Response to spin-isospin transverse probes and pionlike excitations in finite nuclei

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We treat the response in a finite nucleus to spin-isospin sensitive probes in the transverse spin channel. Since linear momentum is here not a good quantum number, we adopt a formalism suggested by Toki and Weise, which incorporates an expansion in partial waves of good nuclear total angular momentum. The pion self-energy Π is nonlocal in momentum space; iterations of this quantity give the response function R, which is obtained by solving a matrix integral equation exactly (using numerical methods). We use R to renormalize the matrix element of a transverse spin-isospin probe and find large effects near the critical momentum region ($q \sim 2$–$3\,m_\pi$) and almost no effects for a small momentum transfer. We also find very important effects of the nonlocality in momentum space. We apply our formalism to the level with $J^P=1^+$; $T=1$ in ^{12}C. The results are compared with currently used approximations: the local density approximation, the infinite nuclear matter approximation, and the approximations of Toki and Weise. All these approximations fail to reproduce the exact results, especially for $q \rightarrow 0$ and near the critical momentum region. Using the infinite nuclear matter approximation, we find the best agreement for the equivalent constant density $\bar{\rho}=0.08$ fm^{-3} for low q; this agreement is not found, however, for any particular value of $\bar{\rho}$ when the value of q is either tiny or big. The approximation of Toki and Weise cannot be reliably used in our case, except possibly for $q \approx m_\pi$. The finite-nucleus results obtained are compared with the corresponding longitudinal response. The great similarity of the last two quantities, in contrast with Fermi gas estimates, is partially supported by new experimental data.

I. INTRODUCTION

The excitation of pionlike levels and the response of nuclear systems to spin-isospin sensitive probes has received considerable attention in the last years.[1–6] Some authors[2,7] have limited their treatment of this problem to the context of infinite nuclear matter, while others present finite-nucleus treatments.[8–11] There have also been studies of spin-isospin strength distribution effects in intermediate-energy reactions using the local density approximation.[12,13] In this work we study in detail the response to spin-isospin sensitive probes in the transverse spin channel and the excitation of pionlike levels in a finite nuclear system, and attempt to establish the degree of accuracy and the domain of validity of various approximations that have been introduced in the context of treatments for finite systems. (The longitudinal spin channel has been studied elsewhere.[11])

The conservation of linear momentum, characterizing the infinite nuclear matter, is no longer valid for finite nuclei. Instead, the response function (obtained by summation over particle-hole states) can be treated with well-defined angular momentum. The nonlocality in momentum space is a crucial feature in our subsequent treatment. We focus on pionlike excitations ($J^P=0^+$; $T=0 \rightarrow J^P=0^-,1^+,2^-,\ldots$; $T=1$) obtained by one-particle—one-hole (1p-1h) excitations, where the interaction is taken to be one-pion exchange with a repulsive term represented by the Migdal parameter g'.[22] In calculating the pion self-energy we adopt the formalism used generally, namely, we sum over 1p-1h excitations taking into account ring diagrams only. The formalism is applied to the study of the nuclear response to a transverse probe $\vec{\sigma}\times\hat{q}e^{i\vec{q}\cdot\vec{r}}\tau_\lambda$, where q is the momentum transfer to the nucleus. This response is studied for the $J^P=1^+$, $T=1$ excitation in ^{12}C for low, intermediate, and high momentum transfers ($q \simeq 0$–600 MeV/c). This range includes the critical momentum region (2–$3\,m_\pi$) as well as the $q \rightarrow 0$ region where the quenching related to the effects studied here is supposed to play a role in $M1$ and Gamow-Teller transitions.

We now give some short remarks on the importance of the operator studied here, in order to make our subsequent results more intelligible. The transverse-channel operator has been analyzed in the context of pionlike excitation for electron inelastic scattering. The (e,e′) reaction can be used for $M1$ excitations via the $(\vec{\sigma}\times\hat{q})\tau_3$ operator; the 1^+, $T=1$ state in ^{12}C at 15.11 MeV has been explored in detail at high momentum transfers by Sagawa *et al.*,[14] by Delorme *et al.*,[15] and by Toki and Weise[16] from various points of view. (The transverse nuclear response has been measured with inelastic electron scattering,[17] and studied for infinite nuclear matter in the sum rule approach by Alberico, Ericson, and Molinari[18,19] at the quasifree peak region.) The $(\vec{\sigma}\times\hat{q})\tau_\lambda$-type operator also appears in the nucleon-nucleon scattering amplitude used, for example, by Toki and Weise[8,16,4] for the analysis of the (p,p′) reaction. This reaction has been proposed, and was later applied,[20] for the excitation of pionlike levels. No tendency towards large critical effects in the longitudinal channel is found, but effects pertinent to the high value of g' ($g' \approx 0.7$) are probably existing. The transverse spin operator also appears in the amplitude of photopion reactions.[21] It is also worthy of note that a transition operator of the transverse-type is suitable for studying spin-isospin strength distribution effects in nuclei. For infinite nuclear matter this kind of operator is not properly aligned with the driving force for the pionic mode. It will therefore not be an appropriate tool to investigate the remnants of the so-called pionic soft mode behavior (which could have occurred at central nuclear densities for less repulsive short range correlations in the N-N interaction) in heavy nuclei. On the other hand, large surface effects make this restriction less severe for small nuclei (such as the ^{12}C nucleus studied here).

The paper is organized as follows: The required formalism is presented in Sec. II. We then study the nuclear response for low and medium (Sec. III B) and for high (Sec. III C) momentum transfers. In Sec. IV we give an overall comparison of numerical results obtained for the finite nucleus for an LDA treatment and for an infinite-matter approximation, as well as a comparison with the approximations proposed by Toki and Weise for handling finite systems. We find that our exact method is needed for a quantitative treatment of finite nuclei. For semi-quantitative estimates in the critical region one obtains more or less equivalent results from the approximate approaches noted above. In the low momentum region the finite nucleus treatment is needed even for semiquantitative studies, at least for light nuclei. The experimental response is discussed in Sec. V. We show that the transverse and longitudinal responses are very similar, in contrast with nuclear matter results, and this conclusion is in agreement with recent experimental (p,p′) data.[24]

II. FORMALISM

The formalism presented here is similar to that of Toki and Weise[8] and was already developed in our earlier paper.[11] We give here some formal comments to make the subsequent material more self-contained and understandable. We first deal with the pion self-energy in the context of finite nuclei. Iterations of the quantity will later give us the nuclear response function. The latter is then used for the renormalization of spin-isospin operators. The various quantities that enter are pertinent without reference to the character of the outside probe, that is, regardless of whether the probe is longitudinal, $\vec{\sigma}\cdot\hat{q}$, or transverse, $\vec{\sigma}\times\hat{q}$.

A. The pion self-energy for finite nuclei

This quantity is the finite-nucleus analog of the lowest-order pion self-energy Π^0 in infinite nuclear matter, presented diagrammatically in Fig. 1. It can be obtained by defining the tensor self-energy $\overleftrightarrow{\Pi}^0$ (or, in terms of spherical-component indices, $\Pi^0_{\eta\rho}$) such that for an excitation energy ω,

$$\langle \vec{q}\,' \,|\, \Pi^0(\omega) \,|\, \vec{q}\,\rangle = \hat{q}\,'\cdot\overleftrightarrow{\Pi}^0(\vec{q}\,',\vec{q};\omega)\cdot\hat{q}\ , \tag{1}$$

where we find for the static ($\omega=0$) tensor (see Ref. 11 for details)

$$\Pi^0_{\mu\nu}(\vec{q}\,',\vec{q}) = \sum_{J,M}\sum_{L,M_L}\sum_{L',M_L'}\left[\sum_{\mathrm{ph}(J)} Q^{JL}_{\mathrm{ph}}(q')\frac{-2}{E_\mathrm{p}-E_\mathrm{h}}Q^{*JL'}_{\mathrm{ph}}(q)\right]$$
$$\times Y_{LM_L}(\hat{q}\,')Y^*_{L'M_L'}(\hat{q})(-1)^{\mu}\langle LM_L 1-\mu \,|\, JM\rangle\langle L'M_L' 1\nu \,|\, JM\rangle\ . \tag{2}$$

In Eqs. (1) and (2), $\vec{q}$ and $\vec{q}\,'$ are the linear momenta of the incoming and outgoing pions, respectively, $E_\mathrm{p}-E_\mathrm{h}$ is the energy of the virtual particle-hole state, and the summation includes nucleon-hole and Δ-isobar-hole states (when the particle is a Δ, the energy $E_\mathrm{p}-E_\mathrm{h}$ includes the Δ-nucleon mass difference $2.2m_\pi$). The particle-hole state of good quantum numbers J and T is obtained by L-S coupling, restricting the summation over ph(J) to 1p-1h configurations of a definite angular momentum number J. The form factors are given by

$$Q^{JL}_{\mathrm{Nh}}(q) = f(q^2)\frac{q}{m_\pi}F^{JL}_{\mathrm{Nh}}(q) \tag{3a}$$

and

$$Q^{JL}_{\Delta\mathrm{h}}(q) = f^*(q^2)\frac{q}{m_\pi}F^{JL}_{\Delta\mathrm{h}}(q)\ , \tag{3b}$$

where m_π is the pion mass, $f(q^2)$ includes a form factor with cutoff Λ, namely,

$$f(q^2) = f\frac{\Lambda^2-m_\pi^2}{\Lambda^2-\omega^2+\vec{q}^{\,2}}\ , \tag{4}$$

and we use the values $\Lambda=1000$ MeV and $f^*(q^2)/f(q^2)=2$ throughout. The quantities $F^{JL}_{\mathrm{Nh}}(q)$ and $F^{JL}_{\Delta\mathrm{h}}(q)$ are

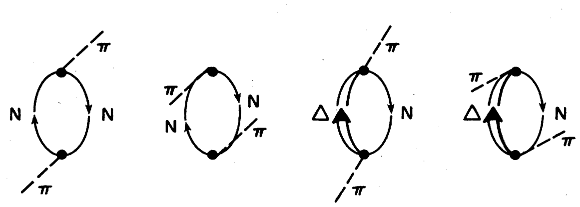

FIG. 1. Nucleon-hole and Δ(1236)-hole excitations ring diagram contribution to the pion self-energy in nuclear matter.

$$F_{\rm ph}^{JL}(q)=\eta_{\rm ph}[(2L+1)(2S+1)(2j_{\rm p}+1)(2j_{\rm h}+1)]^{1/2}\begin{Bmatrix} l_{\rm p} & s_{\rm p} & j_{\rm p} \\ l_{\rm h} & \frac{1}{2} & j_{\rm h} \\ L & S & J \end{Bmatrix}$$

$$\times(-i)^L[4\pi(2l_{\rm h}+1)]^{1/2}\langle L\,0 l_{\rm h} 0\,|\,l_{\rm p} 0\rangle \int_0^\infty dr\, r^2 j_L(qr)R_{\rm p}(r)R_{\rm h}(r)\;, \tag{5}$$

with $s_{\rm N}=\frac{1}{2},\eta_{\rm Nh}=2$ and $s_\Delta=\frac{3}{2},\eta_{\Delta{\rm h}}=\frac{4}{3}$ (see Oset *et al.*[4]); $S=1$ is implied in Eq. (5), and $R_{\rm p(h)}(r)$ are the radial wave functions (here taken in a harmonic oscillator basis).

The pion self-energy in a finite system can be decomposed as in Toki and Weise,[8]

$$\langle \vec{q}\,'\,|\,\Pi^0(\omega)\,|\,\vec{q}\,\rangle=\sum_J \frac{2J+1}{4\pi}\Pi_J^0(q',q;\omega)P_J(\hat{q}\,'\cdot\hat{q})\;, \tag{6}$$

where we use their notation: the partial wave $\Pi_J^0(q',q;\omega)$ is

$$\Pi_J^0(q',q;\omega)=\sum_{LL'} a_{JL}[\hat{\Pi}_J(q',q;\omega)]_{LL'}a_{JL'}\;, \tag{7}$$

the 2×2 matrix $\hat{\Pi}_J$ being given by

$$[\hat{\Pi}_J(q',q;\omega)]_{LL'}=-\sum_{{\rm ph}(J)} Q_{\rm ph}^{JL}(q')[(E_{\rm p}-E_{\rm h}-\omega)^{-1}+(E_{\rm p}-E_{\rm h}+\omega)^{-1}]Q_{\rm ph}^{*JL'}(q)\;, \tag{8}$$

and $a_{JL}=\langle J\;0\;1\;0\,|\,L\;\;0\rangle$ and $L,L'=J\pm1$. In developing our formalism we use $\Pi_{\mu\nu}^0$ (rather than Π_J^0) and thus get results for the most general spin components; in this respect our treatment is different from that of Toki and Weise, who focus primarily on longitudinal ($\vec{\sigma}\cdot\hat{q}$ and $\vec{\sigma}\cdot\hat{q}\,'$) coupling in constructing their response function.

In Eq. (8) the summation over ph states refers to states of definite J, and typically goes up to 10—$30\hbar\omega$ throughout the Periodic Table. As already hinted, we shall calculate for low-lying levels, and thus use $\omega=0$. A convenient base of states is that of the harmonic oscillator. Further details are given in Ref. 11.

The characteristic behavior of the matrix partial wave $\Pi_J^0(q',q)$ can be summarized as follows: (a) The contribution of the high-lying excitations for medium and large q values is very important, and amounts to as much as 200—300%. (b) The diagonal partial wave in momentum space $[\hat{\Pi}_J(q,q)]$ exhibits a pronounced peak (as a function of q). (c) The self-energy partial waves show large nonlocality in momentum space; they have a clear peak at $q=q'$ when $q\approx2$—$3\,m_\pi$, and nondiagonal (in momentum space) peaks for $q\lesssim m_\pi$ or $q\gtrsim 3m_\pi$. (d) The dominant sign of these partial waves is negative with relatively small positive parts when q' is either much smaller or much larger than q. This behavior is changed, however, for $q\lesssim m_\pi$ or $q\gtrsim 3m_\pi$, where we find large positive values of $\hat{\Pi}_J(q,q')$. (e) The Δ-isobar contribution to the self-energy is similar in shape to the nucleon one, and amounts at its peak to 10—25% of the corresponding nucleon contribution.

B. The response function to spin-isospin sensitive probes in finite nuclei

We seek the finite-nucleus equivalent of the iterated self-energy of Fig. 1. For the case of infinite nuclear matter this iteration results[1,7] in the diamesic function renormalization, which represents the many body random phase approximation (RPA) renormalization of the Fermi-gas self-energy. Including momentum space nonlocalities we use[8] an integral equation for the response function R, that replaces the operator geometric series of the infinite nuclear matter treatment:

$$\langle \vec{q}\,'\,|\,R(\omega)\,|\,\vec{q}\,\rangle=\langle \vec{q}\,'\,|\,\Pi(\omega)\,|\,\vec{q}\,\rangle+\int \frac{d^3k}{(2\pi)^3}\langle \vec{q}\,'\,|\,\Pi(\omega)\,|\,\vec{k}\,\rangle D(k,\omega)\langle \vec{k}\,|\,R(\omega)\,|\,\vec{q}\,\rangle\;. \tag{9}$$

In Eq. (9), D is the particle-hole interaction including a one-pion exchange (OPE) potential and a term for short-range effects, represented by the spin-isospin Migdal parameter g'.[22] Using the conventions of Sec. II A, one finds a partial wave expansion of the spherical tensorial response function which is equivalent to Eqs. (1) and (2). The partial wave matrix $\hat{R}_J(q,q')$ satisfies the integral equation

$$[\hat{R}_J(q',q)]_{LL'}=[\hat{\Pi}_J(q',q)]_{LL'}+\int_0^\infty \frac{k^2dk}{(2\pi)^3}\sum_{\lambda\lambda'}[\hat{\Pi}_J(q',k)]_{L\lambda}\left[a_{J\lambda}\frac{-1}{k^2+m_\pi^2}a_{J\lambda'}+\frac{g'}{k^2}\delta_{\lambda\lambda'}\right][\hat{R}_J(k,q)]_{\lambda'L'}\;, \tag{10}$$

where $\lambda,\lambda'=j\pm1$. This formula is correct both for the longitudinal and the transverse spin coupling.

We note in passing that the onset of the static ($\omega=0$) pionic soft mode occurs where

$$R_J(q',q)=\sum_{\lambda,\lambda'} a_{J\lambda}[\hat{R}_J(q',q)]_{\lambda\lambda'}a_{J\lambda'}$$

diverges. This happens for $g'\approx 0.32$ in our case in the critical momentum region ($q\approx 2-3\,m_\pi$), a number much below the currently accepted value of $g'\approx 0.7$. (We note, however, that this critical behavior is predicted to occur for any value of g', provided the effective nuclear density is sufficiently high; critical phenomena are the results of an interplay between the value of g' and the nuclear density.) In the following we present results for g' in the region 0.7—0.4, which roughly spans the range from the currently accepted value to a value that gives pion condensation for central nuclear densities.

In dealing with finite nuclei, we are interested in the degree of nonlocality in $\hat{R}_J(q,q')$. We find that $\hat{R}_J$ follows $\hat{\Pi}_J$ in the sense of having a diagonal peak in momentum variables for $q\approx 2m_\pi$ and a nondiagonal peak for $q\approx m_\pi$ and $q\approx 4m_\pi$. We also note that the OPE term in Eq. (10) results in a coupling which gives rise to large nondiagonal elements of the 2×2 matrix $\hat{R}_J$.

C. The renormalized matrix element of a transverse probe

The formal comments of this subsection [Eqs. (11)—(13)] are identical to material[8] in the literature, but are required here to make the subsequent results intelligible and self-contained. We consider the renormalized matrix element of a transverse spin-isospin operator (valid for $J^P=1^+,2^-,\ldots;\ T=1$)

$$\mathcal{T}^{(\mathrm{ren})}_{fi_\mu}(\vec{q})=\langle \mathrm{ph}(JM;TM_T)\,|\,(\vec{\sigma}\times\hat{q})_\mu e^{i\vec{q}\cdot\vec{r}}\tau_\lambda\,|\,0\rangle_{(\mathrm{ren})}$$

$$=\delta_{T1}\delta_{m_T\lambda}(-i)\sum_{LL'}\int_0^\infty \frac{k^2dk}{(2\pi)^3}F^{*JL}_{\mathrm{ph}}(k)[\hat{\epsilon}_J^{-1}(k,q)]_{LL'}b_{JL'}\langle JM-\mu 1\mu\,|\,JM\rangle Y^*_{JM-\mu}(\hat{q})\ , \tag{11}$$

where $F^{*JL}_{\mathrm{ph}}(k)$ are given in Eq. (5),

$$b_{JL}=\begin{cases}\left[\dfrac{J+1}{2J+1}\right]^{1/2}, & L=J-1\\[2ex] \left[\dfrac{J}{2J+1}\right]^{1/2}, & L=J+1\end{cases} \tag{12}$$

and the diamesic matrix function is a solution of the equation

$$\int_0^\infty \frac{k^2dk}{(2\pi)^3}\hat{\epsilon}_J(q,k)\hat{R}_J(k,q')=\hat{\Pi}_J(q,q')\ . \tag{13}$$

Defining the reduced element T_J by the relation

$$\mathcal{T}^{(\mathrm{ren})}_{fi_\mu}(\vec{q})=\delta_{T1}\delta_{M_T\lambda}T_J^{(\mathrm{ren})}(q)\langle JM-\mu 1\mu\,|\,JM\rangle Y^*_{JM-\mu}(\hat{q}) \tag{14}$$

we can write it as a sum of the P-subspace contribution (the subspace of configurations treated directly—no renormalization effects) and a second part, generated by the renormalization effects:

$$T_J^{(\mathrm{ren})}(q)=-i\left\{\sum_L F^{*JL}_{\mathrm{ph}}(q)b_{JL}+\sum_{L,L'}\int_0^\infty \frac{k^2dk}{(2\pi)^3}F^{*JL}_{\mathrm{ph}}(k)\left[\sum_\lambda \hat{D}_J(k)_{L\lambda}\hat{R}_J(k,q)_{\lambda L'}\right]b_{JL'}\right\}\ . \tag{15}$$

D. The P and Q subspaces

A complete calculation of the excitation of pionlike modes, using a sufficiently large model space, would automatically include the precursor enhancements without a special treatment. In the present approach—and in most similar investigations[1–13] —we study the renormalization of a spin-isospin sensitive probe by higher configurations when only a small P subspace is explicitly treated. The problem we face is to avoid the double counting of configurations that are included in the P subspace and that also contributed in the iterations for $\hat{\Pi}_J$ and $\hat{R}_J$. Toki and Weise[8] calculate the pion self-energy and the diamesic renormalization in the residual $Q=1-P$ subspace only. As discussed in detail in Ref. 11, this is not a completely satisfactory procedure, and a systematic expansion in the off-diagonal interaction terms connecting the two spaces is required. In such an expansion the prediagonalization (P subspace) result is the zeroth-order term while the renormalization treated in Q subspace only is a first order result. This problem has not yet been fully treated in the literature, and we believe that the results of the exact approach will lie in the region between the $P+Q$ and the Q results.

III. NUMERICAL RESULTS AND DISCUSSION

A. General comments

The results presented were calculated for ^{12}C for the residual Q space, where the $0\hbar\omega$ configuration (we take advantage of the fact that this is an almost pure configuration) $(1p_{3/2}^{-1})(1p_{1/2})$ is the P subspace. (Calculations performed in the complete $P+Q$ space yield quite unreasonable results in disagreement with our expectations from previous studies[8,11] and from nuclear matter calculations of the present renormalization. This, however, is not always the case: See Ref. 11 for a study of the $J^P=0^-$ level in ^{16}O where the complete space calculations are needed. The present case is presumably a consequence of the overly large contribution of the $0\hbar\omega$ configuration.) In this case the harmonic oscillator variable is $\alpha=129.5$ MeV, and the summation over particle-hole configurations goes up to $10\hbar\omega$ (in the momentum range considered). These high principal quantum numbers result in serious numerical complications, and care must be taken when handling the high-lying p-h states. (Calculating these we found it necessary to use double precision in various CDC computers.)

In order to get a solution of the integral equation [Eq. (10)] we needed a total of ~ 31 Simpson integration points (a somewhat lower number of Gaussian points is needed). These numbers are smaller for $q\approx q'$, higher when q and q' are appreciably different, and also depend on the magnitude of q and on the proximity to critical behavior.

B. The low and intermediate momentum region

We first look into the very low to medium momentum transfer range. As indicated in Ref. 4, it is customary to assume that when the P space incorporates essentially all the important excitations of purely nucleonic character, the diamesic function builds from Δ-hole configurations only, in the long wavelength limit ($q,q' \ll m_\pi$). This should imply[4] a quenching of all spin-isospin dependent phenomena at low momentum transfer, at least for heavy nuclei. This mechanism has been suggested to understand the systematic reduction of strength in Gamow-Teller and magnetic isovector transitions as compared to shell-model calculations (see Oset, Toki, and Weise,[4] and references therein). Pionlike excitations may thus be able to emphasize the role of Δ degrees of freedom in nuclei, even in the long wavelength limit. (We note that recent studies[25] of isoscalar versus isovector $0^+\to 1^+$ transitions, as well as transitions to high-spin states where precursor phenomena in finite nuclei are not expected at all, indicate that other sources of quenching in both isoscalar and isovector channels should exist.)

In Table I we present results for the renormalized reduced element $T_J^{(\text{ren})}$ [see Eqs. (14) and (15)] compared to the corresponding nonrenormalized quantity $T_J^{(0)}$. We actually find that most of the renormalization is contributed by the lowest Δ-hole configurations; other, more high-lying ones, including higher nucleon-hole configurations, begin to contribute appreciably at $q \gtrsim 100$ MeV/c. The effect of those high configurations alone (no $0\hbar\omega$-Δ-hole configurations included) is demonstrated in Table I by $iT_J^{(H\text{-ren})}$, and is evidently very small for $q<100$ MeV/c. Table I also shows that the renormalization effects are completely negligible for $q\to 0$. *We do not find the above-mentioned quenching for this light nucleus.* This is especially interesting since in corresponding infinite-nuclear-matter or local-density-approximation calculations one finds a constant quenching for $q\to 0$. We shall discuss this point in detail in Sec. IV.

TABLE I. Calculational results for the renormalized matrix element of the transverse probe $\vec{\sigma}\times\hat{q}e^{i\vec{q}\cdot\vec{r}}$ in the residual Q subspace for $J^P=1^+$; $T=1$ in ^{12}C. The value of g' is 0.7, and the notation is explained in the text.

q (MeV/c)	$iT_J^{(0)}$	$iT_J^{(\text{ren})}$	$iT_J^{(H\text{-ren})}$
10	16.34	16.31	
30	16.05	15.61	
50	15.48	14.51	15.44
70	14.66	13.04	
100	13.05	10.73	12.49
150	9.70	6.52	8.25

C. The high momentum region

Results for the renormalized matrix element compared to the nonrenormalized one are given in Table II. The main contribution in this kinematical region is from very high-lying configurations (up to $\sim 10\hbar\omega$). The contribution of the $0\hbar\omega$-Δ-hole configurations is of the order of 20% only for these high momentum transfers. (For $q=350$ MeV/c and $g'=0.7$, for example, we find that $iT_J^{(\text{ren})}$ is -1.07 for the full renormalization case, while it has the value of -1.30 when the lowest Δ-hole configurations are not included.) The importance of nonlocality of the response function in momentum space is evident. In the vicinity of the minimum of $T_J^{(0)}$, the renormalized quantity $T_J^{(\text{ren})}$ collects strength from other regions. This is especially the case for $q\approx 2$–$3\,m_\pi$, i.e., for the critical momentum region. The reason for this is the proximity

TABLE II. The renormalized matrix element of the transverse probe $\vec{\sigma}\times\hat{q}e^{i\vec{q}\cdot\vec{r}}$ in the residual Q subspace for $J^P=1^+$, $T=1$ in ^{12}C and for high momenta. The notation is explained in the text.

q			$iT_J^{(\text{ren})}$	
(MeV/c)	$iT_J^{(0)}$	$g'=0.40$	$g'=0.55$	$g'=0.70$
200	6.3	3.3	3.4	3.2
250	3.4			0.6
300	1.4	-0.9	-0.8	-0.9
350	0.2			-1.1
400	-0.3	-0.9	-0.8	-0.6
500	-0.3			0.2
600	-0.13			0.25

of a phase transition which could have occurred for $g' \simeq 0.4$ (namely, pion condensation or nuclear critical opalescence), and seems to enhance nonlocal effects. We recall that one might expect that an operator such as $M1$, being proportional to $\vec{\sigma}\times\hat{q}$ (i.e., not aligned with the pion source operator $\vec{\sigma}\cdot\hat{q}$), is not an appropriate tool for looking into pionic modes. It has been argued,[4] however, that this is correct for heavy nuclei, but the relatively large surface of light nuclei makes such restrictions less severe in the latter case. The moderate effects found here compared with the longitudinal probe case,[11] and, in particular, the relatively weak dependence on the crucial parameter of the precritical effects, namely, g', do not encourage the study of spin-isospin strength distribution effects with transverse probes.

This behavior of the renormalized matrix element can be qualitatively explained on the basis of general arguments given by Osterfeld, Suzuki, Speth, and Krewald (see, e.g., Ref. 27). The elementary $M1$ operator does not depend on the radial coordinate. It is thus only sensitive to Q-space ph excitations which have the same radial wave functions as those of the P space. The nuclear operator, on the other hand, does have a radial dependence which, in fact, becomes more and more important when the momentum transfer q is increased. This can be seen, for instance, in Eqs. (5), (10), and (15). Thus, the effect of renormalization is expected to be more and more significant with increasing values of q. For light nuclei these effects will be very small for low q; this result is not produced in a nuclear matter calculation. For the case of ^{12}C only the $0\hbar\omega$ Δh excitation can contribute at very low q, while higher $(2\hbar\omega,\ldots)$ excitations enter only for higher values of q. The actual shape of the finite-nucleus response is a result of the finiteness of the system and the surface, which is not present in the case of nuclear matter (Sec. IV).

IV. COMPARISON OF NUMERICAL RESULTS WITH SOME OTHER APPROACHES

In this section we compare the results of our treatment with some other numerical results based on approximations commonly used in the current literature. These approximations include: (a) the simplest approximation of nuclear matter with a constant, effective nuclear density $\bar{\rho}$ (not necessarily the central density ρ_0); (b) the more satisfactory local density approximation (LDA); and (c) the approximations of Toki and Weise.[8]

We present an overall comparison of the ratio

$$\mathscr{R}_J(q) = T_J^{(\text{ren})}(q)/T_J^{(0)}(q)$$

for the $J^P = 1^+$, $T=1$ level of ^{12}C studied in this work.

A. The local density approximation

The use of the LDA is based on introducing a density with radial dependence, taken here in the Fermi form

$$\rho(r) = \frac{3}{4\pi c^3}\left\{\left[1+\frac{\pi^2 t^2}{c^2}\right]\left[1+\exp\left[\frac{r-c}{t}\right]\right]\right\}^{-1}, \qquad (16)$$

where c denotes the nuclear half-density radius, and $a=4.40t$ is the nuclear surface thickness. This density determines a local Fermi momentum

$$p_F(\rho) = [\tfrac{3}{2}\pi^2 A\rho(r)]^{1/3} \qquad (17)$$

which then appears in the Lindhard function[23] used for spin-isospin operator renormalization in the Fermi-gas model.[7] This assumes that the nuclear density is slowly varying so that it is meaningful at each point to assign a local Fermi momentum, determined from the nuclear density at that point, and then to calculate in the corresponding Fermi gas. Naturally this is not necessarily a very satisfactory approximation, and we wish here to study its validity by comparing it to the treatment appropriate to a finite system.

Using the LDA in the Fermi-gas model for the operator renormalization we have[7] for the static case

$$[(\vec{\sigma}\times q)_j e^{i\vec{q}\cdot\vec{r}}\tau_\lambda]_{\text{ren}} = (\vec{\sigma}\times\hat{q})_j \tau_\lambda e^{i\vec{q}\cdot\vec{r}}/[1+g'U(q,0)]\,, \qquad (18)$$

where the renormalization is given in terms of

$$\epsilon'(k,0) = 1+g'U(k,0) = 1+g'\frac{f^2}{m_\pi^2}\left[\frac{\Lambda^2-m_\pi^2}{\Lambda^2+k^2}\right]^2\left[\frac{2M^*p_F}{\pi^2}L\left[\frac{k}{2p_F}\right]+\frac{32}{9}\frac{A\rho(r)}{\omega_\Delta}\right]. \qquad (19)$$

In Eq. (19), $\omega_\Delta = 2.2m_\pi$ is the Δ-nucleon mass difference, M^* is the effective nucleon mass taken as 0.8 times the free nucleon mass, and

$$L(X) = \tfrac{1}{2}\left[1+\frac{1-X^2}{2X}\ln\left[\left|\frac{1+X}{1-X}\right|\right]\right]. \qquad (20)$$

Using this renormalization, the ratio $\mathscr{R}_J(q)$ is given by

$$\mathscr{R}_J(q) = \left[\sum_L F_{\text{ph}}^{*JL}(\epsilon',q)b_{JL}\right]\Bigg/\left[\sum_L F_{\text{ph}}^{*JL}(q)b_{JL}\right]. \qquad (21)$$

The quantities b_{JL} and $F_{\text{ph}}^{*JL}(q)$ of Eq. (21) are defined in Eqs. (12) and (5), while $F_{\text{ph}}^{*JL}(\epsilon',q)$ is obtained from $F_{\text{ph}}^{*JL}(q)$ upon replacement of the nonrenormalized radial integral in Eq. (5) by the renormalized one

$$\int_0^\infty dr\, r^2\epsilon'^{-1}[\rho(r)]j_L(qr)R_{\text{p}}^*(r)R_{\text{h}}(r)\,.$$

Similar applications of the LDA, where the radial dependence (in configuration space) of $\rho(r)$ is taken into account, are given in Refs. 12, 13, and 26.

In the very low momentum region one uses only the Δ-isobar part of the renormalization in Eqs. (18)–(20). This

TABLE III. Numerical results for the ratio $\mathscr{R}_J(q)$ calculated for the $J^P=1^+$; $T=1$ level of ^{12}C. The results refer to a finite nucleus (FN) and to an LDA calculation. (The dependence on g' is relatively weak for the former, and we thus give the FN results only for $g'=0.7$). The parameters for the LDA case were taken as $a=2.29$ fm and $c=2.36$ fm.

q			LDA	
(MeV/c)	FN	$g'=0.40$	$g'=0.55$	$g'=0.70$
50	0.94	0.88	0.85	0.82
100	0.82	0.88	0.84	0.81
150	0.67	0.58	0.50	0.44
200	0.5	0.57	0.49	0.43
300	−0.6	0.47	0.38	0.31
400	2.0	1.02	0.98	0.93
500	−0.7	0.75	0.68	0.61
600	−2.0	0.68	0.60	0.52

may be obtained by dropping the first term in the square brackets of Eq. (19). This results in an almost constant ratio $\mathscr{R}_J$, which turns out to be 0.88, 0.85, and 0.82 for $g'=0.4$, 0.55, and 0.70, respectively. This feature is essentially different from our finite nucleus results (see Sec. III B), in which $\mathscr{R}_J(q) \underset{q\to 0}{\longrightarrow} 1$.

In Table III we give a detailed comparison of the ratio $\mathscr{R}_J(q)$ for finite nuclei (in the column denoted by FN) and for the LDA. Since there is no simple way to separate the low-lying nucleon-hole configurations from the high-lying ones in ϵ' of Eq. (19), we have included the full nucleon Lindhard function for $q \geq 150$ MeV/c. This lowers the results for $q < 300$ MeV/c by about 30%, and has little effect for $q \geq 400$ MeV/c. However, it is not an important source of error, since we have found earlier[11] that LDA results agree much better with finite nucleus calculations in the residual Q subspace than they do with the $P+Q$ results, although the Lindhard functions include, in principle, the complete particle-hole space. A discussion of our LDA results is given in the following subsection.

B. Nuclear matter with a constant density

Closely related to the LDA is the use of a constant nuclear density, $\bar{\rho}$, as the simplest approximation to a finite nucleus. The density $\bar{\rho}$ may be different from the central one ρ_0, since it is well known that light nuclei have a relatively low density on the average. Results for $\bar{\rho}=0.17$, 0.14, 0.11, and 0.08 nucleons/fm^3 are shown in Fig. 2, and are also compared with the LDA results. The agreement is not good, partly due to the different weighing of the various parts of the nucleus and partly due to the behavior of the harmonic oscillator wave functions (which give rise to a minimum in the form factor at $q \approx 2\alpha$). At very low momenta the LDA agrees best with $\bar{\rho}=0.08$ fm^{-3} (rather than $\bar{\rho}=\rho_0=0.17$ fm^{-3} which is frequently used in nuclear matter approximate calculations; the corresponding number found[11] for ^{16}O was $\bar{\rho}=0.12$ cm^{-3}). The $\bar{\rho}$ approximation also gives a constant ratio $\mathscr{R}_J(q)$ for $q \to 0$; this ratio turns out to be 0.78, 0.72, and 0.66 for $\bar{\rho}=0.17$ fm^{-3}; and 0.88, 0.84, and 0.81 for $\bar{\rho}=0.08$, for $g'=0.4$, 0.55, and 0.7, respectively, taken as representative cases.

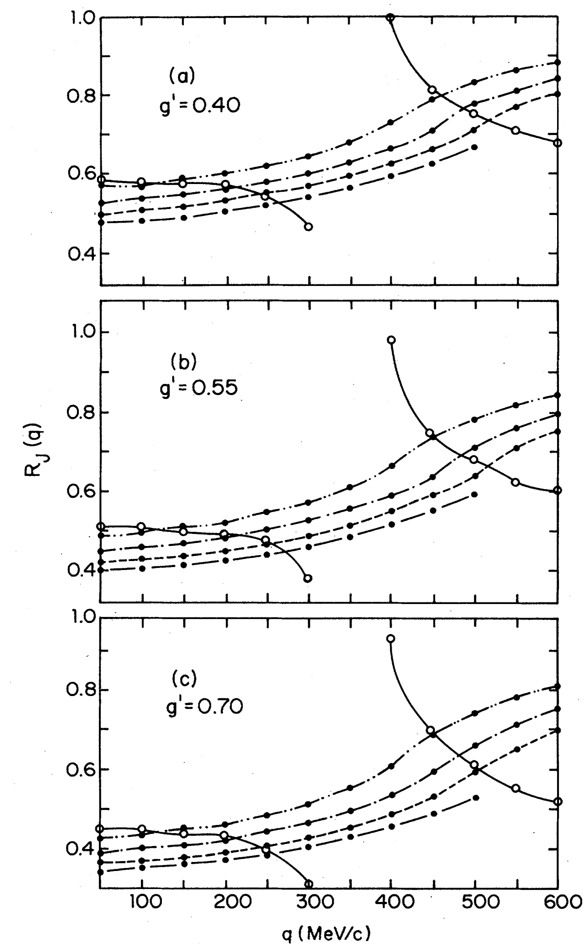

FIG. 2. The ratio $\mathscr{R}_J(q)$ as a function of q and with g' as a parameter. The results refer to constant density calculations with $\bar{\rho}=0.17$ (long-dashed line), 0.14 (short-dashed line), 0.11 (dashed-dotted line), and 0.08 fm^{-3} (dashed-double-dotted line). A comparison with the corresponding LDA results is shown (full line). The parameters for the LDA calculation are as in Table III, where a comparison with the finite nucleus results is given.

It is not surprising that the LDA and $\bar{\rho}$ results are appreciably different from the exact finite-nucleus results, since in the transverse case one expects large effects of the finiteness of the nuclear system, such as important surface effects, and thus the results are greatly modified by using a finite system formalism in the calculations.

C. The approximations of Toki and Weise

Reference 8 applies several approximations to the calculation of the response function in the spin-isospin channel for finite nuclei. In this subsection we apply these approximations and compare the results with our exact calculations of Sec. III.

Beginning with the pion self-energy, Toki and Weise assume (for $q \gtrsim m_\pi$)

$$\hat{\Pi}_J(q,q') = \hat{\Pi}_J(\bar{q},\bar{q}) d_J(q-q') , \tag{22}$$

where $\bar{q}$ is the mean value of q and q', and the distribution d_J satisfies $d_J(0)=1$. This function is chosen as

$$d_J(k)=j_0(kr_J)\ , \tag{23}$$

where r_J^{-1} measures the momentum-space degree of nonlocality generated by the finiteness of the system. Turning back to Ref. 11, we note that this is a very unsatisfactory approximation. For the present 1^+ case we found a number of values for r_J in the range 1.89—3.78 fm and use the intermediate one $r_J\cong 2.5$ fm. For the diamesic renormalization matrix Toki and Weise[8] find

$$\hat{\epsilon}_J(\bar{q})=1-\frac{\bar{q}^2\gamma_J}{8\pi^2 r_J}\hat{D}_J(\bar{q})\hat{\Pi}_J(\bar{q},\bar{q})\ , \tag{24}$$

where $\gamma_J\approx 0.9$ for light nuclei and $\hat{D}_J(k)$ is the particle-hole interaction matrix which has already been introduced in Eqs. (9) and (10),

$$[\hat{D}_J(k)]_{LL'}=a_{JL}\frac{-1}{k^2+m_\pi^2}a_{JL'}+\frac{g'}{k^2}\delta_{LL'}\ . \tag{25}$$

Some numerical tests of these approximations are reported in Ref. 11; we have performed two other numerical comparisons directly related to the present transverse case. The most careful one consists of replacing the exact self-energy by the distribution given in Eq. (23), but solving exactly the integral equation (10) for this new distribution. This results, in addition to a systematic and severe underestimation of the response function, also in results radically different from the exact ones (although for $q\approx m_\pi$ the agreement is quite good). The numbers are given in Table IV, where we also show some results pertaining to the use of a δ-function approximation for the momentum space nonlocality. In this approximation, used by Toki and Weise[8] for heavy nuclei, the distribution of Eq. (22) is taken to be

$$d_J(q-q')\approx\frac{\pi}{r_J}\delta(q-q')\ . \tag{26}$$

Although this approximation is used by Toki and Weise for large nuclei, we test it here *just for completeness.* Under this approximation the ratio $\mathscr{R}_J(q)$ is given by

$$\left\{\sum_{LL'}F_{\rm ph}^{*JL}(q)[\hat{\epsilon}_J^{-1}(q)]_{LL'}b_{JL'}\right\}\Big/\left\{\sum_L F_{\rm ph}^{*JL}(q)b_{JL}\right\}\ .$$

We note that this approximation gives completely unsatisfactory results, stressing once again the importance of nonlocal effects and a complete finite nucleus treatment, especially for this present case of a transverse probe and a small nucleus.

The approximation of Toki and Weise were found to be unsuccessful in both the longitudinal and the transverse channels. The reason for this, as emerges from calculations in which we have checked the importance of the longitudinal versus transverse pieces of the 1p-1h interaction (including the ρ exchange contribution), is as follows. Although the Bessel function distribution may *look* rather similar to the actual exact shape of $[\hat{\Pi}_J(q,q')]_{LL'}$, it does not give correct results for the renormalized matrix element in both channels. This is the case because this approximation brings about spurious components which are aligned with the driving (longitudinal or transverse) interaction, and have large effects. On the other hand, a careful and correct approximation (for a specific channel) would not cause any serious errors even when it differs appreciably from the actual distribution in momentum space, provided that the differences are orthogonal to the dominant channel.

TABLE IV. Results related to the approximations of Toki and Weise for the renormalized matrix element of the transverse probe $\vec{\sigma}\times\hat{q}e^{i\vec{q}\cdot\vec{r}}$ in the residual Q subspace for $J^P=1^+$; $T=1$ in ^{12}C. The results, given for $g'=0.7$, refer to the Bessel function distribution of Eq. (23) and to a δ-function distribution (explained in the text) and should be compared to the exact results of Table II.

q (MeV/c)	$iT_J^{(0)}$	$iT_J^{(\rm ren)}$ Bessel	$iT_J^{(\rm ren)}$ δ function
150	9.7	6.8	6.8
350	0.2		0.0
400	−0.3	0.9	−0.1
500	−0.3	1.5	−0.1

V. THE EXPERIMENTAL RESPONSE

The data of Haji-Saeid *et al.*[20] did not show any critical effects in (p,p′) at 800 MeV, where the $J^P=1^+$, $T=1$ level in ^{12}C was studied. Although we have shown here that the approximate finite nucleus calculations used by these authors are not accurate, the results of Ref. 20 obviously indicate very small critical effects, as do other experimental data.[29]

More direct comparison with our calculations can be obtained from measurements of the longitudinal and transverse responses themselves. This has been carried out in a very recent experiment by Carey *et al.*[24] Their measurement is for the quasifree peak at momentum transfer $q=350$ MeV/c. Nevertheless, a comparison with our calculation can still be made at the semiquantitative level, bearing in mind the following points: (a) The quasifree peak region is much less sensitive to critical spin-isospin phenomena than the low-lying pionlike excitations we deal with, especially because of the relatively high energy transfers (of order $q^2/2M_N$) involved in quasifree excitations. (b) Carey *et al.*[24] compare their results with the Fermi gas calculations of Alberico, Ericson, and Molinari[2,18,19] which give the longitudinal and transverse responses relative to the free Fermi gas case, and correspond to the ratios $\mathscr{R}_J^{(L,T)}(q)$ given in Ref. 11 and in the present work. (c) The large negative values of $\mathscr{R}_J^{(L,T)}$ at $q\sim 250$—350 MeV/c are the result of a nonlocal collection of strength from a low minimum occurring in the nonrenormalized matrix elements $L_J^{(0)}(q)$ and $T_J^{(0)}(q)$, and are thus model dependent and may not appear in the quasifree region.

In Fig. 3 we present the ratios $\mathscr{R}_J^{(L)}$ and $\mathscr{R}_J^{(T)}$ against q. The figure shows the great similarity in the behavior of the longitudinal and transverse responses, in contrast with Fermi gas estimates (where the longitudinal response

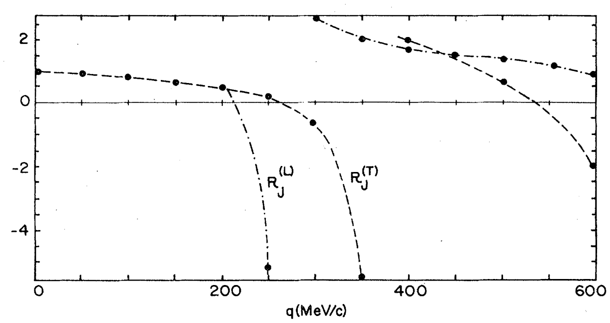

FIG. 3. The exact finite nucleus ratios $\mathscr{R}_J^{(L,T)}(q)$ for the longitudinal and transverse renormalized responses. The value of g' is 0.7.

displays the characteristic enhancement, while the transverse response is quenched). Our calculations thus make it perfectly possible that in the quasifree peak region the ratio $\mathscr{R}_J^{(L)}/\mathscr{R}_J^{(t)}$ would be unity in an exact finite nucleus calculation, as the authors of Ref. 24 find for Pb at $q=350$ MeV/c. This does not mean that spin-isospin strength distribution effects are not present. On the other hand, these effects are not the only important correction to the model (P) space results, as discussed in Refs. 30 and 25.

VI. SUMMARY AND CONCLUSIONS

In this work we have analyzed spin-isospin strength distribution effects for the transverse spin channel $\vec{\sigma}\times\hat{q}e^{i\vec{q}\cdot\vec{r}}$ and for a small nucleus, where surface effects are expected to be strong. Using exact finite nucleus formalism and numerical procedures, we have obtained results for low, medium, and high momentum transfers and have examined in detail some approximate approaches of common use in the current literature. Although these calculations are complicated and require a lot of computer time, one has to apply this exact method for studying these effects, especially for $q\rightarrow 0$ and for $q\approx 2-3\,m_\pi$. We have also found that transverse probes are not good candidates for studying spin-isopin strength distribution effects.

A major modification which we now intend to put into our formalism is the effect of the ρ-meson exchange on spin-isospin strength distribution phenomena. This can be done by modifying the particle-hole interaction [Eq. (25)] in such a way that it includes the ρ-meson exchange term explicitly, as suggested by Delorme *et al.*[15] and by Toki and Weise.[16] We note, however, that for $q\rightarrow 0$ only the repulsive part (g') plays a role in the particle-hole interaction used here, and that for a finite nucleus the OPE term is important in both the longitudinal and the transverse channels. (Moreover, some new considerations[28] may prove that the ρ contribution to the N-N interaction is rather weak, a fact that would make its explicit appearance in our formulae unnecessary.) It remains now to see how this modification would actually affect our present finite nucleus results, which are intended to serve as a guide for further study along these lines.

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