Isobaric mass systematics for $A \leq 60$

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The isobaric multiplet mass equation and existing spectroscopic information have been used to establish systematics for $E_C^{(1)}$ and $E_C^{(2)}$ of the nuclear Coulomb energy and extend the concept of isobaric multiplets in a coherent manner up to $A=60$. Collateral information on masses of $T_z=-1,-2$ nuclei and radii is also presented.

I. INTRODUCTION

The potential describing the Coulomb interaction between nucleons can be written as a sum of scalar, vector, and tensor operators in isospin space. For a state involving A nucleons with total isospin T and $T_z=(N-Z)/2$, the Coulomb energy derived from first-order perturbation theory is written as[1]

$$E_C(A,T,T_z)=E_C^{(0)}(A,T)-T_zE_C^{(1)}(A,T)+[3T_z^2-T(T+1)]E_C^{(2)}(A,T)\ , \qquad (1)$$

where $E_C^{(0)}$, $E_C^{(1)}$, and $E_C^{(2)}$ are the isoscalar, isovector, and isotensor coefficients. $E_C^{(1)}$ and $E_C^{(2)}$ can be related to the b and c coefficients of the isobaric multiplet mass equation (IMME),

$$M(A,T,T_z)=a(A,T)+b(A,T)T_z+c(A,T)T_z^2\ , \qquad (2)$$

by the following relations:[2]

$$E_C^{(1)}(A,T)=\Delta_{\rm nH}-b(A,T)\ , \qquad (3a)$$

$$E_C^{(2)}(A,T)=c(A,T)/3\ . \qquad (3b)$$

$\Delta_{\rm nH}=(782.339\pm0.017)$ keV is the neutron-hydrogen mass difference.[3]

II. ISOVECTOR AND ISOTENSOR COEFFICIENTS FOR $A \lesssim 45$

From data dispersed throughout the literature on $T=1$, $\frac{3}{2}$, and 2 multiplets organized into a 1984 compilation,[4] it has been shown that any T_z^3 term in Eq. (2) is zero within new smaller limits except in the now classic $A=9$ case. It has thus been possible to accurately determine the coefficients of the quadratic IMME for many multiplets and establish well-defined macroscopic systematics for the quantities $E_C^{(1)}$ and $E_C^{(2)}$.

The plot of $E_C^{(1)}$ vs A is shown in Fig. 1. Jänecke's earlier observation[2] that $E_C^{(1)}$ depends linearly on A (or $A^{2/3}$) between values of A of twice the nucleon number for certain shell closures appears to be verified. Prolongation to $A=60$ as discussed in Sec. III also reveals a possible slope change at $A=56$, i.e., at twice the number of nucleons closing an $f_{7/2}$ shell. The overall form of the curve in Fig. 1 follows the prediction[2]

$$E_C^{(1)}(A,T)=0.6e^2A^{2/3}/r_0\ , \qquad (4)$$

where r_0 is the nuclear radius parameter. Figure 2 shows the dependence of $E_C^{(2)}$ on A for triplets, quartets, and quintets.

One can demonstrate a novel use of these curves with the isospin mixing in ^{8}Be. The $T=0+1$ mixing in the 16.626(3) and 16.922(3) MeV levels of ^{8}Be analog to the 2^+ ^{8}Li(g.s.) has been pointed out.[5,6] A quadratic IMME fit to this data for the $T=1$ triad ^{8}Li, ^{8}Be, and ^{8}B, assuming only the 16.626 MeV state in ^{8}Be as analog, yields $E_C^{(2)}=122(1)$ keV. Similarly, using the 16.922 keV ^{8}Be as analog gives $E_C^{(2)}=23(1)$ keV. These two values, identified by "C" in Fig. 2, fall away from a short extrapolation describing the systematics $E_C^{(2)}$ vs A. Thus with this well-defined ^{8}Be test case, it is seen that isospin mixing between two levels causes deviation from the $E_C^{(2)}$ curve. Conversely, it should be expected that deviation from the curve will be indicative of isospin mixing since one expects that the center of gravity of the isotensor term should follow the systematics.

III. COEFFICIENTS FOR HIGHER A

As one goes to higher A, the $N>Z$ pressure of the line of nuclear stability renders the study of $T_<$ nuclei ex-

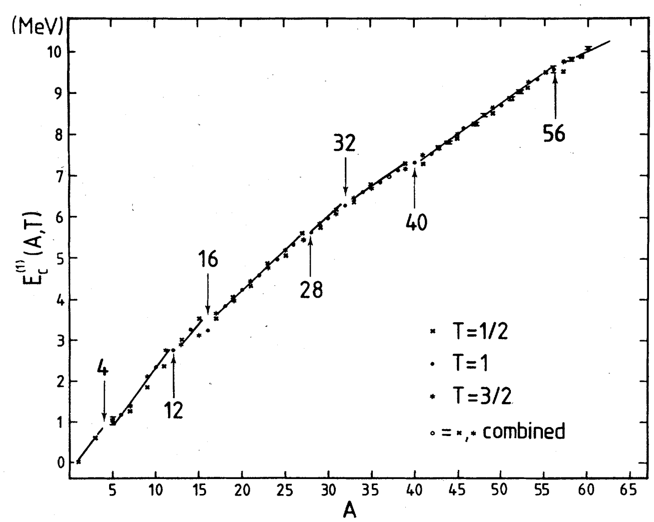

FIG. 1. Isovector term of the Coulomb energy $E_C^{(1)}$ vs A. Errors less than ±50 keV have not been indicated.

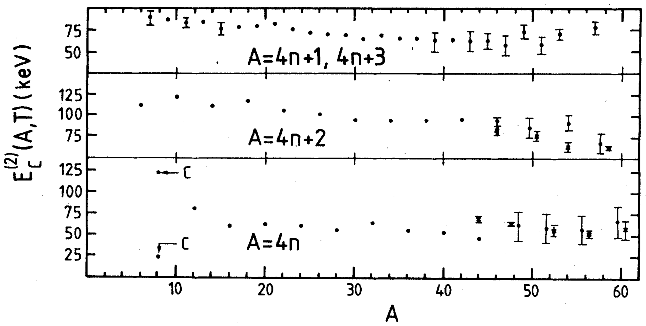

FIG. 2. Isotensor term of the Coulomb energy $E_C^{(2)}$ vs A. $E_C^{(2)}$ for the 16.626 and 16.922 MeV, $T=0+1$, $J^\pi=2^+$ states of ^{8}Be are indicated by "C." IMME fits with Wapstra's masses are denoted by dots. The crosses are from fits where the $T_z=-1$ mass was calculated from Coulomb energy differences. Errors less than ± 3 keV have not been indicated.

tremely difficult. Thus for experimental reasons, and also because of the lack of a solid theoretical or empirical guide, the IMME has not been used much for $A \gtrsim 40$. Nevertheless by making use of the quadratic IMME, existing spectroscopic data, and Wapstra's revised mass values,[3] it is possible to correlate the data and propose members for complete higher A multiplets.

For identifying triplets in the mass range $A=44$–60, we consider the cases where two members $T_z=0,1$ have been at least tentatively characterized in T and J^π, and use Wapstra's mass (or Harchol's[7] for $A=60$) for the $T_z=-1$ member. (See also the next paragraph for an alternative way of obtaining the mass of $T_z=-1$ nuclei.) This information is given in Table I. Energy levels for $A=44$ are from Ref. 8, and for $A=46$–60 are from the most recent Nuclear Data Sheets. Once these three data are taken at face value, the IMME fit (Table II) uniquely determines the $E_C^{(1)}$ and $E_C^{(2)}$ that extend the curve in Fig. 1 and the $4n$ and $4n+2$ curves in Fig. 2 up to $A=60$. The large error bars for $A \geq 44$ reflect the cautious uncertainty on the $T_z=-1$ nucleus quoted by Wapstra.

Instead of using Wapstra's mass value for the $T_z=-1$ member of a triplet, there is also available an empirical method based on Coulomb displacement energies that can in fact give calculated uncertainties smaller than those quoted by Wapstra. A fit of the formula[2] for the Coulomb displacement energy

TABLE I. Properties of $4n$ and $4n+2$ multiplets.

A	J^π	T_z	Nucleus	Mass excess (MeV)	Level (MeV)
44	2^+	-1	V	$-23.800(100)$	g.s.
		0	Ti	$-30.949(4)$	6.600(4)
		1	Sc	$-37.8144(29)$	g.s.
48	4^+	-1	Mn	$-29.220(100)$	g.s.
		0	Cr	$-37.028(12)$	5.790(10)
		1	V	$-44.472(3)$	g.s.
52	6^+	-1	Co	$-34.300(100)$	g.s.
		0	Fe	$-42.679(14)$	5.652(8)
		1	Mn	$-50.7034(24)$	g.s.
56	4^+	-1	Cu	$-38.500(200)$	g.s.
		0	Ni	$-47.466(11)$	6.436(3)
		1	Co	$-56.0380(25)$	g.s.
60	2^+	-1	Ga	$-39.870(100)$	g.s.
		0	Zn	$-49.305(32)$	4.880(30)
		1	Cu	$-58.3438(26)$	g.s.
46	0^+	-1	Cr	$-29.460(30)$	g.s.
		0	V	$-37.0749(15)$	g.s.
		1	Ti	$-44.1253(14)$	g.s.
50	0^+	-1	Fe	$-34.470(60)$	g.s.
		0	Mn	$-42.6260(16)$	g.s.
		1	Cr	$-50.2579(16)$	g.s.
54	0^+	-1	Ni	$-39.210(50)$	g.s.
		0	Co	$-48.0093(15)$	g.s.
		1	Fe	$-56.2508(14)$	g.s.
58	0^+	-1	Zn	$-42.210(100)$	g.s.
		0	Cu	$-51.459(4)$	0.203(3)
		1	Ni	$-60.2251(15)$	g.s.

TABLE II. IMME coefficients.

A	J^π	a (MeV)	b (MeV)	c (keV)	d (keV)	χ^2
44	2^+	−30.949(10)	−7.007(50)	141.8(51)		
46	0^+	−37.0750(15)	−7.333(15)	282(15)		
48	4^+	−37.027(12)	−7.626(50)	182(51)		
50	0^+	−42.6259(16)	−7.894(30)	262(30)		
52	6^+	−42.679(14)	−8.202(50)	178(52)		
54	0^+	−48.0100(15)	−8.520(25)	279(25)		
56	4^+	−47.466(11)	−8.770(100)	197(101)		
58	0^+	−51.459(4)	−9.006(50)	240(50)		
60	2^+	−49.305(32)	−9.237(50)	198(59)		
39	$\frac{7}{2}^-$	−24.131(25)	−6.354(67)	186(33)		
41	$\frac{3}{2}^+$	−26.0616(24)	−6.6199(46)	191.8(31)		0.0004
		−26.0616(37)	−6.6198(58)	192(10)	−0.1(71)	
43	$\frac{7}{2}^-$	−28.582(26)	−6.833(67)	188(33)		
45	$\frac{7}{2}^-$	−30.695(20)	−7.202(50)	191(25)		
47	$\frac{5}{2}^-$	−34.185(26)	−7.423(67)	173(33)		
49	$\frac{7}{2}^-$	−36.710(21)	−7.829(53)	221(27)		
51	$\frac{7}{2}^-$	−39.829(25)	−8.008(67)	175(33)		
53	$\frac{7}{2}^-$	−42.524(13)	−8.437(22)	218(17)		0.04
		−42.522(18)	−8.443(28)	209(45)	7(32)	
57	$\frac{7}{2}^-$	−46.446(28)	−8.909(43)	208(24)		

TABLE III. Mass excesses calculated for $T_z=-1$ nuclei. Corresponding values from Wapstra can be found in Table I.

Pair (J^π) $M_{Z_>}$-$M_{Z_<}$	ΔE_C (MeV) from Eq. (5)	Calculated $M_{Z_>}$ (MeV) This work	Sherr (Ref. 9)	Harchol *et al.* (Ref. 7)
^{44}V-^{44}Ti(2^+)	8.059(16)	−23.673(16)	−23.822	−23.830(50)
^{48}Mn-^{48}Cr(4^+)	8.608(16)	−29.202(20)	−29.303	−29.280(100)
^{52}Co-^{52}Fe(6^+)	9.142(17)	−34.319(22)	−34.314	−34.330(100)
^{56}Cu-^{56}Ni(4^+)	9.662(18)	−38.586(23)		−38.550(100)
^{60}Ga-^{60}Zn(2^+)	10.171(19)	−39.916(37)		−39.870(100)
^{46}Cr-^{46}V(0^+)	8.335(16)	−29.522(16)		−29.530(50)
^{50}Fe-^{50}Mn(0^+)	8.876(17)	−34.532(17)		−34.490(70)
^{54}Ni-^{54}Co(0^+)	9.404(18)	−39.387(18)		−39.320(50)
^{58}Zn-^{58}Cu(0^+)	9.918(18)	−42.323(20)		−42.290(80)

TABLE IV. Properties of $A=4n+1$ and $4n+3$ multiplets. Brackets indicate calculations by the IMME. The levels marked with asterisks are from Ref. 10.

A	J^π	T_z	Nucleus	Mass excess (MeV)	Level (MeV)
39	$\frac{7}{2}^-$	$-\frac{3}{2}$	Sc	−14.180(200)	g.s.
		$-\frac{1}{2}$	Ca	[−20.907(66)]	[6.370(66)]
		$\frac{1}{2}$	K	−27.2609(23)	6.546(2)

TABLE IV. (*Continued*).

A	J^{π}	T_z	Nucleus	Mass excess (MeV)	Level (MeV)
		$\frac{3}{2}$	Ar	−33.242(5)	g.s.
41	$\frac{3}{2}^{+}$	$-\frac{3}{2}$	Ti	−15.700(40)	g.s.
		$-\frac{1}{2}$	Sc	−22.7037(43)	5.940(4)
		$\frac{1}{2}$	Ca	−29.3236(24)	5.815(2)
		$\frac{3}{2}$	K	−35.5600(12)	g.s.
43	$\frac{7}{2}^{-}$	$-\frac{3}{2}$	V	−17.920(200)	g.s.
		$-\frac{1}{2}$	Ti	[−25.119(94)]	[4.202(94)]
		$\frac{1}{2}$	Sc	−31.9527(47)	4.235(4)
		$\frac{3}{2}$	Ca	−38.4084(16)	g.s.
45	$\frac{7}{2}^{-}$	$-\frac{3}{2}$	Cr	−19.460(150)	g.s.
		$-\frac{1}{2}$	V	[−27.046(50)]	[4.829(53)]
		$\frac{1}{2}$	Ti	−34.2467(57)	−4.760(5)*
		$\frac{3}{2}$	Sc	−41.0693(15)	g.s.
47	$\frac{5}{2}^{-}$	$-\frac{3}{2}$	Mn	−22.650(200)	g.s.
		$-\frac{1}{2}$	Cr	[−30.430(48)]	[4.124(50)]
		$\frac{1}{2}$	V	−37.8493(52)	4.155(5)*
		$\frac{3}{2}$	Ti	−44.9317(11)	g.s.
49	$\frac{7}{2}^{-}$	$-\frac{3}{2}$	Fe	−24.470(160)	g.s.
		$-\frac{1}{2}$	Mn	[−32.740(54)]	[4.871(59)]
		$\frac{1}{2}$	Cr	−40.5689(57)	4.760(5)*
		$\frac{3}{2}$	V	−47.9563(13)	g.s.
51	$\frac{7}{2}^{-}$	$-\frac{3}{2}$	Co	−27.420(200)	g.s.
		$-\frac{1}{2}$	Fe	[−35.782(67)]	[4.436(69)]
		$\frac{1}{2}$	Mn	−43.7887(25)	4.451(2)
		$\frac{3}{2}$	Cr	−51.4483(16)	g.s.
53	$\frac{7}{2}^{-}$	$-\frac{3}{2}$	Ni	−29.410(180)	g.s.
		$-\frac{1}{2}$	Co	−38.250(19)	4.390(6)
		$\frac{1}{2}$	Fe	−46.688(15)	4.256(15)
		$\frac{3}{2}$	Mn	−54.6872(16)	g.s.
57	$\frac{7}{2}^{-}$	$-\frac{3}{2}$	Zn	−32.610(130)	g.s.
		$-\frac{1}{2}$	Cu	[−41.940(48)]	[5.440(304)]
		$\frac{1}{2}$	Ni	−50.847(20)	5.230(20)
		$\frac{3}{2}$	Co	−59.3425(15)	g.s.

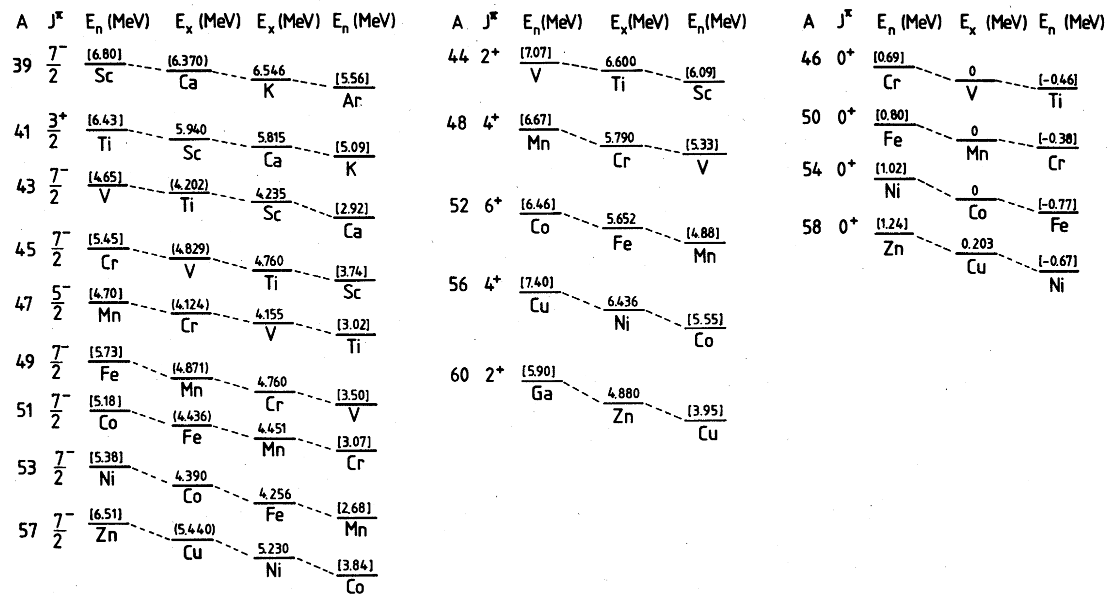

FIG. 3. Isobar diagrams for $4n+1$, $4n+3$, $4n$, and $4n+2$ multiplets. Values in brackets are nuclear energies. Values in parentheses are excitation energies on the nuclear energy scale calculated from the IMME and ground state masses.

$$\Delta E_C = k_1(\bar{Z}/A^{1/3}) + k_2 \quad (5)$$

between neighboring analog pairs has been made for the $T_z=0,1$ nuclei of Table I to determine the constants $k_1=1.490(2)$ MeV and $k_2=-1.437(10)$ MeV. $\bar{Z}$ is the average charge between the pair. Then by using a different $\bar{Z}$ corresponding to the $T_z=-1,0$ members of the same multiplet, one obtains the mass of the $T_z=-1$ member from the $T_z=0$ reference nucleus. These results for $44 \le A \le 60$ are given in Table III together with other values for these same nuclei.[9,7]

For identifying quartets in the mass range $A=39$–57, the procedure was similar to that just described for the triplets. Data are from the same sources. The levels marked with asterisks in Table IV are from Ref. 10, with errors estimated from neighboring levels. In the $A \ge 39$ quartets, the $T_z=\frac{1}{2},\frac{3}{2}$ members are experimentally known and with Wapstra's mass used for the $T_z=-\frac{3}{2}$ member, the IMME determines the mass of the $T_z=-\frac{1}{2}$ analog. Subtracting Wapstra's mass yields the excitation energy of the analog level in the $T_z=-\frac{1}{2}$ nucleus. These data are reported in Table IV. The deduced $E_C^{(1)}$ and $E_C^{(2)}$ are plotted in Figs. 1 and 2. The new multiplets have been represented according to the convention of Ajzenberg-Selove[6] on the isobaric diagrams in Fig. 3.

TABLE V. Mass excesses of $T_z=-2$ nuclei.

Nucleus $T_z=2$	Mass excess of $T_z=2$ reference nucleus (MeV)	Nucleus $T_z=-2$	Mass excess of $T_z=-2$ nucleus (MeV): Wapstra (Ref. 3)	This work[a]	This work[b]	Jänecke-Garvey-Kelson (Ref. 14)	Liran-Zeldes (Ref. 14)
^{8}He	31.597(8)	^{8}C	35.095(25)	35.547(8)	35.105(18)		
^{10}Li	33.830(250)	^{10}N	39.700(400)	40.018(250)		33.25	
^{12}Be	25.077(15)	^{12}O	32.060(40)	33.014(15)	32.061(47)	33.05	
^{14}B	23.664(21)	^{14}F	33.610(400)	33.637(21)		33.38	
^{16}C	13.694(4)	^{16}Ne	23.989(22)	23.331(19)	24.001(22)	24.67	
^{18}N	13.117(20)	^{18}Na	25.320(400)	25.319(22)		25.57	
^{20}O	3.7956(19)	^{20}Mg	17.572(27)	17.512(14)	17.536(21)	17.40	17.80
^{22}F	2.830(30)	^{22}Al	18.040(90)	18.090(30)		17.95	18.31
^{24}Ne	−5.950(10)	^{24}Si	10.760(30)	10.779(13)	10.756(16)	10.74	10.81
^{26}Na	−6.906(16)	^{26}P	11.260(300)	11.234(17)		11.20	11.04
^{28}Mg	−15.0188(21)	^{28}S	4.130(160)	4.360(8)	4.205(20)	4.41	4.09
^{30}Al	−15.890(40)	^{30}Cl	4.840(300)	4.850(40)		4.84	4.23
^{32}Si	−24.0780(13)	^{32}Ar	−2.180(50)	−2.126(16)	−2.201(8)	−2.20	−2.19
^{34}P	−24.5579(12)	^{34}K	−1.480(300)	−1.452(6)		−1.46	−1.65

TABLE V. (*Continued*).

Nucleus $T_z=2$	Mass excess of $T_z=2$ reference nucleus (MeV)	Nucleus $T_z=-2$	Mass excess of $T_z=-2$ nucleus (MeV): Wapstra (Ref. 3)	This work[a]	This work[b]	Jänecke-Garvey-Kelson (Ref. 14)	Liran-Zeldes (Ref. 14)
^{36}S	−30.66444(25)	^{36}Ca	−6.440(40)	−6.466(16)	−6.481(13)	−6.45	−6.21
^{38}Cl	−29.79823(15)	^{38}Sc	−4.460(300)	−4.489(18)		−4.66	−4.49
^{40}Ar	−35.0399(13)	^{40}Ti	−9.064(11)	−9.024(9)		−9.01	−8.89
^{42}K	−35.0234(15)	^{42}V	−8.220(300)	−8.176(13)		−8.02	−7.96
^{44}Ca	−41.4691(16)	^{44}Cr	−13.220(180)	−13.441(200)		−13.59	−13.33
^{46}Sc	−41.7587(15)	^{46}Mn	−12.470(400)	−12.426(62)		−12.66	−12.56
^{48}Ti	−48.4869(11)	^{48}Fe		−17.983(200)		−18.24	−17.97
^{50}V	−49.2197(15)	^{50}Co		−17.644(120)		−17.79	−17.76
^{52}Cr	−55.4152(16)	^{52}Ni		−22.607(200)		−22.69	−22.83
^{54}Mn	−55.5540(18)	^{54}Cu		−21.474(100)		−21.88	−21.92
^{56}Fe	−60.6041(15)	^{56}Zn		−25.528(400)		−25.98	−25.74
^{58}Co	−59.8443(18)	^{58}Ga		−23.820(200)		−24.45	−24.25
^{60}Ni	−64.4707(15)	^{60}Ge		−27.523(200)		−28.21	−27.99

[a]With b calculated from IMME fit to $T=1$ nuclei (see Table VII of Ref. 4).
[b]With b calculated from IMME fit to $T=2$ nuclei (see Table VIII of Ref. 4).

IV. MASSES OF $T_z=-2$ NUCLEI AND RADII

Next we turn to the prediction of the mass excesses of even A, $T_z=-2$, proton-rich nuclei using IMME coefficients. This can be done accurately (Table V) if b coefficients from a $T=2$ multiplet are available. In the present work, it was found that another method appears to give quite good results. The method is different from that of Gul,[11] who calculated the mass of a $T_z=-2$ nucleus to an estimated overall accuracy of 150 keV. Attempts to improve his accuracy by using more recent b values,[4] restricted A ranges, or the complete theoretical form[12]

$$b=-0.6[(A-1)e^2/r_0A^{1/3}]+\Delta_{nH} \quad (6)$$

did not appreciably better his results.

In the alternative procedure, the IMME coefficients for a known $T=1$ triplet of a given A are calculated. Then the same b coefficient is used in the equation[13]

$$M(A,Z,-T_z)=M(A,Z-2T_z,T_z)-2T_zb \quad (7)$$

to calculate the mass excess of the $T_z=-2$ nucleus. The mass excesses thus obtained and from other sources[3,14] are given in Table V, where the quoted uncertainties on our values arise from the fitted b coefficients and the value of the $M(A,Z-2T_z,T_z)$ reference nucleus.[3] The consistent results obtained in most cases with the b transposed from one T to another may signify that the b values do not vary as much between different T as they do with excitation within a given T (Ref. 15).

Nuclear charge radii $r_0(b)$ obtained from the b coefficients in Table II and Eq. (6) are represented in Fig. 4 as a function of A. The previously reported trend[12] is continued.

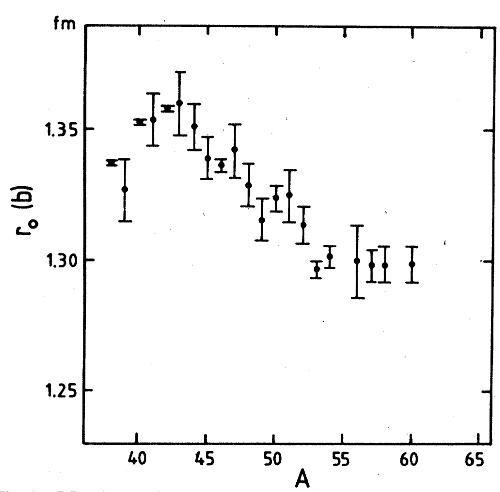

FIG. 4. Nuclear charge radius parameter $r_0(b)$ as a function of A.

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