

Structures of $N = 88$ and $N = 90$ isotones in the interacting boson approximation

D. S. Chuu*

Department of Physics, State University of New York at Buffalo, Amherst, New York 14260

C. S. Han

Department of Electrophysics, National Chiao Tung University, Hsinchu, Taiwan, Republic of China

S. T. Hsieh

Department of Physics, National Tsing Hua University, Hsinchu, Taiwan, Republic of China

M. M. King Yen

Department of Nuclear Engineering, National Tsing Hua University, Hsinchu, Taiwan, Republic of China

(Received 5 March 1984)

Effective interacting boson approximation Hamiltonians are used to describe the energy spectra of even-even nuclei for $N = 88$ and $N = 90$ isotones. For each N , a single effective interacting boson approximation Hamiltonian reproduced energy levels very well. A unified $E2$ transition operator is searched to reproduce the observed $B(E2)$ values. The quadrupole moments of the first 2^+ level and the two-proton separation energy in each case are calculated. Satisfactory agreements are obtained.

I. INTRODUCTION

The interacting boson approximation (IBA), initially introduced by Arima and Iachello,¹⁻⁷ has been rather successful in describing the collective properties of several medium and heavy nuclei. In the first approximation, only pairs with angular momentum $L = 0$ (called s boson) and $L = 2$ (called d boson) are considered. This model has associated with it an inherent group structure. In spite of its simplicity, it is capable of providing a beautiful theoretical explanation of the observed spectra exhibited by many nuclei.

In recent years, several systematic studies⁸⁻¹⁹ of even-even nuclei have been performed within the framework of the IBA model. A phenomenological analysis¹³ for even Sm isotopes used a mass dependent boson Hamiltonian and a unified effective charge to reproduce the low-lying spectra and the intra-ground band $B(E2)$ behavior of the chain of even Sm isotopes. On the contrary, Castanos *et al.*¹⁷ showed the possibility of providing accurate fits to the low-lying spectra of series of even-even heavy isotopes without any explicit boson number dependence in the Hamiltonian. A more recent work¹⁹ found that it

would seem to be unlikely to obtain a unified Hamiltonian as well as a unified $E2$ transition operator in passing from the SU(5) limit to the SU(3) limit, at least within the IBA-I model, where one does not distinguish neutron bosons from proton bosons. Since all of these previous works are performed for a series of isotopes, it is interesting to see what the result is of a study of a series of isotones.

The purpose of this work is twofold. First we want to present systematic calculations of the $N = 88$ and 90 isotones within the framework of the IBA model. Second, and most importantly, we want to investigate whether it is possible to have a unified Hamiltonian and an $E2$ transition operator for the isotones which have spectra intermediate between the SU(5) limit and the SU(3) limit.

II. THE MODEL

For both the $N = 88$ and $N = 90$ isotone series, the closed shell at $Z = 50$ and $N = 82$ is taken to be inert. Extra core nucleons are treated as active bosons. The two-body effective Hamiltonians between bosons can have at least two forms¹⁵:

$$\begin{aligned}
 H = & \epsilon_s (s^\dagger \cdot \bar{s}) + \epsilon_d (d^\dagger \cdot \bar{d}) + \sum_{L=0,2,4} \frac{1}{2} (2L+1)^{1/2} c_L \{ [d^\dagger \times d^\dagger]^{(L)} \times [\bar{d} \times \bar{d}]^{(L)} \}^{(0)} \\
 & + \frac{1}{2^{1/2}} \bar{v}_2 \{ [d^\dagger \times d^\dagger]^{(2)} \times [\bar{d} \times \bar{s}]^{(2)} + [d^\dagger \times s^\dagger]^{(2)} \times [\bar{d} \times \bar{d}]^{(2)} \}^{(0)} \\
 & + \frac{1}{2} \bar{v}_0 \{ [d^\dagger \times d^\dagger]^{(0)} \times [\bar{s} \times \bar{s}]^{(0)} + [s^\dagger \times s^\dagger]^{(0)} \times [\bar{d} \times \bar{d}]^{(0)} \}^{(0)} \\
 & + u_2 \{ [d^\dagger \times s^\dagger]^{(2)} \times [\bar{d} \times \bar{s}]^{(2)} \}^{(0)} + \frac{1}{2} u_0 \{ [s^\dagger \times s^\dagger]^{(0)} \times [\bar{s} \times \bar{s}]^{(0)} \}^{(0)}, \tag{2.1}
 \end{aligned}$$

TABLE I. Parameters of the IBA Hamiltonian, in MeV.

	ϵ	c_0	c_2	c_4	u_0	u_2	\tilde{v}_2	\tilde{v}_0
$N=88$	0.239	-0.161	0.107	0.061	0.0	0.039	0.002	-0.063
$N=90$	0.077	0.025	-0.040	0.062	0.0	0.086	-0.077	-0.122

where ϵ_s and ϵ_d are the single-boson energies, and c_L , \tilde{v}_L , and u_L describe the two-boson interaction. Also,

$$H' = \epsilon'' n_d + a_0 (P^\dagger \cdot P) + a_1 (L \cdot L) + a_2 (Q \cdot Q) + a_3 (T_3 \cdot T_3) + a_4 (T_4 \cdot T_4), \quad (2.2)$$

where

$$\begin{aligned} n_d &= (d^\dagger \cdot \tilde{d}), \\ P &= \frac{1}{2} (\tilde{d} \cdot \tilde{d}) - \frac{1}{2} (\tilde{s} \cdot \tilde{s}), \\ L &= \sqrt{10} [d^\dagger \times \tilde{d}]^{(1)}, \\ Q &= [d^\dagger \times \tilde{s} + s^\dagger \times \tilde{d}]^{(2)} - \frac{1}{2} \sqrt{7} [d^\dagger \times \tilde{d}]^{(2)}, \\ T_3 &= [d^\dagger \times \tilde{d}]^{(3)}, \\ T_4 &= [d^\dagger \times \tilde{d}]^{(4)}. \end{aligned}$$

A least squares search for the nine parameters in Eq. (2.1) and one for the six parameters in Eq. (2.2) are carried out to fit the observed energy spectra. All levels with reliable spin assignments are included up to the point that the first level with an uncertain spin assignment appears. A total of 68 levels for $N=88$ isotones and 85 levels for $N=90$ isotones are included in the actual calculations.

In fitting the energy spectra of $N=88$ isotones, we first determine the six parameters of H' that best reproduce the ground state (g.s.) band excitations. This set is in turn used as an initial set for fitting all the energy levels. The overall rms deviation for the final fit is 116 keV with the following parameters (in MeV): $\epsilon''=0.21079$, $a_0=-0.00263$, $a_1=0.00441$, $a_2=0.00323$, $a_3=0.04534$, and $a_4=0.02304$. We also search for the nine parameters of H that can fit the energy spectra of $N=88$ isotones. The rms deviation of the final best fit is 106 keV. The

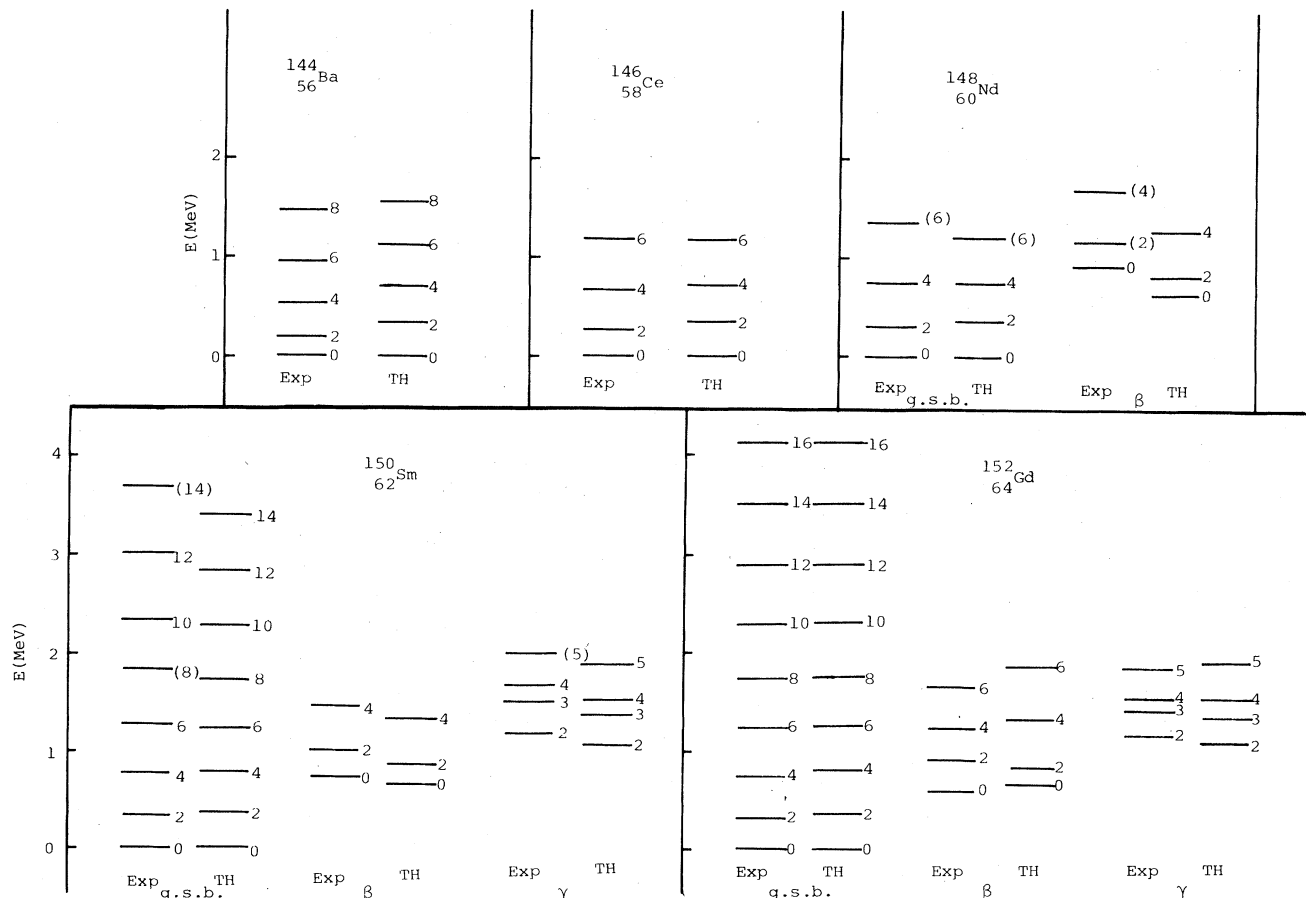


FIG. 1. Experimental and calculated energy spectra for isotones ^{144}Ba , ^{146}Ce , ^{148}Nd , ^{150}Sm , and ^{152}Gd .

same procedure is also carried out for $N=90$ isotones. The overall rms deviation for 85 levels is 147 keV for H' and 103 keV for H .

Table I lists the parameters obtained in the final search. Since we are concerned with excitations only, the effect of ϵ_s can be absorbed in ϵ_d . Therefore we list the single boson energy difference $\epsilon = \epsilon_d - \epsilon_s$ instead. The parameter u_0 is kept at zero because it can be absorbed into the parameters ϵ and c_L .⁸ Besides, our calculation also shows that the result is rather insensitive to u_0 . Thus we are left with seven parameters in reality. The single boson energy ϵ for $N=88$ isotones is three times larger than that for $N=90$ isotones. This is because the energies of the first 2^+ states of $N=88$ isotones are almost three times higher than those of $N=90$ isotones. Since the single boson energy plays an important role in fitting the energy spectra, this great difference in ϵ makes the other interaction parameters differ quite a bit for the two series.

III. ENERGY SPECTRA

Eight even-even isotones with $144 \leq A \leq 158$ and $148 \leq A \leq 160$ are considered for $N=88$ and $N=90$,

respectively. We present the results calculated with the Hamiltonian H for both series of isotones.

A. The $N=88$ isotones

Figure 1 shows the comparison of the experimental and calculated spectra for lighter mass $N=88$ isotones. Levels with spin values enclosed in parentheses are not included in the least squares fit. In general, the fit is quite good. The worst fit is the first excited 0^+ level at 917 keV of the ^{148}Nd nucleus which yields a discrepancy of 292 keV. For the nucleus ^{150}Sm the asymmetric rotor model²⁰ with a different nonaxial parameter (γ) has been employed to explain known individual $B(E2)$ values and the low lying energy level. Our calculated levels in the g.s. band agree well with the observed ones, especially the low lying states with spins 2, 4, and 6. Since we have included more high spin states in the calculation, our results seem to be more reasonable. The excellent agreement of the high spin states in ^{152}Gd is also very impressive. The low-lying levels of ^{152}Gd show many similarities to those of the isotone ^{150}Sm . Almost every level with an assigned

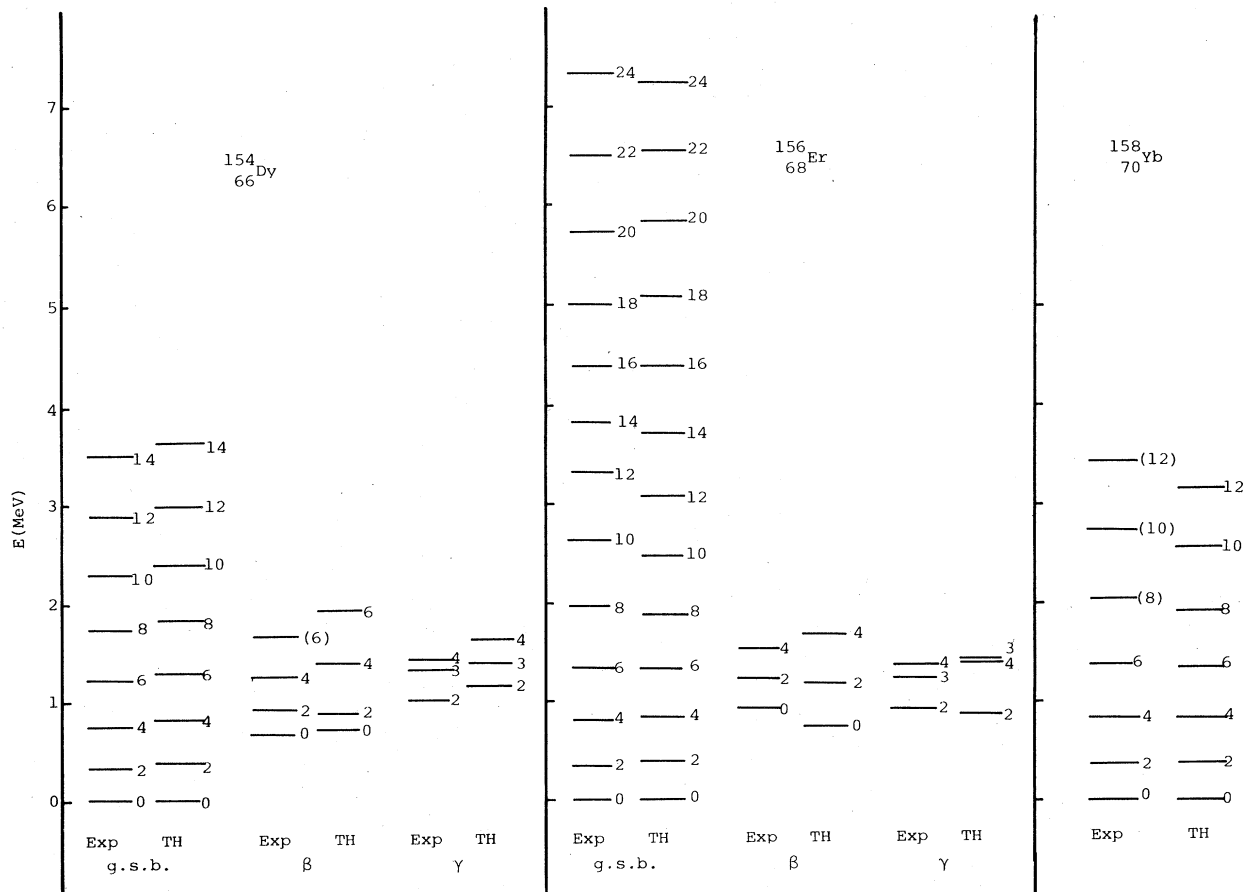


FIG. 2. Experimental and calculated energy spectra for isotones ^{154}Dy , ^{156}Er , and ^{158}Yb .

spin and parity has a partner in ^{150}Sm with an excitation energy differing less than 100–150 keV from that in ^{152}Gd .

Figure 2 lists the calculated levels and their experimental values for the other three heavier $N=88$ isotones. The levels of ^{154}Dy have been studied before²¹ using the $^{146}\text{Nd}(^{12}\text{C}, xn, \gamma)$ reaction. Our theoretical predictions for the energy spectra of ^{154}Dy are consistent with the previously proposed level scheme. For the ^{156}Er nucleus, the predictions for energy levels in the g.s., β , and γ bands are in good agreement with the observed ones. A second side band with odd spin and even parity states up to 21^+ in the nuclei ^{156}Er has been observed previously.^{22,23} However, our calculations (not shown in the figure) cannot reproduce these levels reasonably. Earlier works²⁴ have been performed to predict the energy levels of ^{156}Er by particle rotor models.^{25,26} These calculations reproduced the g.s. band well, however, very poor agreement was obtained for the states with odd spin and even parity.

The wave functions for $N=88$ isotones show some regularities. In general, for any nucleus, the wave functions are more dispersive for smaller spin states, and the wave functions for odd spin states are more concentrated than those for even spin states. For a particular spin state

in the $N=88$ isotone series, the wave functions are more dispersive for heavier isotones. For levels with the same spin the wave functions are more concentrated for higher excitation energy levels, except for small spins like 0, 2, and 3, for which the higher the excitation, the more dispersive the wave function.

B. The $N=90$ isotones

Figure 3 shows the comparison of the experimental spectra and their theoretical counterparts for lighter $N=90$ isotones. In general, the fitting is quite good, except the 3_1^+ state of the ^{152}Sm nucleus, which is 291 keV lower than the observed one. The level structure of ^{152}Sm has been studied²⁸ with various spectroscopic techniques using the $(\alpha, 2n\gamma)$ reaction. Rotational bands were identified up to spin 14. These high spin states are reproduced quite well. Van Isacker *et al.*¹⁸ presented an extended version of the IBA model to the even-even Gd isotopes. Besides s and d bosons, they also considered s' , d' , and g bosons as elementary building blocks. Even though they included these complicated refinements, their theoretical

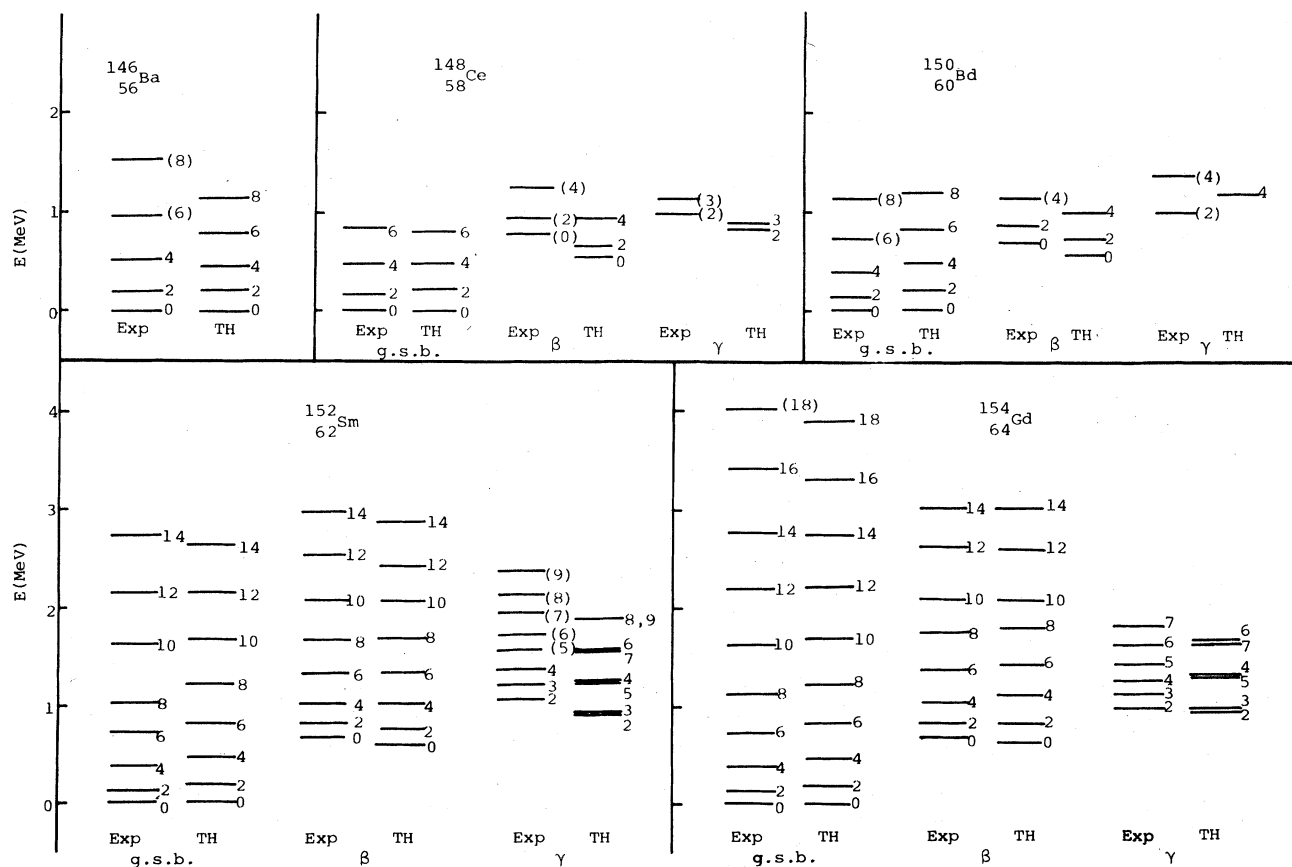


FIG. 3. Experimental and calculated energy spectra for isotones ^{146}Ba , ^{148}Ce , ^{150}Nd , ^{152}Sm , and ^{154}Gd .

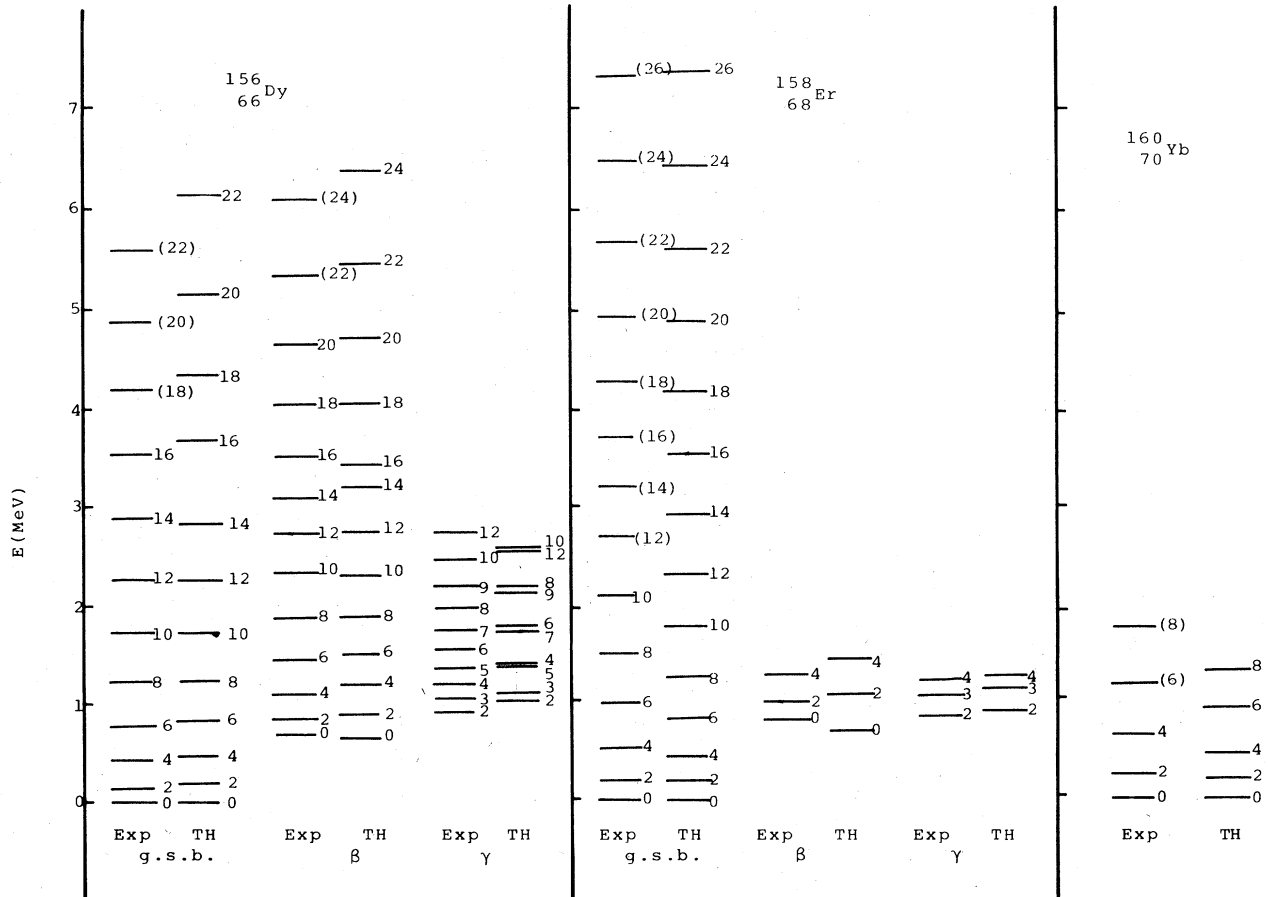


FIG. 4. Experimental and calculated energy spectra for isotones ^{156}Dy , ^{158}Er , and ^{160}Yb .

predictions for the energy levels of the ground, β , and γ bands in ^{154}Gd are almost as good as ours. However, the second excited $K^\pi=0^+$ band and the $K^\pi=4^+$ band can be explained in their calculation as arising from the coupling of an s' boson and a g boson.

Figure 4 presents the calculated levels and their experimental counterparts for three other heavier isotones with $N=90$. In general, the fitting is consistent with the observed values. High spin states of ^{156}Dy have been identified²⁷ by the inverse reaction $^{24}\text{Mg}(^{136}\text{Xe}, 4n)^{156}\text{Dy}$. A super band up to spin 30 and a β band up to spin 26 are identified. In our model, we have only 12 valence bosons which can pair to a maximum total spin of 24. For the ^{158}Er nucleus, the energy levels are also identified up to spin 32. We have 13 active bosons for the ^{158}Er nucleus, which can pair to a maximum spin of only 26. In spite of the simplification of our model, we are able to predict the energy spectra with spins up to 26. To make a full description of the energy spectra for ^{156}Dy and ^{158}Er , one has to include the core excitation or higher angular momentum bosons (e.g., g bosons) in the IBA model.

The wave functions for $N=90$ isotones are more

dispersive than those for $N=88$ isotones. Most of the levels in $N=90$ isotones possess dominant components with an intensity of roughly 60%. In general, for levels with the same spin value, the wave functions are more dispersive for the levels with lower excitation energies.

IV. $E2$ transitions and quadrupole moments

In order to investigate how the Hamiltonian reflects other physical properties of the nuclear system, we employ the wave functions obtained from the diagonalization of H to calculate the $E2$ transitions and quadrupole moments of 2_1^+ states.

To be consistent with the analysis of energies, we have used the more general $E2$ operator:

$$T^{(2)} = \alpha(d^\dagger s + s^\dagger \tilde{d})^{(2)} + \beta(d^\dagger \tilde{d})^{(2)}.$$

The quadrupole moments for the L^+ state are defined as

$$Q_{L^+} = [16\pi/5(2L+1)]^{1/2} (LL20 | LL)(L || T^{(2)} || L).$$

For $L=2$, we have

$$Q_{2+} = \left[\frac{32\pi}{175} \right]^{1/2} (2||T^{(2)}||2).$$

A. For $N=88$ isotones

In order to match the experimental $B(E2)$ values, we first searched for the suitable values of α which can best reproduce the observed $B(E2)$ values within the ground band. It is found that the values of α for each nucleus

TABLE II. $B(E2)$ values (in e^2b^2) and their branching ratios in $N=88$ nuclei.

Nucleus	$J_i \rightarrow J_f$	Expt.	$B(E2)$ values in e^2b^2		
			This work	Previous works	
^{144}Ba	$2_1 \rightarrow 0_1$	0.314 ^a	0.20	0.454 ^a	
^{146}Ce	$2_1 \rightarrow 0_1$	0.157 ^a	0.24	0.341 ^a	
^{148}Nd	$2_1 \rightarrow 0_1$	0.258 ^b	0.29		
^{150}Sm	$2_1 \rightarrow 0_1$	0.274 ^c	0.34	0.275 ^c	0.41 ^d
	$4_1 \rightarrow 2_1$	0.53 ^a	0.54	0.51 ^c	0.73 ^d
	$4_1 \rightarrow 2_2$	0.0106 ^c	0.0104	0.139 ^c	0.18 ^d
	$2_2 \rightarrow 0_1$	0.0036 ^c	0.006	0.02 ^c	0.008 ^d
	$2_3 \rightarrow 0_1$	0.0088 ^c	0.0004	0.02 ^c	0.015 ^d
	$2_2 \rightarrow 2_1$	0.27 ^c	0.54	0.181 ^c	0.12 ^d
	$2_3 \rightarrow 2_1$	0.0387 ^c	0.002	0.024 ^c	0.029 ^d
	$2_2 \rightarrow 0_1/2_1$	0.084 ^c	0.011	0.11 ^c	0.068 ^d
	$2_2 \rightarrow 4_1/2_1$	0.5 ^c	0.015	0.43 ^c	0.85 ^d
	$3_1 \rightarrow 2_1/4_1$	0.29 ^c	0.09	1.09 ^c	0.54 ^e
	$4_2 \rightarrow 2_1/4_1$	0.05 ^c	0.023	0.056 ^c	0.0054 ^d
	$4_2 \rightarrow 3_1/2_2$	3.7 ^c	0.10	0.53 ^c	2.13 ^e
$3_1 \rightarrow 2_2/2_1$	24 ± 5 ^c	32.15	4.34 ^c	16.8 ^e	
^{152}Gd	$2_1 \rightarrow 0_1$	0.33 ^f	0.40	0.33 ^g	0.33 ^h
	$2_1 \rightarrow 0_2$	0.17 ± 0.04 ^f	0.072	0.625 ^g	0.74 ^h
	$4_1 \rightarrow 2_1$	0.64 ^f	0.65	0.62 ^g	0.69 ^h
	$4_1 \rightarrow 2_2$	0.096 ^f	0.011	0.06 ^g	
	$6_1 \rightarrow 4_1$	0.95 ^f	0.79	0.76 ^g	
	$2_2 \rightarrow 2_1$	0.077 ^f	0.64	0.164 ^g	0.006 ^h
	$2_2 \rightarrow 0_1$	0.0014 ^f	0.0072	0.026 ^g	0.009 ^h
	$2_2 \rightarrow 0_2$	0.21 ^f	0.024	0.21 ^g	
	$2_2 \rightarrow 0_2/2_1$	< 1.6 ⁱ	0.04		
	$2_2 \rightarrow 0_1/2_1$	0.021 ⁱ	0.011		
	$4_2 \rightarrow 2_2/4_1$	6.0 ± 1.5 ⁱ	1.11		
$6_2 \rightarrow 4_2/6_1$	4.4 ± 4.8 ⁱ	2.11			
^{154}Dy	$2_2 \rightarrow 0_1/2_1$	0.015 ⁱ	0.011		
	$4_2 \rightarrow 2_2/4_1$	5.1 ± 0.5 ⁱ	1.11		
	$6_2 \rightarrow 4_2/6_1$	14 ± 2 ⁱ	2.16		
^{156}Er	$2_1 \rightarrow 0_1$	0.33 ^j	0.52	0.33 ^j	
	$4_1 \rightarrow 2_1$	0.53 ^j	0.86	0.66 ^j	
	$6_1 \rightarrow 4_1$	1.03 ^j	1.23	0.99 ^j	

^aReference 30.

^bReference 31.

^cReference 32.

^dReference 33.

^eReference 20.

^fReference 34.

^gReference 42.

^hReference 35.

ⁱReference 36.

^jReference 37.

vary from a minimum value of 0.117 for ^{146}Ce to a maximum value of 0.184 for ^{144}Ba if we keep the ratio $\beta/\alpha = -1.323$ as used by Scholten *et al.*¹³ We adopt an average value of $\alpha = 0.145$ for all isotones and vary β to calculate the $B(E2)$ values including the intra-ground band and those for cross band transitions. Table II lists the $B(E2)$ values and their branching ratios for $N = 88$ isotones using $\alpha = 0.145$ and $\beta = -0.15$. Previous calculations are also included for comparison. In general, our predictions are quite good in comparison with the observed and other theoretical values, especially for the tran-

sition within the ground band. For the cross band transitions the agreement is also satisfactory.

We also calculated the quadrupole moments for 2_1^+ states. Only two pieces of experimental data^{31,38} are available. Our predicted values of Q_{2^+} yield -0.41 e b for ^{148}Nd and -0.43 e b for ^{150}Sm nuclei, which are much smaller than the observed values of -1.36 e b and -1.31 e b, respectively. This has to do with the fact that the two parameters α and β in the quadrupole operator are determined from fitting the intra-ground band and cross band $B(E2)$ transitions. Also we have used a unified Hamil-

TABLE III. $B(E2)$ values (in e^2b^2) in $N = 90$ nuclei.

Nucleus	$J_i \rightarrow J_f$	Expt.	$B(E2)$ values in e^2b^2	
			This work	Previous works
^{146}Ba	$2_1 \rightarrow 0_1$	0.277 ^a	0.43	0.506 ^a
^{148}Ce	$2_1 \rightarrow 0_1$	0.445 ^a	0.52	0.578 ^a
^{150}Nd	$2_1 \rightarrow 0_1$	0.53 ^b	0.63	
^{152}Sm	$2_1 \rightarrow 0_1$	0.67 ^c	0.75	0.673 ^c
	$4_1 \rightarrow 2_1$	1.017 ^c	1.00	0.98 ^c
	$6_1 \rightarrow 4_1$	1.2 ^d	0.97	1.193 ^c
	$8_1 \rightarrow 6_1$	1.39 ^d	0.83	1.373 ^c
^{154}Gd	$2_1 \rightarrow 0_1$	0.773 ^f	0.87	0.773 ^f
	$4_1 \rightarrow 2_1$	1.178 ^f	1.18	0.110 ^f
	$6_1 \rightarrow 4_1$	1.39 ^f	1.17	0.119 ^f
^{156}Dy	$2_1 \rightarrow 0_1$	3.79 ^g	1.01	
	$4_2 \rightarrow 2_2$	1.29 ^h	0.75	
	$6_2 \rightarrow 4_2$	1.42 ^h	1.07	
	$8_2 \rightarrow 6_2$	1.49 ^h	1.01	
	$10_2 \rightarrow 8_2$	1.53 ^h	0.81	
	$12_2 \rightarrow 10_2$	1.56 ^h	0.56	
	$14_2 \rightarrow 12_2$	1.58 ^h	0.27	
$16_1 \rightarrow 14_1$	0.34 ^h	0.39		
^{158}Er	$2_1 \rightarrow 0_1$	0.55 ⁱ	1.16	0.55 ⁱ
	$4_1 \rightarrow 2_1$	0.87 ⁱ	1.59	1.10 ⁱ
	$6_1 \rightarrow 4_1$	1.14 ⁱ	1.63	1.65 ⁱ
	$8_1 \rightarrow 6_1$	1.16 ⁱ	1.53	2.20 ⁱ
	$10_1 \rightarrow 8_1$	1.12 ± 0.5^j	1.35	1.20 ± 0.56^j
	$12_1 \rightarrow 10_1$	$> 0.9^j$	1.12	$\geq 1^j$
	$14_1 \rightarrow 12_1$	0.77 ± 0.18^j	0.87	0.8 ± 0.19^j
	$16_1 \rightarrow 14_1$	1.35 ± 0.48^j	0.61	1.4 ± 0.35^j
	$18_1 \rightarrow 16_1$	$> 0.6^j$	0.35	$> 0.6^j$

^aReference 30.

^bReference 39.

^cReference 32.

^dReference 40.

^eReference 41.

^fReference 18.

^gReference 42.

^hReference 43.

ⁱReference 37.

^jReference 44.

TABLE IV. Quadrupole moments Q_{2+} (in $e b$) for $N=90$ isotones.

Nucleus	Q_{2+} in $e b$		
	Expt.	This work	Theo. Previous work
^{150}Nd	-2.0^a	-1.72	-1.475^a
^{152}Sm	-1.67^b	-1.95	-1.693^b
^{154}Gd	-1.82^c	-2.19	

^aReference 45.

^bReference 46.

^cReference 47.

tonian and $E2$ operator in the calculation. This is different from the previous calculation,¹³ where a mass-dependent interaction is used. A better Q_{2+} value can be obtained by using some larger values of α and β , which of course will change the results of $B(E2)$.

B. For $N=90$ isotones

The same procedures have also been applied to the $N=90$ isotones. The value of α and of β which can best reproduce the observed $B(E2)$ values for transitions within ground and within β bands is 0.226 and -0.240 , respectively. Table III lists the calculated and observed $B(E2)$ values; other theoretical values are also included for comparison. We have also calculated the transitions between different bands; the result is not as satisfactory as in the case within the same band.

Table IV lists the quadrupole moments for the first 2^+ states in the nuclei ^{150}Nd , ^{152}Sm , and ^{154}Gd . The monopole and quadrupole charge distributions of ^{154}Gd were investigated recently⁴⁷ by muonic atom K and L x-ray measurements. Our theoretical prediction for Q_{2+} of ^{154}Gd is in reasonable agreement with that observation.

V. Two-proton separation energy

We have also calculated other properties of the isotone chains discussed above. Rather than directly discuss the

binding energies, it is more convenient to investigate the two proton separation energies, which are related to the binding energies by⁴⁸

$$S_{2\pi} = E_B(Z, N) - E_B(Z - 2, N);$$

the binding energy for a fixed neutron number can be written as

$$E_B(Z, N) = E^{(c)} + A_{\pi} N_{\pi} + B_{\pi} \frac{1}{2} N_{\pi} (N_{\pi} - 1) + E_0(N_{\pi}),$$

where N_{π} is the number of proton bosons in the valence shell. The constant $E^{(c)}$, and A_{π} and B_{π} are characteristic for each major shell, and E_0 is the contribution to the binding energy due to deformation. Using the above equations, we calculated the separation energies with $A_{\pi} = 24.58$ MeV and $B_{\pi} = -2.05$ MeV for $N=88$ isotones, and $A_{\pi} = 36.76$ MeV and $B_{\pi} = -2.1$ MeV for $N=90$ isotones. Table V lists comparisons of the experimental and calculated separation energies for two protons. The agreement is excellent.

VI. DISCUSSION AND CONCLUSIONS

In this paper we report effective Hamiltonian IBA calculations for two series of isotones with $N=88$ and $N=90$, respectively. We find that accurate fits to the energy spectra can be obtained by using an effective Hamiltonian without any explicit boson number dependence in its parameters. Castanos *et al.*¹⁷ have also obtained a single effective IBA Hamiltonian that can accurately reproduce the spectra of a whole series of isotopes. Table VI lists the renormalized parameters $\tilde{\epsilon}$ and \tilde{c}_L given by⁸

$$\tilde{\epsilon} = \epsilon - u_0(N - 1) + \frac{1}{\sqrt{5}} u_2(N - 1),$$

$$\tilde{c}_L = c_L + u_0 - \frac{2}{\sqrt{5}} u_2.$$

The columns labeled IBA contain the parameters from the calculation of Castanos *et al.* The columns labeled IBM contain the parameters projected from the IBM calculation^{8,13} of the Sm isotopes. The parameters $\tilde{\epsilon}$ and \tilde{c}_2 are similar, but our \tilde{c}_4 is smaller. Our result for \tilde{c}_0 is more

TABLE V. Two-proton separation energies for $N=88$ ($A_{\pi} = 24.58$ MeV, $B_{\pi} = -2.05$ MeV) and for $N=90$ ($A_{\pi} = 36.76$ MeV, $B_{\pi} = -2.11$ MeV).

$N=88$	$S_2(N_{\pi})$ (MeV)		$N=90$	$S_2(N_{\pi})$ (MeV)	
	Theo.	Expt.		Theo.	Expt.
^{144}Ba	20.46	20.56			
^{146}Ce	18.40	18.22	^{148}Ce	19.80	19.83
^{148}Nd	16.34	16.23	^{150}Nd	17.68	17.45
^{150}Sm	14.24	14.22	^{152}Sm	15.55	15.66
^{152}Gd	12.23	12.23	^{154}Gd	13.43	13.52
^{154}Dy	10.18	10.27	^{156}Dy	11.32	11.40
^{156}Er	8.13	8.12	^{158}Er	9.06	9.08
^{158}Yb	6.07	5.86	^{160}Yb	7.06	7.10

TABLE VI. The renormalized parameters $\tilde{\epsilon}$ and \tilde{c}_L (in MeV) for ^{150}Sm and ^{152}Gd .

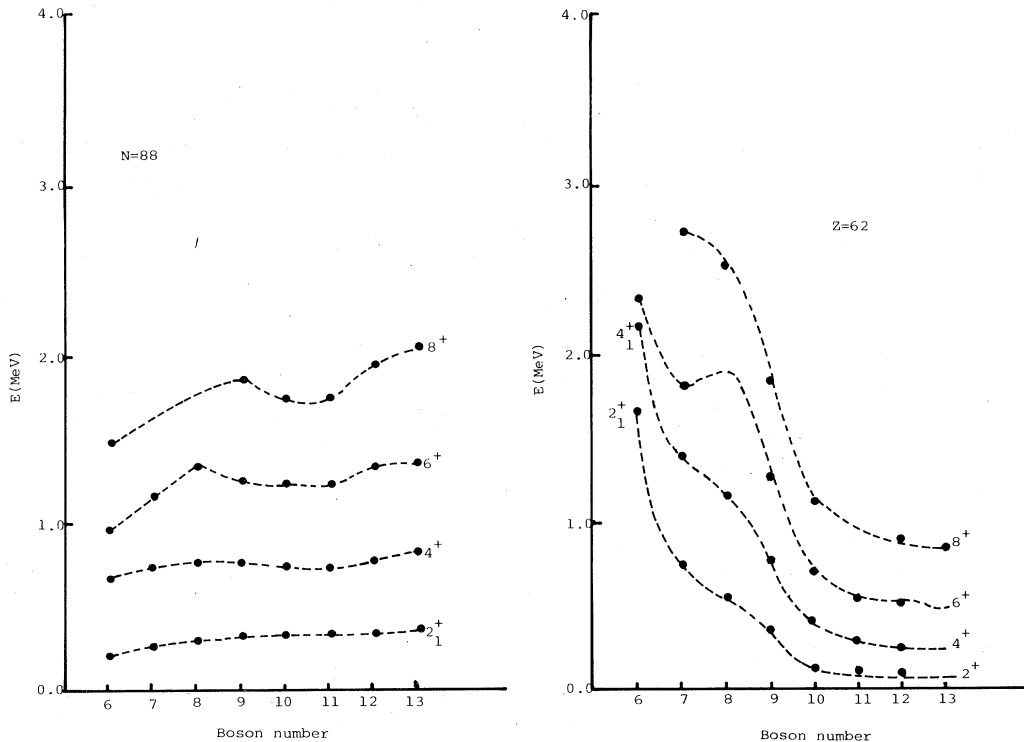
	$\tilde{\epsilon}$			\tilde{c}_0			\tilde{c}_2			\tilde{c}_4		
	This work	IBA	IBM	This work	IBA	IBM	This work	IBA	IBM	This work	IBA	IBM
^{150}Sm	0.360	0.345	0.380	-0.196	-0.156	0.38	0.072	0.128	0.110	0.026	0.076	0.068
^{152}Gd	0.360	0.353		-0.196	-0.160		0.072	0.052		0.026	0.069	

attractive than that of IBM. This discrepancy can be traced to the fact that our IBA fits include more high-spin levels up to $I=16$, whereas in IBM calculations only levels up to $I=8$ are included. From an earlier least-squares fit to the Sm isotopes, which only include spins up to 4^+ , Castanos *et al.* have found that the only significant change due to high-spin states occurs for \tilde{c}_0 .¹⁷ Our result agrees with their conclusion.

Our calculations show that the energy spectra in $N=88$ isotones can be reproduced almost equally well with either the Hamiltonian H or H' . It seems to be in contrast to a previous work,¹⁹ in which the energy spectra of samarium isotopes can be reproduced well only with Hamiltonian H . These two results are not contradictory, since for Sm isotopes, the chain of nuclei is passing from the SU(5) limit (the lighter isotopes) to the SU(3) limit (the heavier isotopes). This is very different from the $N=88$ isotones, in which all nuclei lie in a region inter-

mediate between the SU(5) and the SU(3) limit. Figure 5 presents low lying levels of g.s. bands for $N=88$ and $Z=62$ nuclei. The level structures for the g.s. band in $N=88$ isotones are almost independent of the boson numbers. On the contrary, the energies of levels with the same spin in $Z=62$ isotopes decrease drastically with increasing mass and become almost constant for heavier isotopes.

We have also calculated $E2$ transitions for our nuclear system. For $N=88$ isotones, it is found that a unified $E2$ operator not only can reproduce the transitions within the ground band, but also reasonably describes the transitions for cross bands. In $N=90$ isotones, a unified $E2$ operator is obtained within the ground and β bands. This is also in contrast to the previous work for Sm isotopes,¹⁹ in which a restricted quadrupole operator is used and the value of α is found to be mass dependent. Again, this has to do with the fact of variation of nuclear structure within isotones as discussed in the preceding paragraph.

FIG. 5. Low-lying level structures of the ground-state band for $N=88$ and $Z=62$ nuclei.

- *On sabbatical leave from National Chiao Tung University, Hsinchu, Taiwan, Republic of China.
- ¹A. Arima and F. Iachello, *Phys. Rev. Lett.* **35**, 1069 (1975).
 - ²A. Arima and F. Iachello, *Ann. Phys. (N.Y.)* **99**, 253 (1976).
 - ³A. Arima and F. Iachello, *Ann. Phys. (N.Y.)* **111**, 201 (1978).
 - ⁴A. Arima and F. Iachello, *Phys. Rev. Lett.* **40**, 385 (1978).
 - ⁵A. Arima and F. Iachello, *Ann. Phys. (N.Y.)* **123**, 468 (1977).
 - ⁶A. Arima, T. Otsuka, F. Iachello, and I. Talmi, *Phys. Lett.* **66B**, 205 (1977).
 - ⁷T. Otsuka, A. Arima, F. Iachello, and I. Talmi, *Phys. Lett.* **76B**, 139 (1978).
 - ⁸O. Scholten, Ph.D. thesis, University of Groningen, 1980.
 - ⁹O. Scholten, in *Interacting Bosons in Nuclear Physics*, edited by F. Iachello (Plenum, New York, 1979), pp. 17–35.
 - ¹⁰G. Puddu, O. Scholten, and T. Otsuka, *Nucl. Phys.* **A348**, 109 (1980).
 - ¹¹R. Bijker, A. E. L. Dieperink, O. Scholten, and R. Spanoff, *Nucl. Phys.* **A344**, 207 (1980).
 - ¹²P. D. Duval and B. R. Barrett, *Phys. Rev. C* **23**, 492 (1981).
 - ¹³O. Scholten, F. Iachello, and A. Arima, *Ann. Phys. (N.Y.)* **115**, 325 (1978).
 - ¹⁴R. F. Casten and J. A. Cizewski, *Nucl. Phys.* **A309**, 477 (1978).
 - ¹⁵A. Arima and F. Iachello, *Annu. Rev. Nucl. Part. Sci.* **31**, 75 (1981).
 - ¹⁶P. Van Isacker and G. Puddu, *Nucl. Phys.* **A348**, 125 (1980).
 - ¹⁷O. Castanos, P. Federman, A. Frank, and S. Pittel, *Nucl. Phys.* **A379**, 61 (1982).
 - ¹⁸P. Van Isacker, K. Heyde, M. Waroquire, and G. Wenes, *Nucl. Phys.* **A380**, 383 (1982).
 - ¹⁹M. M. King Yen, S. T. Hsieh, H. C. Chiang, and D. S. Chuu, *Phys. Rev. C* **29**, 688 (1984).
 - ²⁰K. K. Gupta, V. P. Varshney, and D. K. Gupta, *Phys. Rev. C* **26**, 685 (1982).
 - ²¹K. Krien, R. A. Naumann, J. O. Rasmussen, and I. Rezanka, *Nucl. Phys.* **A209**, 572 (1973).
 - ²²H. Beuscher, W. F. Davidson, R. M. Lieder, and C. Mayer-Boricke, *Z. Phys.* **263**, 201 (1973).
 - ²³A. W. Sunyar, E. Der Mateosian, O. C. Kistner, A. Johnson, A. H. Lumpkin, and P. Thieberger, *Phys. Lett.* **62B**, 283 (1976).
 - ²⁴C. Flaum and D. Cline, University of Rochester, Nuclear Structure Research Laboratory Report 103, 1975.
 - ²⁵J. Kumlinde and Z. Szymanski, *Phys. Lett.* **36B**, 157 (1971).
 - ²⁶F. S. Stephens and R. S. Simon, *Nucl. Phys.* **A183**, 257 (1972).
 - ²⁷D. Ward, H. R. Andrews, and O. Hausser, *Nucl. Phys.* **A332**, 433 (1979).
 - ²⁸J. Konijn, J. B. R. Berkhout, W. H. A. Hesselink, J. J. Van Ruijven, P. Van Nes, H. Verheul, F. W. N. De Boer, C. A. Fields, E. Sugarbaker, P. M. Walker, and R. Bijker, *Nucl. Phys.* **A373**, 397 (1982).
 - ²⁹*Table of Isotopes*, 7th ed., edited by C. M. Lederer and V. S. Shirley (Wiley, New York, 1978).
 - ³⁰J. B. Wilhelmy, S. G. Thompson, R. C. Jared, and E. Cheifetz, *Phys. Rev. Lett.* **25**, 1122 (1970).
 - ³¹P. A. Crowley, J. R. Kerns, and J. X. Saladin, *Phys. Rev. C* **3**, 2049 (1971).
 - ³²T. Tamura, K. Weeks, and T. Kishimoto, *Phys. Rev. C* **20**, 307 (1979).
 - ³³S. K. Bhardwaj, K. K. Gupta, J. B. Gupta, and D. K. Gupta, *Phys. Rev. C* **27**, 872 (1983).
 - ³⁴N. R. Johnson, I. R. Lee, F. K. McGowan, T. T. Sugihara, S. W. Yates, and M. W. Guidry, *Phys. Rev. C* **26**, 1004 (1982).
 - ³⁵K. Kumar and J. B. Gupta, *Nucl. Phys.* **A304**, 295 (1978).
 - ³⁶W. W. Bowman, T. T. Sugihara, and F. R. Hamiter, *Phys. Rev. C* **3**, 1275 (1971).
 - ³⁷R. M. Diamond, F. S. Stephens, W. H. Kelly, and D. Ward, *Phys. Rev. Lett.* **22**, 546 (1969).
 - ³⁸J. J. Simpson, D. Eccleshall, M. J. L. Yates, and N. J. Freeman, *Nucl. Phys.* **A94**, 177 (1967).
 - ³⁹H. W. Kugel, R. R. Borchers, and R. Kalish, *Nucl. Phys.* **A186**, 513 (1972).
 - ⁴⁰R. M. Diamond, F. S. Stephens, K. Nakai, and R. Nordhagen, *Phys. Rev. C* **3**, 344 (1971).
 - ⁴¹S. Davydov and V. L. Ovcharenko, *Yad. Fiz.* **3**, 1011 (1966) [*Sov. J. Nucl. Phys.* **3**, 740 (1966)].
 - ⁴²T. Grottdal, K. Nybø, T. Thorsteinsen, and B. Elbek, *Nucl. Phys.* **A110**, 385 (1968).
 - ⁴³H. R. Andrews, D. Ward, R. L. Graham, and J. S. Geiger, *Nucl. Phys.* **A219**, 141 (1974).
 - ⁴⁴D. Ward, H. R. Andrews, J. S. Geiger, and R. L. Graham, *Phys. Rev. Lett.* **30**, 493 (1973).
 - ⁴⁵H. S. Gertzman, D. Cline, H. E. Gove, and P. M. S. Lesser, *Nucl. Phys.* **A151**, 282 (1971).
 - ⁴⁶R. J. Powers, P. Barreau, B. Bihoreau, J. Miller, J. Morgenstern, J. Picard, and L. Roussel, *Nucl. Phys.* **A316**, 295 (1979).
 - ⁴⁷D. B. Laubacher, Y. Tanaka, R. M. Steffen, E. B. Shera, and M. V. Hoehn, *Phys. Rev. C* **27**, 1772 (1983).
 - ⁴⁸A. Arima and F. Iachello, *Phys. Rev. C* **16**, 2085 (1977).