Coexistence in Ge isotopes and two-neutron transfer

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Generalized two-state model wave functions are used to fit the cross section ratio $\sigma(0_2^+)/\sigma(\mathrm{g.s.})$ for two-nucleon transfer reactions, in particular, ^{70,72,74}Ge(t,p) and ^{72,74,76}Ge(p,t). We fit these ratios with just one parameter which is restricted to lie within a narrow range. We also are able to calculate the ground state and 0_2^+ state wave functions for ^{68}Ge and ^{78}Ge.

I. INTRODUCTION

In both two-neutron stripping and pickup on even isotopes of Ge, the cross section for populating the first excited state varies dramatically[1] from isotope to isotope. The excitation energy[2] of this excited 0^+ state also varies considerably. These effects are understood in a qualitative way, but attempts to reproduce them quantitatively within a simple model have met with very limited success. The most recent attempt,[3] using the interacting boson model (IBM), reproduces the trend as a function of mass number A, but detailed agreement is still absent. The aim of the present work is to identify a minimum set of assumptions necessary to produce satisfactory fits to the data.

II. EXPERIMENTAL DATA

Figure 1 displays low-lying 0^+ states[2] in the even Ge isotopes. The (t,p) and (p,t) reactions have been performed[4–10] on ^{70,72,74,76}Ge. Thus (p,t) and (t,p) can be directly compared for three cases, viz., $A_< = 70$, 72, and 74. In ^{70,72,74}Ge, we discuss results for the g.s. and the first excited 0^+ states at 1.216, 0.691 55, and 1.486 MeV, respectively. In ^{76}Ge, for reasons discussed later, we consider both excited states at 1.911 and 2.901 MeV.

The cross section ratios

$$\frac{\sigma(^{A+2}\mathrm{Ge(p,t)}^{A}\mathrm{Ge}_{0_2^+})}{\sigma(^{A+2}\mathrm{Ge(p,t)}^{A}\mathrm{Ge}_{\mathrm{g.s.}})}$$

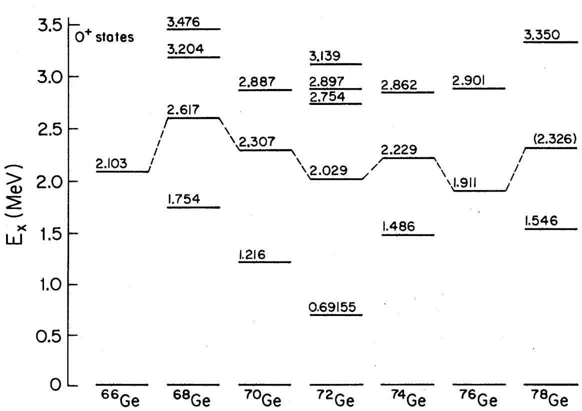

FIG. 1. Low lying 0^+ levels of even Ge isotopes.

and

$$\frac{\sigma(^{A}\mathrm{Ge(t,p)}^{A+2}\mathrm{Ge}_{0_2^+})}{\sigma(^{A}\mathrm{Ge(t,p)}^{A+2}\mathrm{Ge}_{\mathrm{g.s.}})}$$

(denoted by P_A^2 and T_A^2, respectively) are summarized in Table I. Two-nucleon transfer on a 0^+ target leading to a 0^+ residual state proceeds via an $L=0$ angular momentum transfer, and therefore forward-angle data are crucial for a careful measurement of P_A^2 and T_A^2. Measurements at the smallest angles available ($\theta_{\mathrm{c.m.}} = 5°, 4°$) are given in the second and seventh columns of Table I. Also given in the table are cross-section ratios for other angles, and angle-integrated ratios. The (t,p) cross sections were measured at the University of Pennsylvania, and so uncertainties in the numbers are readily available. On the other hand, we must estimate the uncertainties in the (p,t) numbers. Since angle-integrated cross-section ratios for $L=0$ shapes should be similar to peak cross-section ratios, we first estimate the uncertainty in P_A^2 by comparing ratios at fixed angle with ratios of angle-integrated yields. When these ratios are different, we use the average and estimate the uncertainty from the span of values, with a minimum uncertainty of 5%.

To compensate for Q-value effects, DWBA calculations were performed with the code DWUCK,[11] using optical-model parameters from Ref. 12, and a bound-state (BS) wave function of the form

$$\psi_{\mathrm{BS}} = \frac{-N}{\sqrt{2(\frac{9}{2})+1}}(1g_{9/2})^2 + \frac{N}{\sqrt{2(\frac{5}{2})+1}}(1f_{5/2})^2 + \frac{N}{\sqrt{2(\frac{3}{2})+1}}(2p_{3/2})^2 + \frac{N}{\sqrt{2(\frac{1}{2})+1}}(2p_{1/2})^2 ,$$

where $N = 0.992$ for normalization. Calculations were performed for all beam energies at which the experimental numbers were measured. Ratios of the DWBA cross sections are summarized in Table II. To investigate configuration dependencies in the DWBA calculations, we also ran the code DWUCK (Ref. 11) for bound-state wave functions of pure $(1g_{9/2})^2$ and pure $(2p_{1/2})^2$ and used these results to estimate uncertainties in the DWBA ratios. The final Q-corrected ratios, along with their uncertainties to be used in this analysis, are summarized in Table III.

TABLE I. Experimental cross section ratios for $2n$ pickup and stripping on even Ge isotopes.

	P_A^2					T_A^2	
	Forward[b]	Second maximum		Angle integrated		Forward[g]	Angle[h]
A[a]	angle	20 MeV[c]	35.4 MeV[d]	26 MeV[e]	35.4 MeV[f]	angle	integrated
70	0.068		0.052	0.071		0.002	0.0072
72	0.28	0.32	0.23	0.29	0.26	0.20	0.20
74	0.002[i]		≤ 0.016	0.016	≤ 0.008	0.05	0.045

[a] A is the target for (t,p) and the residual nucleus for (p,t).
[b] Reference 4, $E_p = 26$ MeV, $\theta = 5°$.
[c] Reference 5, $\theta = 34°$.
[d] Reference 6, $\theta = 20°$.
[e] Reference 4, $10° \leq \theta \leq 60°$.
[f] Reference 6, $10° \leq \theta \leq 45°$.
[g] References 8–10, $E_t = 15.0$ MeV, $\theta = 4°$.
[h] Reference 7, $10° \leq \theta \leq 60°$.
[i] Reference 4, $\theta = 7.5°$.

TABLE II. DWBA calculated cross section ratios for $2n$ pickup and stripping on Ge isotopes using ψ_{BS} given in the text.

	P_A^2					T_A^2	
	Forward	Second maximum		Angle integrated		Forward	Angle
A	angle	20 MeV	35.4 MeV	26 MeV	35.4 MeV	angle	integrated
70	1.004		0.960	0.870		1.023	1.064
72	1.045	0.989	1.058	0.952	0.998	1.011	1.113
74	1.137		1.152	0.931	1.012	0.966	1.127

TABLE III. Corrected ratio for Q value $=$Expt/DWBA.

	P_A^2					T_A^2	
	Forward[a]	Second maximum		Angle integrated		Forward[a]	
A	angle	20 MeV	35.4 MeV	26 MeV	35.4 MeV	angle	Integrated
70	0.068 ± 0.004		0.054	0.082		0.002 ± 0.0005	0.0068
72	0.27 ± 0.016	0.32	0.22	0.30	0.26	0.20 ± 0.016	0.18
74	0.0018 ± 0.0014		≤ 0.014	0.017	0.0079	0.052 ± 0.0057	0.040

[a] These numbers were used in our analysis.

III. THE MODEL

In the past, a two-state model for describing the ground and 0_2^+ states in the even-even Ge isotopes has had considerable success, considering the simplicity of the model. In particular, Vergnes[13] and van den Berg *et al.*[14] used the proton wave functions,

$$\psi_{\text{g.s.}}(^A\text{Ge}) = \alpha_A (p\tfrac{3}{2})_0^4 + \beta_A (p\tfrac{3}{2})_0^2 (f\tfrac{5}{2})_0^2 \,,$$
$$\psi_{0_2^+}(^A\text{Ge}) = \beta_A (p\tfrac{3}{2})_0^4 - \alpha_A (p\tfrac{3}{2})_0^2 (f\tfrac{5}{2})_0^2 \qquad (1)$$

to explain some properties in various direct reactions. In addition, Duval *et al.*[3] used the IBM and wave functions derived from a mixing Hamiltonian of the form

$$H_{\text{mix}} = \alpha (S_\pi^\dagger S_\pi' + S_\pi'^\dagger S_\pi)^{(0)} + \beta (d_\pi^\dagger \tilde{d}_\pi' + d_\pi'^\dagger \tilde{d}_\pi)^{(0)}$$

to calculate energies and electromagnetic properties of levels in the Ge isotopes. They also qualitatively reproduce the trend of the (p,t) and (t,p) cross-section ratios. Neither model, however, reproduces the cross-section ratios to any satisfactory degree of accuracy.

In this paper we generalize those results by writing

$$\psi_{\text{g.s.}}(^A\text{Ge}) = \alpha_A \varphi_g^A + \beta_A \varphi_e^A \,,$$
$$\psi_{0_2^+}(^A\text{Ge}) = \beta_A \varphi_g^A - \alpha_A \varphi_e^A \,. \qquad (2)$$

Properties of φ_g^A and and φ_e^A will emerge below. We note that 0_2^+ here refers to that excited 0^+ state of ^{A}Ge which mixes with the ground state. It usually is, but need not be, the first excited 0^+ state.

We assume simple relationships among the $2n$ transfer

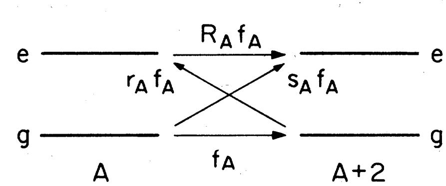

FIG. 2. Basis state definitions of r_A, R_A, and s_A.

amplitudes connecting the basis states, as depicted in Fig. 2. To be specific, we let the amplitude connecting φ_g^A and φ_g^{A+2} be f_A. We then write the amplitude for connecting φ_e^A and φ_e^{A+2} as $R_A f_A$. For the "cross" transitions $e \leftrightarrow g$, we use $r_A f_A$ and $s_A f_A$. All previous models have assumed $R_A = 1.0$ and $s_A = r_A = 0$.

With these definitions and the wave functions of Eq. (2), we obtain the following expression for the (p,t) ratios:

$$P_A^2 = \left[\frac{\chi_{A+2} + s_A - \chi_A \chi_{A+2} r_A - \chi_A R_A}{\chi_A \chi_{A+2} + \chi_{A+2} r_A + \chi_A s_A + R_A} \right]^2 . \tag{3}$$

Similarly for the (t,p) ratios

$$T_A^2 = \left[\frac{\chi_A + r_A - \chi_A \chi_{A+2} s_A - \chi_{A+2} R_A}{\chi_A \chi_{A+2} + \chi_{A+2} r_A + \chi_A s_A + R_A} \right]^2 . \tag{4}$$

Throughout, A is the nucleon number of the lighter nucleus; χ_A is α_A/β_A; and R_A, r_A, and s_A are as defined in Fig. 2. We seek first solutions in which r_A, s_A, and R_A are slowly varying functions of A. If they are independent of A we can eliminate the χ_A's from Eqs. (3) and (4), leaving us with the following result:

$$\begin{aligned}(R - rs)(R+1)^2[(R-1)^2 + (r+s)^2]\{(R-rs)[(T_{A-2}T_A - P_A P_{A-2})^2 + (T_{A-2} + T_A - P_A - P_{A-2})^2] \\ + (R+1)^2 (T_{A-2} - P_A)(P_{A-2} - T_A)\} \\ + [(R-1)^2 + (r+s)^2](s-r) \sum_{K=1}^{13} g_K(r,s,R) f_K(T_{A-2}, T_A, P_A, P_{A-2}) = 0 ,\end{aligned} \tag{5}$$

where the $g_K(r,s,R)$ are well defined polynomial functions of r,s,R, and the $f_K(T_{A-2}, T_A, P_A, P_{A-2})$ are well defined polynomial functions of $T_{A-2}, T_A, P_A, P_{A-2}$. An immediate solution of Eq. (5) occurs for $(R-1)^2 + (r+s)^2 = 0$, i.e., $R=1$, $s=-r$. However, these values would require $P_A^2 = T_A^2$ for all A, a condition not satisfied by the data. So we must dismiss this possibility and Eq. (5) becomes

$$\begin{aligned}(R-rs)(R+1)^2\{(R-rs)[(T_{A-2}T_A - P_A P_{A-2})^2 + (T_{A-2} + T_A - P_A - P_{A-2})^2] + (R+1)^2 (T_{A-2} - P_A)(P_{A-2} - T_A)\} \\ + (s-r) \sum_{K=1}^{13} g_K(r,s,R) f_K(T_{A-2}, T_A, P_A, P_{A-2}) = 0 .\end{aligned} \tag{6}$$

This equation represents a surface in r,s,R space and is still difficult to solve by hand. If, however, we make the additional reasonable assumption that $r=s$, then Eq. (6) becomes

$$(R - r^2)(R+1)^2 [R - r^2 + K_A (R+1)^2] = 0 , \tag{7}$$

where

$$K_A = \frac{(T_{A-2} - P_A)(P_{A-2} - T_A)}{(T_{A-2}T_A - P_A P_{A-2})^2 + (T_{A-2} + T_A - P_A - P_{A-2})^2} . \tag{8}$$

The roots $R - r^2 = 0$ and $R + 1 = 0$ lead to inconsistencies that are both physical and mathematical in nature, leaving the result,

$$r^2 = R + K_A (R+1)^2 . \tag{9}$$

Hence, if r and R are to be independent of A, then so must be K_A. With the available data for ^{70,72,74}Ge(t,p) and ^{72,74,76}Ge(p,t) we can calculate K_{72} and K_{74}. From Eq. (8) we see that K_A depends on linear powers of T_A and P_A, whereas the measurable quantities are T_A^2 and P_A^2. So out of all 64 possible sign combinations in the square roots, we search for a combination which gives $K_{72} = K_{74}$ within the uncertainty of the experimental numbers. More shall be said about this point in Sec. IV.

Using Eqs. (3), (4), and (9) one can show that

$$\chi_A = \frac{-r - K_A (T_A + P_A Z_A)(R+1)}{1 + K_A (1 + Z_A)(R+1)} = \frac{R + K_A (1 + Z_A)(R+1)}{-r + K_A (T_A + P_A Z_A)(R+1)} , \tag{10}$$

where

$$Z_A = \frac{T_A - P_{A-2}}{T_{A-2} - P_A} . \tag{11}$$

In addition Eqs. (3) and (4) with $s_A = r_A = r$ and $R_A = R$ leads easily to the result

$$\chi_{A+4} = \frac{(T_{A+2} - P_A) + (1 + P_A T_{A+2})\chi_A}{(1 + P_A T_{A+2}) - (T_{A+2} - P_A)\chi_A} \tag{12}$$

independently of r, s, and R. This, after much algebra, results in an additional constraint on the P_A's and T_A's given by

$$L_A = 1 , \tag{13}$$

where

$$L_A = \frac{Z_{A+2}(1 + T_A^2)}{Z_A (1 + P_A^2)} , \tag{14}$$

a result which also follows from Eq. (9) and the physical requirement that (after correcting for kinematic factors)

$$\sigma(^A\mathrm{Ge(t,p)}^{A+2}\mathrm{Ge_{g.s.}}) = \sigma(^{A+2}\mathrm{Ge(p,t)}^A\mathrm{Ge_{g.s.}}) .$$

TABLE IV. Comparison of experimental results to theory for P_A^2's and T_A^2's. Note the following: P_{74}^2 experimental value is the average between forward angle and all angle integrated data on Table III.

	Experimental		Calculated[a]	
A	P_A^2	T_A^2	P_A^2	T_A^2
70	0.068 ± 0.004	0.0020 ± 0.0005	0.068	0.0020
72	0.27 ± 0.016	0.20 ± 0.016	0.27	0.20
74	0.0090 ± 0.0072	0.052 ± 0.0057[b]	0.013	0.052[b]
	0.0090 ± 0.0072	0.025 ± 0.0034[c]	0.0039	0.025[c]

[a]For any value of R in the region $0.75 \le R \le 1.33$.
[b]Using $E_{76}=1.911$ MeV.
[c]Using $E_{76}=2.901$ MeV.

The experimental values measured do give $K_{72}=K_{74}$ and $L_{72}=1$. In addition, Eqs. (9)—(14) with K_A independent of A and $L_A=1$ gives an infinite number of solutions for α_A^2—all given in terms of just the one parameter R.

IV. APPLICATION OF THE MODEL TO GERMANIUM DATA

A. ^{70,72,74,76}Ge

Table III gives the cross-section ratios for the even germanium nuclei assuming that the excited 0^+ state which mixes with the ground state is the first excited 0^+ for each nucleus. From this table we conclude that of the 64 possible sign combinations, only four have $K_{72}=K_{74}$ and $L_{72}=1$. They are, for P_{70}, P_{72}, P_{74}, T_{70}, T_{72}, and T_{74}:

$$---+++\ ,$$

$$-+--+-\ ,$$

$$+-++-+\ ,$$

$$+++---\ .$$

The first and fourth have $K_{72}=-0.2448\pm0.0009$, $K_{74}=-0.2432\pm0.0037$, while the other two give

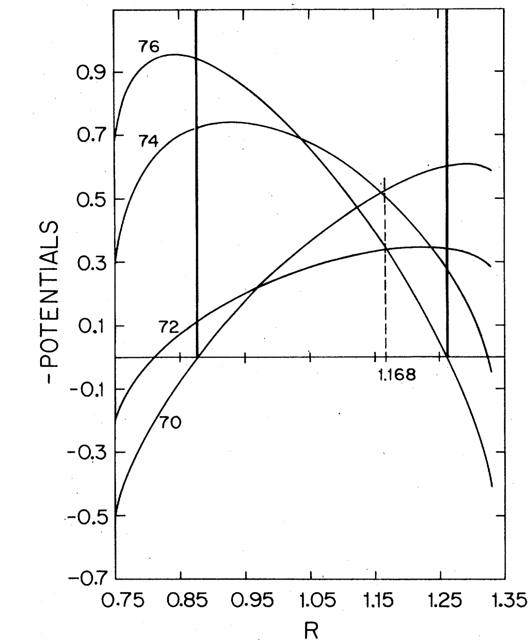

FIG. 4. Plot of $-V_A$ (in MeV) vs R with the physical region where $-V_A>0$.

$K_{72}=11.75\pm2.69$, $K_{74}=9.01\pm5.04$. All four have $L_{72}=1$. Only the small value of K_A contains previous models as a special case. The larger value of K_A requires $r^2=48$ when $R=1$, which we reject as unreasonable. With the small value of K_A, r^2 is 0.02 when $R=1$.

For both remaining sign combinations, r can be positive or negative. Changing all signs, including that of r, merely changes the relative sign of all the α_A/β_A. Hence, these two pairs are equivalent, leaving only two. And they are related by $R\to1/R$, $\varphi_e \leftrightarrow \varphi_g$. So, in the end, we have only one set of input data.

Equation (9) gives the relationship between r and R. It is elliptic for negative K_A with $0.75 \le R \le 1.33$. There exist an infinite number of solutions for α_A^2, the coefficient of φ_g^A in Eq. (2), but they can all be described in terms of the one parameter R. Table IV gives the results of these fits. Figure 3 gives a graph of these solutions vs R. We

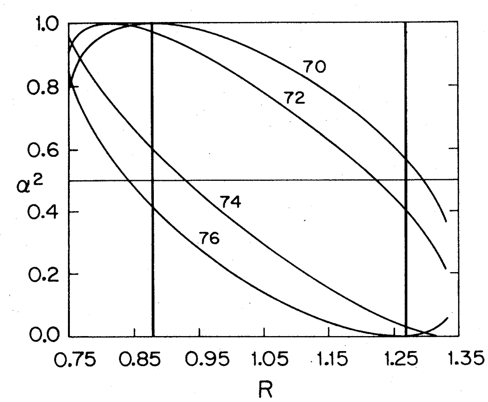

FIG. 3. Plot of α_A^2 vs R with the physical region between the broad vertical lines.

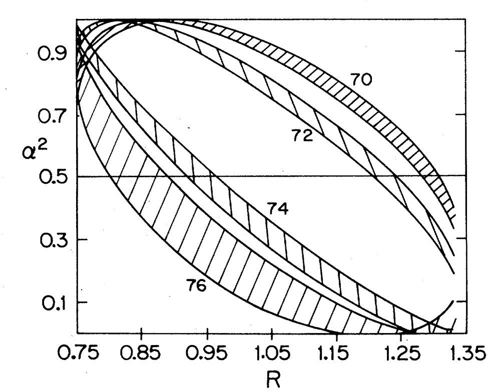

FIG. 5. Plot of α_A^2 vs R with uncertainties.

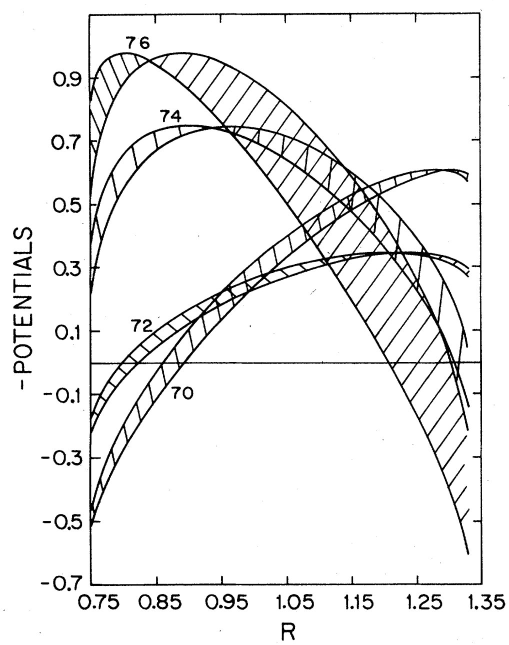

FIG. 6. Plot of $-V_A$ vs R with uncertainties.

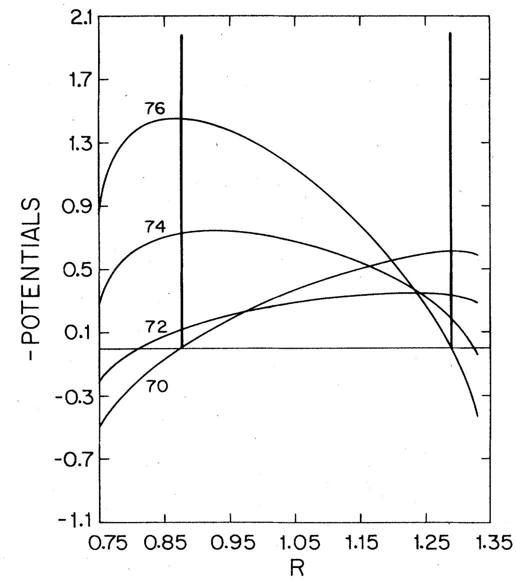

FIG. 8. Plot of $-V_A$ vs R (using $E_{76}=2.901$ MeV).

have arbitrarily chosen our phase convention such that all α_A, β_A are non-negative. In a two-state model, the interaction matrix element

$$\langle \varphi_e^A \mid V \mid \varphi_g^A \rangle \equiv V_A$$

is thus given by $V_A = -E_A \alpha_A \beta_A$ (E_A being the experimental excitation energy difference in nucleus A, between the excited 0^+ state and the ground state). We plot $-V_A = E_A \beta_A \alpha_A$ vs R in Fig. 4. Because physically we can choose α_A and β_A positive in $\psi_{\text{g.s.}}(^A\text{Ge})$, we must have it that $-V_A > 0$; therefore physically we can dismiss solutions for which any $-V_A$ is negative. Thus we must have $0.88 \leq R \leq 1.26$. These bounds are indicated in Figs. 3 and 4 by the broad vertical lines. Also indicated in Fig. 3 is the transition line $\alpha_A^2 = 0.5$. It is clear from the figure that a transition does occur as we move through the germanium isotopes from lighter A to heavier A with $\psi_{\text{g.s.}}(^A\text{Ge})$ being mostly φ_g^A in the lighter isotopes and mostly φ_e^A in the heavier ones. The point at which the transition occurs is dependent on R. In particular, for $0.88 \leq R \leq 0.93$ the transition occurs between ^{74}Ge and ^{76}Ge, for $0.93 \leq R \leq 1.22$ the transition is between ^{72}Ge and ^{74}Ge, while for $1.22 \leq R \leq 1.26$, the transition occurs between ^{70}Ge and ^{72}Ge. Without additional input the parameter R is still a free variable and is restricted only to the interval $0.88 \leq R \leq 1.26$. It might be hoped that at some value of R all the potentials in Fig. 4 would be equal. The V_A's "almost" agree at a common value for $1.12 \leq R \leq 1.24$ and therefore we would expect the transition to occur between ^{72}Ge and ^{74}Ge. Actually, the V_A's agree in pairs very well at $R=1.17$, with $V_{72}=V_{76}$ and $V_{70}=V_{74}$, but $V_{72}=(0.65)V_{70}$, perhaps suggesting a "staggering" dependence of V on A.

A more detailed analysis using the uncertainties in P_A and T_A lead to α_A^2 and $-V_A$ bands instead of curves, and

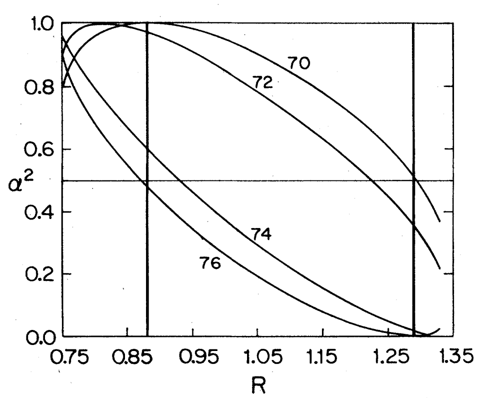

FIG. 7. Plot α_A^2 vs R (using $E_{76}=2.901$ MeV).

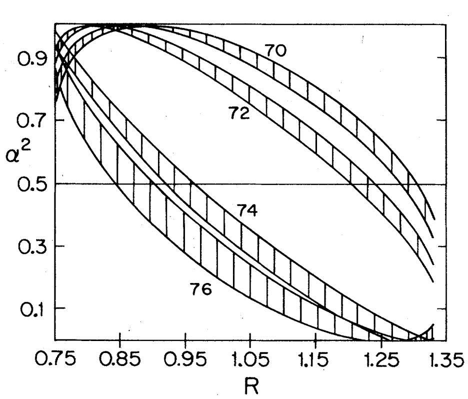

FIG. 9. Plot of α_A^2 vs R with uncertainties (using $E_{76}=2.901$ MeV).

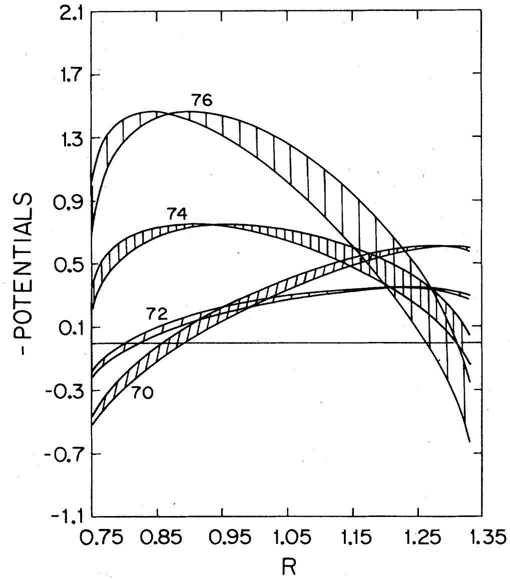

FIG. 10. Plot of $-V_A$ vs R with uncertainties (using $E_{76}=2.901$ MeV).

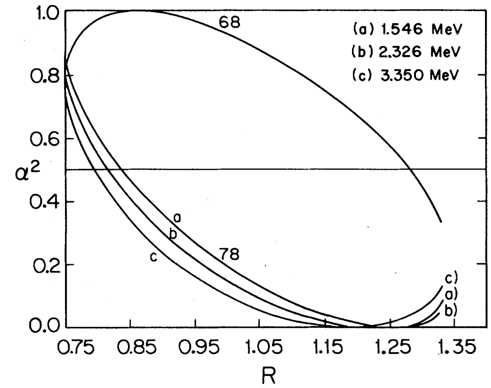

FIG. 11. Plot of α_{68}^2 and α_{78}^2 vs R using (a) $E_{78}=1.546$ MeV, (b) $E_{78}=2.326$ MeV, (c) $E_{78}=3.350$ MeV.

they are given in Figs. 5 and 6. These lead, however, to the same conclusions that a transition occurs and it is most likely to be between ^{72}Ge and ^{74}Ge.

So far in our discussion we have been assuming that it is the *first* excited 0^+ state that arises from mixing with the ground state. That may not necessarily be the case. In Fig. 1 we see an energy level diagram for 0^+ states in the even Ge isotopes. From ^{66}Ge to ^{78}Ge, each isotope has a 0^+ state at about 2.22 MeV as indicated by the dashed lines. These states at about 2.22 MeV could be inert for all the germanium isotopes and not mix with any of the other 0^+ states. If that is the case, then for ^{76}Ge the next candidate to consider with the ground state in a two-state model would be the 0^+ level at 2.901 MeV. The corrected cross-section ratio for this state is $T_{74}^2=0.025 \pm 0.003$.[9] The same scheme as above again leads to a unique sign combination.

Within the uncertainty the values of K_A are still -0.2448 and $L_A=1$, so the relationship between r and R is the same as before. Then Figs. 7 and 8 give the solutions α_A^2 and potentials $-V_A$. With the 2.901 MeV state in ^{76}Ge as the mixing state we now note that three of the four potentials intersect at a common point. Error-analysis results are given in Figs. 9 and 10.

TABLE V. Results on ^{70}Ge(p,t)^{68}Ge and ^{76}Ge(t,p)^{78}Ge.

	P_{68}^2		T_{76}^2	
Experimental	0.0058[a]	0.0367[b]	0.0167[c]	0.0633[d]
DWBA	0.8198	0.9385	0.878	0.8167
Expt/DWBA	0.0071	0.0391	0.0190	0.0775

[a]Using the excited 0^+ state at 1.753 MeV.
[b]Using the excited 0^+ state at 1.546 MeV.
[c]Using the excited 0^+ state at 2.326 MeV.
[d]Using the excited 0^+ state at 3.350 MeV.

B. Wave functions for ^{68}Ge and ^{78}Ge

Knowledge of P_{70}^2, P_{72}^2, P_{74}^2, T_{70}^2, T_{72}^2, and T_{74}^2 were used in the previous section to obtain two-state wave functions for ^{70,72,74,76}Ge ground states and 0_2^+ states. Knowledge of T_{68}^2 and P_{76}^2 of course is unattainable, but P_{68}^2 and T_{76}^2 have been measured in Refs. 15 and 12, respectively, and are given in Table V along with the appropriate DWBA calculations. It is therefore possible from Eq. (12) to calculate χ_{68} and χ_{78} and hence obtain wave functions for ^{68}Ge and ^{78}Ge ground states and 0_2^+ states. Without T_{68}^2 and P_{76}^2 we cannot calculate K_{70} and K_{76} and therefore there is no condition for obtaining the sign of P_{68} and T_{76}. To be consistent with earlier sign assignments, we chose P_{68} negative and T_{76} positive. As shown in Table V we use the 1.753-MeV state for the 0_2^+ state in ^{68}Ge, but it is not exactly clear which state should be chosen as the 0_2^+ state in ^{78}Ge. The 1.546-MeV level seems to contradict the parabolic trend of the admixed excited 0^+ states as one moves from $A=68$ to 78. The 3.350-MeV state would seem to be too far away to mix with the ground state while the 2.326-MeV state would fall in the category of the possible "inert" state proposed in Sec. IV A. Because of this ambiguity we will not attempt to calculate V_{78}; however, the general trend of α_{68}^2 and α_{78}^2 is not affected by the choice of E_{78} as shown in Fig. 11. Again the figure gives the lighter isotope being mostly φ_g^A and the heavier one being mostly φ_e^A.

V. SUMMARY AND CONCLUSIONS

We have applied a generalized two-state model in describing the cross-section ratios for two-nucleon transfer reactions. In particular, we are able to fit the $\sigma(0_2^+)/\sigma(\text{g.s.})$ ratios for ^{70,72,74}Ge(t,p) and ^{72,74,76}Ge(p,t). There exist an infinite number of possible solutions, all given in terms of one parameter, R, which is restricted to lie in a narrow range. In each of these solutions, the structure of the light Ge isotopes is different from that of the heavy ones with the transition occurring between ^{72}Ge and ^{74}Ge for most values of R. In addition, two-nucleon pickup on ^{70}Ge and two-nucleon stripping on ^{76}Ge allow wave functions to be calculated for ^{68,78}Ge. They are in agreement with the concept of a transition having occurred. As a final point it is quite easy to show that $1+4K_A \geq 0$ (i.e., $K_A \geq -\frac{1}{4}$) for any values of the T_A's and P_A's. The experimental results for germanium yield $K_A = -0.2448 \pm 0.0023$, very close to its minimum value. For K_A exactly equal to $-\frac{1}{4}$ we get $R=1$ and $r=0$, a result implied by Vergnes[13] in his choice for φ_g^A and φ_e^A. This limit, of course, cannot be correct because it requires $T_A^2 = P_A^2$, a result not satisfied by experiment. We emphasize that the present model accounts for the Ge(p,t) and (t,p) ratios *without* at all specifying the nature of φ_g^A and φ_e^A.

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[1] M. N. Vergnes *et al.*, Phys. Lett. **72B**, 447 (1978).

[2] *Table of Isotopes,* 7th ed., edited by C. Michael Lederer and Virginia Shirley (Wiley, New York, 1978).

[3] P. D. Duval, D. Goutte, and M. Vergnes, Phys. Lett. **124B**, 297 (1983).

[4] F. Guilbault *et al.*, Phys. Rev. C **16**, 1840 (1977).

[5] G. C. Ball *et al.*, Nucl. Phys. **A231**, 334 (1974).

[6] A. C. Rester, J. B. Ball, and R. L. Auble, Nucl. Phys. **A346**, 371 (1980).

[7] C. Lebrun *et al.*, Phys. Rev. C **19**, 1224 (1979).

[8] S. Mordechai, H. T. Fortune, R. Middleton, and G. Stephans, Phys. Rev. C **19**, 1733 (1979).

[9] S. LaFrance, S. Mordechai, H. T. Fortune, and R. Middleton, Nucl. Phys. **A307**, 52 (1978).

[10] S. Mordechai, H. T. Fortune, R. Middleton, and G. Stephans, Phys. Rev. C **18**, 2498 (1979).

[11] P. D. Kunz (private communications).

[12] J. F. Mateja *et al.*, Phys. Rev. C **17**, 2047 (1978).

[13] M. Vergnes, Inst. Phys. Conf. Ser. **49**, 25 (1980).

[14] A. M. van den Berg *et al.*, Nucl. Phys. **A379**, 239 (1982).

[15] F. Guilbault *et al.*, Phys. Rev. C **15**, 894 (1977).