Decay of ¹⁸⁸Tl and observed shape coexistence in the bands of ¹⁸⁸Hg

J. D. Cole,* J. H. Hamilton, A. V. Ramayya, W. Lourens,[†] B. van Nooijen,[†] H. Kawakami,[‡] and L. A. Mink[§] Vanderbilt University, Nashville, Tennessee 37325

E. H. Spejewski, H. K. Carter, R. L. Mlekodaj, and G. A. Leander UNISOR, Oak Ridge Associated Universities, Oak Ridge, Tennessee 37830

L. L. Riedinger and C. R. Bingham University of Tennessee, Knoxville, Tennessee 37816

E. F. Zganjar Louisiana State University, Baton Rouge, Louisiana 70803

J. L. Wood and R. W. Fink Georgia Institute of Technology, Atlanta, Georgia 30332

K. S. Toth

Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830

B. D. Kern

University of Kentucky, Lexington, Kentucky 40504

K. S. R. Sastry University of Massachusetts, Amherst, Massachusetts 01002 (Received 9 April 1984)

The decay of the isotope ¹⁸⁸Tl $(T_{1/2}=71\pm1 \text{ s})$ was studied via conversion-electron, gamma, gamma-gamma, and gamma-conversion-electron measurements. Levels of ¹⁸⁸Hg were deduced. Two bands with quite different deformation were established. One band is built on a near spherical ground state and the other band is built on a well-deformed shape with an excited 0^+_2 band head at 824.5 keV. The bands cross and coexist above and below the crossing point. Their crossing explains the unusual behavior of the moment of inertia of the yrast cascade.

INTRODUCTION

In an earlier Letter,¹ we reported the definite identifica-tion of the isotopes ^{186,188}Tl and the crossing of complete bands of levels built on near-spherical ground states and well-deformed excited 0_2^+ states in ^{186,188}Hg. These and similar data² for ¹⁸⁴Hg combined with in-beam studies³⁻⁶ and optical pumping studies^{7,8} give a clear picture of the onset of large deformation in the ground states of ^{183,185}Hg compared to ¹⁸⁷Hg, and of the coexistence of near-spherical and deformed shapes in 184,185,186,188Hg with overlapping bands of levels built on each shape in the even-A isotopes. These shapes persist with different collective motions observed from spin 0^+ to 8^+ in each band in ¹⁸⁸Hg with mixing at the crossing. While many cases of the crossing of the ground band by rotation aligned structures had been seen at that time¹ (see the review of Ward⁹), these results provided the first clear picture for the crossing of the ground band in heavy nuclei by a band of much larger deformation. In this paper, we report results of our studies of the identification of the isotope ¹⁸⁸Tl and its decay to ¹⁸⁸Hg. Some further details can be found in three theses. $^{10-12}$

EXPERIMENTAL PROCEDURES AND RESULTS

Two different reactions were used to produce 188 TI. The first reaction, 181 Ta(16 O,9n) 188 TI used a 143–145 MeV 16 O beam from the Oak Ridge isochronous cyclotron (ORIC). The second reaction, 180 W(14 N,6n) 188 TI, used a beam of 184–188 MeV 14 N ions from the ORIC. The target for this reaction was produced by bonding 180 W (enriched to 92%) to a carbon mesh. The products from the reactions were captured and ionized for mass separation in the ion source of the UNISOR mass separator. 13 The mass separated 188 TI ions were extracted from the separator collection box via a drift tube and deposited on a tape in a movable tape transport unit. A source was collected for 200 s and then moved to appropriate detector arrangements in less than one second.

Gamma-ray and conversion-electron data were taken with Ge(Li) and Si(Li) detectors. Singles data were acquired in a spectrum multiscaling mode by taking ten spectra over a period of 200 s for each source. (A new source was collected while the first was counted.) This gave multiscale data for half-life determination and species identification. Both gamma-gamma-time and

30 1267



FIG. 1. Gamma-ray singles spectrum for the decay of ¹⁸⁸Tl and daughter nuclei. The peaks are marked by energy in keV and the decaying nucleus. (a) and (b) are the lower and upper 4096 channels of the spectrum, respectively.



FIG. 2. The conversion electron spectrum for the decay of ¹⁸⁸Tl.

gamma-electron-time coincidence data were taken concurrently with the multiscale spectra, and were accumulated in a list data format. In Figs. 1 and 2 are shown the gamma-ray and conversion electron spectra, respectively; Figs. 3 and 4 show energy gates as examples of the coincidence spectra. The singles and multiscale spectra were analyzed with the code SAMPO,¹⁴ modified to use a Levenberg-Marquardt^{15,16} residue technique. From this analysis, centroids and areas of peaks were extracted giving energies and relative intensities of gamma rays and conversion electrons. These results are summarized in Table I.

The energy scales were calibrated from a linear fit of energies obtained in a separate calibration run of standard sources. The conversion-electron results were used to obtain internal conversion coefficients. These were used to assign the multipolarities to transitions by comparison with plots of theoretical internal conversion coefficients by Rösel *et al.*¹⁷ as illustrated in Fig. 5. A summary of these results is given in Table I where the error limits



FIG. 3. The background subtracted energy gate for the 412.9 keV ground state transition in ¹⁸⁸Hg from the gamma-gamma-time coincidence data.



FIG. 4. The 504.3-keV $(6^+ \rightarrow 4^+)$ transition gate in ¹⁸⁸Hg as in Fig. 4.

Transition	Experimental					
energy	Ir	I_e	Ι. ακ			
(keV)	(%)	(%)	(×100)	Multipolarity ^a		
203.2±0.2	1.3 ±0.1	42.4 ±5.0	99 ±11	M1/E2+E0		
215.7±0.1	0.60 ± 0.1					
247.6±0.1	1.9 ±0.1	8.0 ±0.9	13 ± 2	E 2		
269.4±0.1	1.4 ±0.1					
281.5 ± 0.1	0.80 ± 0.08					
291.7±0.1	4.0 ±0.3	2.8 ± 0.3	2.1 ± 0.5	<i>E</i> 1		
301.2±0.1	5.5 ±0.3	8.6 ±0.9	8.3 ± 1.5	<i>E</i> 2		
326.9±0.1	10.70 ± 0.51	11.40 ± 0.15	4.0 ± 0.4	<i>E</i> 2		
385.8 ± 0.1	3.7 ± 0.3	4.4 ±0.6	3.6 ± 0.9	E 2		
387.5±0.2	0.30 ± 0.05		- J			
398.2±0.2	0.60 ± 0.06					
412.9±0.1	100.0 ±5.0	100.0 ± 8.0	3.05 ± 0.30	E 2		
		29.8 ±2.0	$0.9 \pm 0.2L$	<i>E</i> 2		
417.9±0.1	1.1 ± 0.1	1.1 ± 0.1	3.0 ± 1.0	E 2		
424.1±0.1	3.9 ± 0.3	2.6 ± 0.3	2.0 ± 1.0	E2(E1)		
443.1 ± 0.1	1.9 ±0.2	2.0 ± 0.3	3.2 ± 0.7	<i>E</i> 2		
445.9±0.1	1.0 ±0.1					
450.3±0.1	$0.50 {\pm} 0.05$	2.5 ±0.5	6.8 ± 1.4	E2/M1		
452.7±0.1	2.8 ± 0.2	2.3 ± 0.3	2.9 ± 0.5	E 2		
		1.4 ± 0.2				
460.7±0.1	8.2 ±0.5	7.0 ±0.9	2.6 ± 0.4	<i>E</i> 2		
		1.9 ± 0.3	$0.66 \pm 0.10L$	<i>E</i> 2		
468.2±0.1	5.7 ±0.3	16.4 ± 1.0	8.7 ± 0.6	(M1/E2 + E0)		
		2.3 ± 0.3	$1.3 \pm 0.4L$			
504.3±0.1	26.5 ± 1.6	16.7 ± 1.0	1.9 ± 0.2	<i>E</i> 2		
		4.5 ±0.6	$0.48 \pm 0.08 L$	<i>E</i> 2		
535.0±0.1	1.3 ± 0.1	0.50±0.09	1.3 ± 0.4	<i>E</i> 2		
569.3±0.1	3.9 ± 0.3	2.2 ± 0.2	1.7 ± 0.4	<i>E</i> 2		
574.0±0.1	4.5 ±0.3	2.6 ±0.3	2.4 ± 0.4	(E2)		
592.1±0.1	69.1 ±3.1	31.2 ± 1.0	1.37 ^b	<i>E</i> 2 ^b		
		8.9 +0.9		E2		

 TABLE I. Transition energies, relative transition intensities, relative conversion electron intensities, experimental K-conversion coefficients, and transition multipolarities for the decay of ¹⁸⁸Tl.



FIG. 5. Plot of the experimental K-shell internal conversion coefficients versus energy in keV. The solid curves are theoretical values from Ref. 17.

given are entirely statistical.

The beta feeding to the 0_2^+ state and to the higher spin states indicates that decays of both a high- and a low-spin isomer of ¹⁸⁸Tl are being observed as is the case in ^{186,190}Tl.^{18,19} Apparently the high and low spin isomers of ¹⁸⁸Tl have half-lives within 5% of each other. The results of analysis of the multiscale data for several strong lines



FIG. 6. Plot of counts versus time showing half-lives for the strong lines in the decay of 188 Tl.

INDEL I. (Continueu).							
Transition			Experimental				
energy	I_{γ}	Ie	α_K				
(keV)	(%)	(%)	(×100)	Multipolarity ^a			
622.0±0.2	0.70±0.07						
627.2 ± 0.1	1.7 ± 0.1	1.1 ± 0.4	1.9 ± 0.8	E 2			
645.6±0.2	2.4 ±0.2	1.9 ± 0.8	2.3 ± 0.9	E2/M1			
692.3±0.2	2.5 ± 0.2	0.8 ± 0.3	0.93 ± 0.35	E2			
700.1±0.2	3.3 ± 0.3	2.1 ± 0.2	1.9 ± 0.3	(E2/M1)			
701.7±0.2	0.9 ± 0.2						
714.1±0.2	0.40 ± 0.05	2.3 ± 0.2	$> 12 \pm 3$	>M5,E0+E2+M1			
745.7±0.2	0.50 ± 0.06	*					
764.6±0.1	$0.80 {\pm} 0.08$						
769.8±0.1	2.0 ± 0.2						
772.4 ± 0.1	13.5 ± 0.6	2.9 ± 0.2	0.65 ± 0.11	<i>E</i> 2			
795.2±0.1	11.3 ± 0.6	2.7 ± 0.1	0.73 ± 0.08	E2			
824.5 ± 0.2		8.0 ±0.9	>130 ±10	E0			
		1.7 ± 0.2	$28.0 \pm 3.3L$	E0			
826.7±0.1	2.7 ± 0.2	1.7 ± 0.4	1.8 ± 0.5	M1/E2			
837.8±0.1	1.3 ± 0.1						
873.9±0.1	0.6 ± 0.1						
881.1±0.1	8.6 ± 1.1	1.8 ± 0.2	0.65 ± 0.07	<i>E</i> 2			
904.8±0.1	12.3 ± 0.7	1.4 ± 0.3	0.20 ± 0.04	<i>E</i> 1			
913.2±0.1	0.3 ± 0.1						
928.5±0.1	1.6 ± 0.1						
948.0±0.1	0.5 ± 0.1	1.0 ± 0.3	0.72 ± 0.29	<i>E</i> 2			
1042.0±0.1	3.5 ± 0.2	1.4 ± 0.6	0.47± 0.24	<i>E</i> 2			
1057.8±0.1	1.1 ± 0.1						
1272.6±0.1	0.9 ±0.1						
1445.6±0.1	1.1 ± 0.1						
1477.5±0.1	1.0 ±0.1						

TABLE I. (Continued)

^aThe multipolarities are assigned by comparison of theoretical and experimental α_K by use of the plots of the data in Ref. 17. It should be noted that in the case of the E2 assignments, a possible small M1 admixture cannot be excluded on the basis of the experimental conversion coefficients alone. The E2 assignments mentioned in the table are preferred as discussed in the text. ^bUsed as the theoretical value.

in ¹⁸⁸Hg from the decay of ¹⁸⁸Tl are shown in Fig. 6. The half-life determined from an average of the transitions depopulating the low spin states is 71 ± 1 s. The half-life

ENERGY LEVELS OF ¹⁸⁸Hg

of the electrons of the 824.5 keV, EO transition was found

to be 71 ± 2 s.

The ¹⁸⁸Hg levels deduced from the decay of ¹⁸⁸Tl are shown in Fig. 7. The energy of each level is calculated by using the weighted average of all the gamma rays and electron energies that feed or depopulate that level. In addition to the 2^+ , 4^+ , 6^+ , and 8^+ yrast levels at 412.9, 1005.0, 1509.3, and 1970.0 keV, respectively, which are seen via in-beam studies,⁶ new 0_2^+ , 2_2^+ , 4_2^+ , 6_2^+ , and $(8)_2^+$ levels at 824.5, 881.1, 1208.0, 1777.2, and 2422.8 keV, respectively, were established in this work. The four new levels of lowest spin are given particular attention in this paper because of their importance in establishing shape coexistence in ¹⁸⁸Hg.

In the gamma singles spectrum (Fig. 1) no peak is seen at an energy of 824.5 keV, corresponding to a strong line in the conversion-electron spectrum (Fig. 2). This allows a lower limit of 130 to be set for the conversion coefficient of the 824.5-keV transition; thus, it is assigned as E0. The 0^+_2 level at 824.5 keV is established from the coincidence of this strong E0 transition with mercury x rays, and the absence of a coincidence relation with the 412.9-keV, $2^+ \rightarrow 0^+$ transition, gamma ray. An upper limit on the intensity of a 412-keV transition from the 0^+_2 level to the 2^+ , 412.9-keV level is 3%. The gate on the 824.5-keV transition K-shell electrons in the electron coincidence data shows only the mercury x rays and a strong 511-keV peak that indicates beta feeding of this level. The possible 56.6-keV feeding transition from the 2^+_2 level at 881.1 keV is not seen. There is no evidence for feeding of this level from higher lying states.

The 2^+_2 level at 881.1 keV is established by coincidence spectra obtained with gates on the 881.1- and 468.2-keV transitions to the ground state and to the 2^+ level at 412.9 keV, respectively. The 881.1-keV transition is E2 and the 468.2-keV transition is consistent with pure M1 multipolarity or E2 with a strong E0 component. The E2 decay to the ground state establishes the 2^+ assignment of this level.

The 4^+_2 level at 1208.0 keV has strong coincidence con-



FIG. 7. Energy level scheme for ¹⁸⁸Hg.

nections to the 2_1^+ , 2_2^+ , 4_1^+ , and 6_1^+ levels at 412.9, 881.1, 1005.0, and 1509.3 keV, respectively. Three of the four connecting transitions are E2 with the 203.2-keV transition to the 4_1^+ level having a large E0 or M1 admixture. Thus a 4^+ spin and parity assignment is established.

The 6_2^+ level at 1777.2 keV is supported by coincidence relations with the 772.4- and 569.3-keV transitions to the 4_1^+ and 4_2^+ levels at 1005.0 and 1208 keV, respectively. Both transitions are consistent with pure E2 multipolarity. Although spins of 2-6 are possible, 6 is chosen based upon the systematics of heavier, even-even mercury nuclei and its fit energetically as a member of the ground band.

These four levels and the yrast levels are grouped into two bands based on their energies and decay characteristics. The 2_1^+ , 4_1^+ , 6_2^+ , and $(8)_2^+$ levels at 412.9, 1005.0, 1777.2, and 2422.8 keV, respectively, is the ground band built upon the 0_1^+ ground state. The 2_2^+ , 4_2^+ , 6_1^+ , and 8_1^+ levels at 881.1, 1208.0, 1509.3, and 1970.0 keV comprise the deformed band built upon the 0_2^+ level at 824.5 keV. This division is based upon the B(E2) ratios of these levels. These ratios are given in the right-most column of Table II.

COMPARISON TO ANOTHER WORK

Our work on shape coexistence in ¹⁸⁸Hg was confirmed by Bourgeois *et al.*²⁰ There are some disagreements in spin-parity assignments for the higher levels not associated with the two bands discussed above. They reported the level at 2201 keV as a 7⁻ level, while the present measurements indicate it to be a $(4-6)^+$ level, since the α_K for the 291.7 keV transition indicates *E*1 character. The 8⁻ and

	Two- mixi	band ng ^a		Bohr	Experimental
Transitions	Ι	II	IBM ^b	Hamiltonian ^c	results
$\frac{2_2^+ \rightarrow 2_1^+}{2_2^+ \rightarrow 0_1^+}$	43.0 ^e	10.1°	87.2 ^e	54 ^e	15 ± 3^{d}
$\frac{4_1^+ \rightarrow 2_2^+}{4_1^+ \rightarrow 2_1^+}$	0.2	0.7	0.33 ^e	0.4 ^e	
$\frac{4_2^+ \rightarrow 2_2^+}{4_2^+ \rightarrow 2_1^+}$	77.6	6.5	15.2	16°	81 ±6
$\frac{6_1^+ \rightarrow 4_2^+}{6_1^+ \rightarrow 4_1^+}$	2.5	28.3	0.8 ^e	0.6 ^e	2.7±0.2
$\frac{6_2^+ \rightarrow 4_2^+}{6_2^+ \rightarrow 4_1^+}$	0.88	0.0	1.8	2.9 ^e	1.3±0.1
$\frac{8_2^+ \rightarrow 6_2^+}{8_2^+ \rightarrow 6_1^+}$	69.6 ^e	5.0 ^e	34.0 ^e	85°	45 ±10

TABLE II. Comparison of theoretical and experimental B(E2) ratios for levels in the decay of ¹⁸⁸Tl.

^aReference 37.

^bReference 38, and private communication.

^cReference 36.

^dReference 20.

^eNumbers not previously published.

 9^{-} levels at 2249 and 2471 keV, respectively, reported by Bourgeois *et al.*,²⁰ were populated too weakly to be confirmed in the present work.

The 714.1-keV transition is reported²⁰ as a pure E0, $0_3^+ \rightarrow 0_2^+$ transition. We, however, observe a 714.1-keV gamma ray. Nevertheless, the conversion coefficient of the 714.1-keV transition is larger than M1 and is assigned as E2+E0. It is, of course, possible that the peak is a doublet with one member being pure E0.

Bourgeois et al.²⁰ reported the level at 1239 keV as being 2⁺. Of the three populating transitions (215.7, 535.0, and 837.8 keV) and two depopulating transitions (826.7 and 1239.8 keV), only the multipolarity of the 837.8-keV transition is given. In our data the 1239.8-keV transition is unobserved. If the level is 2⁺, some decay to the ground state is expected. The 826.7-keV transition is assigned as M1/E2 in this work. Although the multipolarity of the 269.4-keV transition is unknown, the decay from a 6⁺ \rightarrow 2⁺ level is unexpected as this would be an E4 transition. Thus our assignment is 4⁺ for the 1239.6keV level. Earlier we had reported¹ a tentative level at 948 keV. In the absence of definite data, that level is no longer included in the level scheme.

COMPARISON WITH THEORY

Several different theoretical studies have been made proposing explanations of the structure in the light mass mercury nuclei. There is the proposal of bubble nuclei,^{21,22} potential energy surfaces with deformation energy from Strutinsky²³⁻³⁰ and Hartree-Fock calculations,³¹⁻³³ and band-mixing³⁴⁻³⁸ calculations. All of these works propose two bands built upon states of different deformation and structure.

All the potential energy calculations give surfaces with two minima at $\beta \simeq -0.1$ for a near spherical oblate shape and at $\beta \simeq 0.28$ for a more strongly deformed prolate shape. These minima agree well with the large and small deformations reported in the in-beam studies.³⁻⁶ A band of levels built upon these two deformations can then be constructed. Figure 8 shows the level energies from a calculation²⁸ of the two bands without mixing, as compared to the the experimental energies determined by this study. The agreement is quite good, the main discrepancy being in the softness of the ground band.

A further test of theory is provided by E2 branching ratios involving both bands. The measured ratios between reduced B(E2) transition rates, assuming pure E2 transitions, are given in the right-hand column of Table II. It has been proposed by Guttormsen³⁷ that these branching ratios are sensitive to whether the intrinsic quadrupole moment has the same or opposite sign in the two mixing bands. One of the assumptions is that the band-mixing Hamiltonian matrix element has the same sign for the different spins. In a phenomenological two-band mixing analysis, he found that the ¹⁸⁸Hg (and ^{184,186}Hg) data were much better described with quadrupole moments of the same sign than opposite sign (calculation I vs calculation II in Table II).

A theoretical picture, where the bands do have the same sign of the quadrupole moment, was recently obtained³⁸



FIG. 8. A comparison of the experimental and theoretical level energies vs I(I+1) for ¹⁸⁸Hg. The theoretical values are from Ref. 28.

on the basis of the interacting boson model (IBM2). The less deformed ground-state band is obtained with one (=82-80/2) proton boson, and the deformed band is obtained with the three proton bosons that would result from the excitation of a proton pair across the Z=82shell gap. These two IBM2 cores are mixed by a phenomenological interaction of the simplest possible form in terms of s and d bosons. With reasonable IBM2 parameters both bands are predominantly prolate, and furthermore the relevant mixing matrix elements do have the same sign. The branching ratios for IBM in Table II agree with experiment about as well as the two-band mixing calculation I.

Theoretical values based upon the potential-energy surface picture, where the quadrupole moments have opposite sign, have been obtained by solving the Bohr Hamiltonian in a potential $V(\beta,\gamma)$ with the appropriate topology. The depths of the two minima and an inertial mass parameter were adjusted phenomenologically to give a reasonable energy spectrum.³⁶ Surprisingly, in view of the above, the resulting "Bohr Hamiltonian" branching ratios in Table II agree with experiment about as well as the ones from IBM and mixing calculation I. In summary, all the theoretical calculations available at the present time reproduce the experimental data moderately well.

CONCLUSION

The 0^+ ground state, the 2^+_1 level at 412.9 keV, the 4^+_1 level at 1005.0 keV, the 6^+_2 level at 1777.2 keV, and the $(8)^+_2$ level at 2422.8 keV are members of a near-spherical band based on their energies, decay characteristics, and the systematics of the energies and level deformations established from lifetimes^{2,4,5} in ^{184,186}Hg. The 0^+_2 , 2^+_2 , 4^+_2 , 6^+_1 , and 8^+_1 levels at 824.5, 881.1, 1208.0, 1509.3, and



FIG. 9. The left half shows the levels in ¹⁸⁸Hg divided into two bands. The band built on the 0^+ level at 824 keV is the deformed band while the other band built upon the ground state (ground band) compares nicely with the ground band in ¹⁹⁰Hg data which is shown on the right half of the figure. The ¹⁹⁰Hg data are from Ref. 18.

1970.0 keV, respectively, are members of a well-deformed band, again, based on their energies, decay properties, and the systematics of the energies and lifetimes^{2,4,5} of this band in 184,186 Hg. Further evidence for a sharp change of

- *Present address: Louisiana State University, Baton Rouge, LA 70803.
- [†]Present address: Reactor Institute IRICKF, University of Delft, Delft, The Netherlands.
- [‡]Present address: Institute for Nuclear Study, University of Tokyo, Tokyo 188, Japan.
- Present address: Arkansas State University, Jonesboro, AR 72401.
- ¹J. H. Hamilton, A. V. Ramayya, E. L. Bosworth, W. Lourens, J. D. Cole, B. van Nooijen, G. Garcia-Bermudez, B. Martin, B. N. Subba Rao, H. Kawakami, L. L. Riedinger, C. R. Bingham, F. Turner, E. F. Zganjar, E. H. Spejewski, H. K. Carter, R. L. Mlekodaj, W. D. Schmidt-Ott, K. R. Baker, R. W. Fink, G. M. Gowdy, J. L. Wood, A. Xenoulis, B. D. Kern, K. J. Hofstetter, J. L. Weil, K. S. Toth, M. A. Ijaz, and K. S. R. Sastry, Phys. Rev. Lett. 35, 562 (1975).
- ²J. D. Cole, J. H. Hamilton, A. V. Ramayya, W. G. Nettles, H. Kawakami, E. H. Spejewski, M. A. Ijaz, K. S. Toth, R. L. Robinson, K. S. R. Sastry, J. Lin, F. T. Avignone, W. H. Brantley, and P. V. G. Rao, Phys. Rev. Lett. 37, 1185 (1976).
- ³D. Proetel, R. M. Diamond, P. Kienle, R. Leigh, K. H. Maier, and F. S. Stephens, Phys. Rev. Lett. **31**, 896 (1973).

shape between the bands is the fact that the 468.2-keV transition between the two 2^+ levels and the 203.2-keV transition between the two 4^+ levels both have large E0 components. This occurs in transitions between bands with different deformations because the E0 matrix element is sensitive to changes in the mean-square radius. One can fit the rotational energy level formula

$$E = E_0 + AI(I+1) + BI^2(I+1)^2$$

to the deformed band by using the 0^+_2 (824.5 keV), 6^+_1 (1509.3 keV), and 8^+_1 (1970.0 keV) level energies to yield A = 0.0170 keV and $B = 1.63 \times 10^{-5}$ keV. The calculated energies are then 825, 927, 1159, 1510, 1964, 2496, and 3078 keV for the deformed band. There is some perturbation at the $2^+, 4^+$ energies from mixing at the crossing. The ground band energies in ¹⁸⁸Hg compare very well with the energies of the ground band¹⁸ in ¹⁹⁰Hg shown in Fig. 9. It was found that when the deformed band becomes yrast, the near spherical ground band is not terminated. Below and above the crossing point the two bands "coexist." As noted earlier³⁹ these data helped confirm the long standing theoretical predictions of Hill and Wheeler,⁴⁰ and later many others, that we should find coexisting bands of levels built on quite different shapes.

ACKNOWLEDGMENTS

We wish to thank the ORIC staff for their help. We also wish to acknowledge the discussion with A. F. Barfield concerning the IBM calculations. UNISOR is a consortium of twelve institutions, supported by them and by the Office of Energy Research of the U.S. Department of Energy under Contract No. DE-AC05-760R00033 with Oak Ridge Associated Universities. This work was also supported through U.S. Department of Energy Contracts DE-AS05-76ER04935, DE-AS-76ER06034, DE-AS05-80ER10599, DE-AS05-76ER04936, and W-7405-eng-26.

- ⁴D. Proetel, R. M. Diamond, and F. S. Stephens, Phys. Lett. **48B**, 102 (1974).
- ⁵N. Rud, D. Ward, H. R. Andrews, R. L. Graham, and J. S. Geiger, Phys. Rev. Lett. **31**, 1421 (1973).
- ⁶D. Proetel, F. S. Stephens, and R. M. Diamond, Nucl. Phys. **A231**, 301 (1974).
- ⁷J. Bonn, G. Huber, H.-J. Kluge, L. Kugler, and E. W. Otten, Phys. Lett. **38B**, 308 (1972).
- ⁸T. Kühl, P. Dabkiewicz, C. Duke, H. Fischer, H.-J. Kluge, H. Kremmling, and E.-W. Otten, Phys. Rev. Lett. **39**, 180 (1977).
- ⁹D. Ward, in *Reactions Between Complex Nuclei*, edited by R. L. Robinson, F. K. McGowan, J. B. Ball, and J. H. Hamilton (North-Holland, Amsterdam, 1974), Vol. 2, p. 417.
- ¹⁰E. L. Bosworth, Ph.D. thesis, Vanderbilt University, 1974.
- ¹¹J. D. Cole, M. S. thesis, Vanderbilt University, 1976.
- ¹²J. D. Cole, Ph.D. thesis, Delft Technological University, 1978.
- ¹³E. H. Spejewski and R. L. Mlekodaj, in *Future Direction in Studies of Nuclei Far From Stability*, edited by J. H. Hamilton, E. H. Spejewski, C. R. Bingham, and E. F. Zganjar (North-Holland, Amsterdam, 1980), p. 63.
- ¹⁴J. T. Routti and S. G. Prussin, Nucl. Instrum. Methods 72, 125 (1969).

- ¹⁵K. Levenberg, Q. Appl. Math. 2, 164 (1944).
- ¹⁶D. W. Marquardt, J. Soc. Ind. Appl. Math. 2, 11 (1963).
- ¹⁷F. Rösel, H. M. Fries, K. Alder, and H. C. Pauli, At. Data Nucl. Data Tables 21, 291 (1978).
- ¹⁸C. R. Bingham, L. L. Riedinger, F. E. Turner, B. D. Kern, J. L. Weil, K. J. Hofstetter, J. Lin, E. F. Zganjar, A. V. Ramayya, J. H. Hamilton, J. L. Wood, G. M. Gowdy, R. W. Fink, E. H. Spejewski, W. D. Schmidt-Ott, R. L. Mlekodaj, H. K. Carter, and K. S. R. Sastry, Phys. Rev. C 14, 1586 (1976).
- ¹⁹J. D. Cole, A. V. Ramayya, J. H. Hamilton, H. Kawakami, B. van Nooijen, W. G. Nettles, L. L. Riedinger, F. E. Turner, C. R. Bingham, H. K. Carter, E. H. Spejewski, R. L. Mlekodaj, W. D. Schmidt-Ott, E. F. Zganjar, K. S. R. Sastry, F. T. Avignone, III, K. S. Toth, and M. A. Ijaz, Phys. Rev. C 16, 2010 (1977).
- ²⁰C. Bourgeois, M. Bouet, A. Caruette, A. Ferro, R. Foucher, J. Fournet, A. Höglund, L. Kotfila, G. Landois, C. F. Liang, B. Merlant, J. Obert, A. Peghaire, J. C. Putaux, J. L. Sarrouy, W. Watzig, A. Wojtasiewicz, and V. Berg, J. Phys. (Paris) 37, 49 (1976).
- ²¹P. Hornshøj, P. G. Hansen, B. Jonson, A. Lindahl, and O. B. Nielsen, Phys. Lett. **43B**, 377 (1973).
- ²²C. Y. Wong, Phys. Lett. **41B**, 451 (1972).
- ²³A. Faessler, U. Götz, B. Slavov, and T. Ledergerber, Phys. Lett. **39B**, 579 (1972).
- ²⁴U. Götz, H. C. Pauli, K. Alder, and K. Junker, Nucl. Phys. A192, 1 (1972).
- ²⁵S. G. Nilsson, J. R. Nix, P. Möller, and I. Ragnarsson, Nucl. Phys. A222, 221 (1974).

- ²⁶F. Dickman and K. Dietrich, Z. Phys. 271, 417 (1974).
- ²⁷P. Möller, S. G. Nilsson, and J. R. Nix, Nucl. Phys. **A229**, 292 (1974).
- ²⁸S. Frauendorf and V. V. Pashkevich, Phys. Lett. **55B**, 365 (1975).
- ²⁹K. Kumar, B. Remaud, P. Auger, J. S. Vaagen, A. C. Rester, R. Foucher, and J. H. Hamilton, Phys. Rev. C 16, 1235 (1977).
- ³⁰I. Ragnarsson, in *Future Directions in Structure of Nuclei far From Stability*, edited by J. H. Hamilton, E. H. Spejewski, C. R. Bingham, and E. F. Zganjar (North-Holland, Amsterdam, 1980), p. 367.
- ³¹M. Cailliau, J. Letessier, H. Flocard, and P. Quentin, Phys. Lett. **46B**, 11 (1973).
- ³²M. Girod and P. G. Reinhard, Phys. Lett. 117B, 1 (1982).
- ³³C. R. Praharaj and S. B. Khadkikar, J. Phys. G 6, 241 (1980).
- ³⁴F. Dickman and K. Dietrich, Z. Phys. 271, 417 (1974).
- ³⁵D. Kolb and C. Y. Wong, Nucl. Phys. A245, 205 (1975).
- ³⁶G. Leander, in Selected Topics in Nuclear Structure, edited by J. Styczen and R. Kulessa (Institute of Nuclear Physics, Jagiellonian University, Krakow, 1978), p. 621.
- ³⁷M. Guttormsen, Phys. Lett. 105B, 99 (1981).
- ³⁸A. F. Barfield, B. R. Barrett, K. A. Sage, and P. D. Duval, Z. Phys. A 311, 205 (1983).
- ³⁹J. H. Hamilton, in *Proceedings of the International Conference on Selected Topics in Nuclear Structure*, edited by V. G. Soloviev (Joint Institute for Nuclear Research, Dubna, USSR, 1976), Vol. 2, p. 303.
- ⁴⁰D. L. Hill and J. A. Wheeler, Phys. Rev. 89, 1102 (1953).