and Colby.⁶ Their method essentially consists of Coulomb penetration factor corrections to the pole residue, an approach which has been found to tend to overestimate the Coulomb distortion in the physical region.⁷

The Coulomb distortion corrections to T_{22} of the Reid potential calculation at $E_{\text{lab}}^d = 11$ MeV were used for obtaining the n-d tensor polarizations from p-d measurements at $E_{\text{lab}}^d = 10$ MeV.³¹ To construct $f(z)$ we have to know both $\sigma(\theta)$ and T_{22} at the same angles, but since the differential cross section at $E_{\text{lab}}^n = 5$ MeV (Ref. 33) was measured at different angles we have fitted T_{22} at these points. The best accuracy in extrapolating the n-d data was achieved with $L_{\text{max}} = 2$, but even here the error was already around 15% which is unavoidable with the present accuracy of the data at this energy. The value of $\eta = 0.0266 \pm 0.0039$ should not be taken too seriously, but it demonstrates, at least, that η extrapolated from n-d data does not significantly differ from the commonly accepted values.

Finally, we have looked at the $\sigma(\theta)$ data³³ to investigate their correlation with the S-state normalization. In Fig. 5 $h(w)$ at $E_{\text{lab}}^n = 12$ MeV is shown together with the experimental points. Again the smallest error in η was found for $L_{\text{max}} = 2$, but in this case the fit to the data is not as good, which is of particular importance in the backward direction. At lower energies, like at $E_{\text{lab}}^n = 5$ MeV, the fit

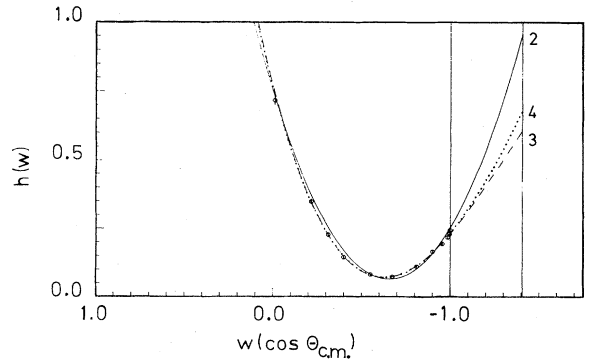


FIG. 5. Polynomial expansion of the function $h(w)$ obtained from fitting the n-d cross section data of Ref. 33 at $E_{\text{lab}}^n = 12$ MeV (circles). Angular variable and notation at the pole line same as in Fig. 2.

with $L_{\max}=2$ is better, which might explain why at this energy $A_s=0.8847\pm 0.0326$ is rather close to the average experimental value,¹ whereas, e.g., at 12 MeV the $L_{\max}=2$ fit neglects part of the very backward structure, thus yielding a much too high result of $A_s=0.9199\pm 0.0109$. In this context one should mention also recent measurements of $\sigma(\theta)$ at $\theta_{c.m.}=180^\circ$ by the Uppsala group.³² At 6 MeV their data point is significantly bigger than the 179° measurement of the Karlsruhe group,³³ whereas at 12 MeV it is the other way around.

V. SUMMARY

To study the method of pole extrapolation in the three-nucleon scattering system we have calculated n-d differential cross sections and tensor polarizations with various N-N potentials that differ mainly in the asymptotic properties of the deuteron. These pseudo-data were then analytically continued to the proton-exchange pole and the pole residues were related to the deuteron asymptotic normalizations. In comparing the extrapolated values with the deuteron asymptotic normalizations of each model employed we have obtained information on the exactness of said method in n-d scattering.

As far as A_s is concerned, the pole extrapolation almost reproduces the aimed for value, the difference being close to only 1%. The discrepancy becomes roughly 5% in the case of η , but here the error in the extrapolation is already 2% not including the error due to the truncation of the polynomial expansion of the pseudo-data. Adding corrections arising from modifications of the pole approximation of the scattering matrix might further help to fill in the gap. Considering the 2% accuracy achieved with the

model data the 3% error following from extrapolating less accurate p-d data²⁰ seems to be rather optimistic.

Continuing recent $\sigma(\theta)$ measurements in n-d scattering to the proton exchange pole we have found at lower energies (<8 MeV) A_s to be roughly compatible with $A_s=0.880$, but the accuracy of the method is only moderate. At higher energies where more partial waves contribute, the precision of the data is not sufficient to guarantee a sensible extrapolation within our approach.

Approximate Coulomb corrections were shown to improve the agreement of n-d Faddeev calculations with p-d tensor polarization measurements. The effect of the Coulomb contributions in the physical region translates into a 20% effect on η at around 5 MeV and shows the typical decreasing behavior with energy. The magnitude of the correction to η , however, turns out to be smaller than found previously in a DWBA-type calculation. Having applied our corrections to T_{22} data of p-d scattering at $E_d=10$ MeV they were combined with the n-d cross section data of the same energy and extrapolated to the pole. The resulting η happens to be close to the current experimental average value. Taking into account the error of about 15% we may conclude only that this set of "n-d" data does not severely conflict with the p-d data on the basis of the asymptotic normalizations of the deuteron.

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