Structure of the 2_1^+ state in ^{12}C and the inelastic form factor

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(Received 24 January 1984)

We study the discrepancy between the measured $0_1^+ \rightarrow 2_1^+$ form factor of ^{12}C and the existing theoretical results, and specify a generator-coordinate state that resolves the discrepancy.

The charge form factors of the transitions between the lowest states in ^{12}C have been studied in a variety of theoretical approaches including random phase approximation (RPA), alpha cluster model (ACM), generator coordinate (GCM), resonating group (RGM), projected Hartree-Fock (PHF), and SU_3 methods (see, e.g., Refs. 1–5). While in the elastic case the experimental form factor can be reproduced rather accurately by any of these methods (except PHF), all of them fail to fit the inelastic form factors of the 2_1^+, 3_1^-, 1_1^-, and 4_1^+ state over the entire range of measured momentum transfer q (cf. Figs. 2.8 and 2.9 of Ref. 1). The discrepancy is particularly severe for the $0_1^+ \rightarrow 2_1^+$ transition at high momentum transfer. Here, the ACM, GCM, RGM, and PHF results fit the experimental form factor reasonably well up to about $q = 2$ fm^{-1} but deviate from the experimental curve for larger q values (cf. Fig. 1). In particular, the calculated form factors fail to reproduce the rapid falloff at intermediate momenta and the shoulder at large momenta, that are characteristic for the experimental curve.

In this Brief Report we study what types of wave functions are capable of resolving the existing discrepancies between the calculated and the experimental $0_1^+ \rightarrow 2_1^+$ form factor. In particular, we attempt to determine modifications of the existing solutions that lead to an improvement in the inelastic form factor at large momentum transfer while leaving the low-momentum part and the elastic form factor unaffected if possible. We ignore here the energy associated with such a wave function since the minimum-energy principle underlying the above variational methods has not led to a satisfactory description of the form factor. Moreover, the energy would require the selection of some specific model Hamiltonian as an additional ambiguity that can be avoided in a fit of the form factors.

The charge form factor[2,3,6,7]

$$|F(q)|^2 = \frac{1}{Z^2}\frac{4\pi}{2J_i+1}\sum_{\lambda=0}^{\infty} |\langle J_f||M^\lambda||J_i\rangle|^2 f^2(q) \qquad (1)$$

is essentially determined by the reduced matrix elements of the multipole terms

$$M_\mu^\lambda = \sum_p j_\lambda(qr_p)\, Y_{\lambda\mu}(\hat{r}_p) \qquad (2)$$

between the initial and final states $|J_i\rangle$ and $|J_f\rangle$. Here, $f^2(q)$ is a correction factor for the finite size of the proton and the center-of-mass motion. The label p denotes the summation over all proton coordinates r_p and the corresponding angles $\hat{r}_p$.

Our selection of suitable methods to describe the initial and final states is guided by the outcome of previous form-factor calculations. Some approaches have to be ruled out because they either fail to fit the elastic form factor (PHF) or the low-momenta part of the inelastic form factor (SU_3), or have a complicated geometrical structure (RGM) that is difficult to improve. Therefore, among the above methods, the ACM and GCM wave functions appear to be the best candidates for the initial and final states $|J_i\rangle$ and $|J_f\rangle$.

The ACM wave function is given by single-nucleon orbits that are taken to be $1s$ states of given width $b = (\hbar/m\omega)^{1/2}$

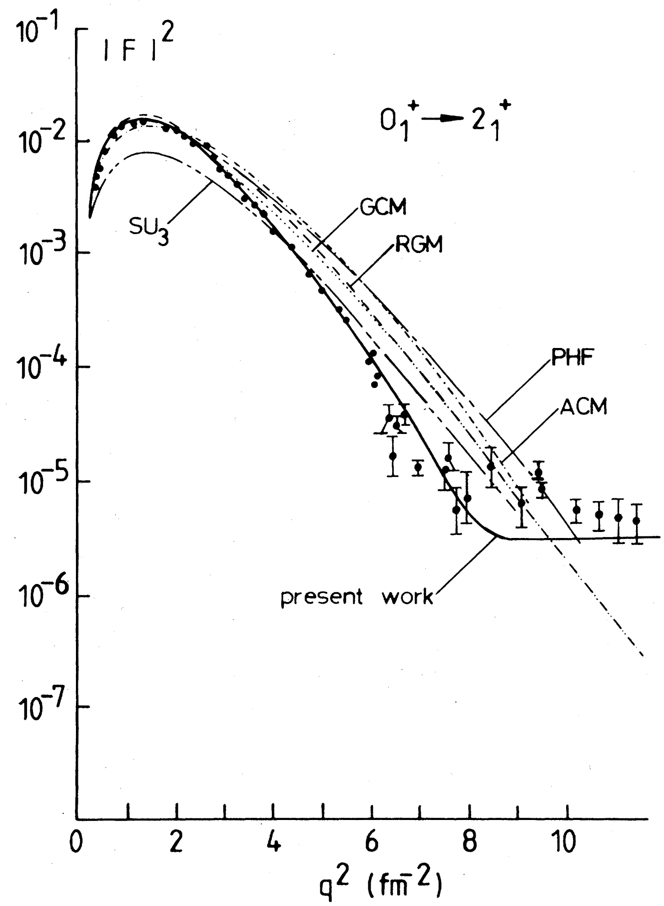

FIG. 1. Charge form factor $|F|^2$ of the $0_1^+ \rightarrow 2_1^+$ transition in ^{12}C as a function of q^2, the square of the momentum transfer. The result of the present work is shown in comparison with the experimental data and the results of previous calculations (taken from Refs. 1 and 2). For the experimental longitudinal form factor see Ref. 17.

with quartet symmetry that are centered around given points $\vec{R}_1, \vec{R}_2, \vec{R}_3$,

$$\phi_i(\vec{x}_i) = (b\sqrt{\pi})^{-3/2}\exp[-(\vec{x}_i-\vec{R}_j)^2/2b^2]\chi_i \tag{3}$$

$$(i=1,\ldots,12;\quad j=1,\ldots,3)\ .$$

Here, i labels the nucleons and j the alpha cluster centers, and χ_i denotes the spin and isospin state of the nucleon. The antisymmetrized ACM state of the whole system is then given by the Slater determinant $|\vec{R}_1,\vec{R}_2,\vec{R}_3,b\rangle$ of the (in general nonorthogonal) single-nucleon states.[2,8] The cluster positions $\vec{R}_1,\vec{R}_2,\vec{R}_3$ and the oscillator constant b are the variational parameters of the system.

As the ACM many-body wave functions have in general neither good parity nor good angular momentum numerical parity and angular-momentum projections are required. The latter is performed by using the Peierls-Yoccoz projection operator

$$P^J_{KM}|\phi(\vec{R}_j)\rangle = \frac{2J+1}{8\pi^2}\int_0^{\pi} d\beta\,\sin\beta\, d^J_{KM}(\beta)\int_0^{2\pi} d\alpha\int_0^{2\pi} d\gamma\, e^{i(M\alpha+K\gamma)}|\phi[R_3(\alpha)R_2(\beta)R_3(\gamma)\vec{R}_j]\rangle\ . \tag{4}$$

With the standard Hamiltonians of the Volkov,[9] Brink-Boeker,[10] or Skyrme type, a variation after projection leads to minimum-energy states where $\vec{R}_1$, $\vec{R}_2$, and $\vec{R}_3$ form an equilateral triangle for both the $J^\pi=0^+$ and 2^+ result.[3,11-15] Only ACM states with such a triangular symmetry and some excited states with a linear configuration have been considered in this Brief Report. They can be specified by the two variational parameters R and b. In addition, we have studied GCM states that are a superposition of triangular ACM states

$$|\phi\rangle \sim \int |R,b\rangle g(R)\,dR. \tag{5}$$

Since the elastic form factor is known to fit the experimental data sufficiently accurately[1-5,16] no attempt has been made to improve the initial state in the $0_1^+\to 2_1^+$ reduced transition matrix element of Eq. (1). All searches for a suitable ACM wave function for the 2^+ state failed. Any changes in the parameter values R or b (regardless of the energy) have led to a $0_1^+\to 2_1^+$ form factor that fits at best the high or low-momenta part but not both regions simultaneously. It turned out that the experimental curve can be fitted over the entire range by a 2^+ state that consists of a superposition of the original ACM state[2] ($R=1.4434$ fm) and a wider ($R=2.0$ fm) triangular ACM state with the same oscillator constant ($b=1.4$ fm) (see Fig. 2).

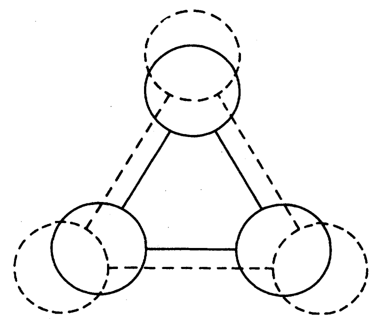

FIG. 2. Schematic picture of the superposition of ACM states which (after antisymmetrization, parity- and angular-momentum projection) fit the form factor. The overlap between the two states is 62%.

The result is plotted in Fig. 1 in comparison with the results of previous calculations. The only noticeable deviations occur at about $q^2=7$ and 9 fm^{-2} where the experimental curve appears to have some additional structure although the error bars are relatively large in these areas. It is conceivable that a more refined GCM state consisting of more than two ACM states with a finer generator coordinate mesh would further improve the agreement. This has not been studied as the data are rather sparse and the deviation is small and may even be insignificant. Apart from that the calculated and experimental form factors agree almost within the error bars even in the region of medium- and high-momentum transfer where all previous calculations failed.

The authors are grateful to Professor A. Faessler, Dr. W. Reuter and Professor G. Wagner for helpful discussions. One of us (M.R.) would also like to thank the Theory Group of the Tübingen University for their hospitality and the Kernforschungsanlage Jülich for their support.

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