Isospin mixing in the decay of the $T_{>}$ giant dipole state

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The magnitude of the isospin mixing in the decay of the $T_>$ giant dipole resonance has been estimated, using the (γ, n) and (γ, p) cross sections available for the medium-weight nuclei ⁶⁰Ni, ⁸⁸Sr, ⁸⁹Y, ⁹⁰Zr, and ⁹²Mo. The deduced values show a fair correspondence with the existing data for mixing between compound states. From these results the mean mixing Coulomb matrix elements between compound states could also be derived.

I. INTRODUCTION

Isospin symmetry breaking in nuclei, generated by the Coulomb interaction, may cause decay of excited states into channels that are otherwise closed. The resulting isospin mixing can then be studied from a comparison of an isospin allowed with an isospin forbidden channel. As far as photonuclear reactions are concerned, isospin mixing in the giant dipole resonance (GDR) can be determined, for self-conjugate nuclei, by means of the Barker and Mann formalism,¹ or from (γ, α) reaction results. However, for non-self-conjugate nuclei (with ground state isospin T), both $T_{<} = T$ and $T_{>} = T + 1$ coherent dipole states are excited. The decay of the coherent $T_{>}$ dipole state (whose existence is at present firmly established²⁻⁴) into $T-\frac{1}{2}$ neutron channels is forbidden by the isospin selection rules. Unlike the $T + \frac{1}{2}$ proton channels, the isospin-allowed $T + \frac{1}{2}$ neutron channels are located at high excitation energies due to the large Coulomb displacement energy for medium-heavy nuclei, so that the $T_{>}$ decay in the neutron channel is much reduced. Consequently, as far as the decay of the $T_{>}$ resonance is concerned, proton and neutron channels can be considered as the isospin allowed and forbidden channels, respectively.

Starting from this point of view we have further analyzed⁵ our existing $^{89}Y(\gamma,p)^{88}Sr$ data, the results of which are being used in this paper in order to deduce an estimate for the isospin mixing parameter and to study the role and nature of the isospin mixing mechanism in the decay of the $T_{>}$ GDR. Simultaneously, we have made an analysis of (be they less accurate) photonuclear data directly available in the literature for the nuclei ^{60}Ni (Ref. 4), ^{88}Sr , ^{90}Zr (Refs. 6 and 7), and ^{92}Mo (Refs. 8 and 7), for which both total (γ ,n) and (γ ,p) cross sections have been measured (or, for ^{60}Ni , could be estimated⁴).

II. DETERMINATION OF THE ISOSPIN MIXING

The isospin mixing parameter ϵ^2 in the $T_>$ GDR can be defined as

 $\epsilon^2 = \lim_{t \to \infty} |b(t)|^2,$

wherein $|b(t)|^2$ represents the probability for a nucleus,

described by

$$|\Psi(t)\rangle = a(t) |T_{>}\rangle + b(t) |T_{<}\rangle$$

to be found in a $T_{<}$ state or channel at a time t, when it was initially (t=0) in a pure $T_{>}$ doorway state $|D(T_{>})\rangle$. The time evolution of the nuclear system, starting from $|D(T_{>})\rangle$, leads to the decay in either a $T_{<}$ or a $T_{>}$ channel when isospin symmetry breaking interactions are present. The defined mixing then represents the probability that the nucleus decays in a $T_{<}$ channel. As such the isospin mixing is generated by all the cou-As such the isospin mixing is generated by an the coupling mechanisms between $T_{>}$ and $T_{<}$ states that may play a role during the decay of the $T_{>}$ doorway state. This doorway state is the coherent $T_{>}$ dipole state generation. erated by means of the isovector E1 excitation. Since this simple $T_{>}$ coherent state decays partly by the direct escape of nucleons and partly by the spreading of its strength and dissipation of its energy over the many more complicated degrees of freedom, some assumptions concerning the isospin mixing mechanism have to be made in order to allow the extraction of ϵ^2 from the experimental data. These are the following:

(a) Mixing is negligible in the direct decay process.

(b) Preequilibrium decay is not important.

(c) After spreading, mixing essentially occurs between compound states.

These assumptions can be made plausible by the following arguments. The magnitude of the isospin mixing is mainly determined by the ratio of the lifetime of the state $(\sim 1/\Gamma)$ to the time needed for mixing. The latter one is caused by the weak Coulomb interaction such that mixing takes a long time in comparison to the short lifetime of the coherent $T_{>}$ state. Thus the $T_{>}$ doorway is damped before appreciable mixing can occur. This argument is confirmed by the results of a (γ, n_0) experiment on ⁶⁰Ni (Ref. 4). The second assumption concerns the emission of nucleons after the spreading of the initial $T_{>}$ dipole state has taken place. During the evolution towards statistical equilibrium, at each stage of increasing complexity nucleons may escape to the continuum. However, for the energies ($\cong 20$ MeV) and the masses ($A \cong 60-90$) under consideration, the spreading widths of the intermediate complicated states are much larger than their escape

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widths.⁹ This means that preequilibrium decay is small, and consequently the nucleus will mainly decay by evaporation of nucleons once statistical equilibrium is reached. The third assumption states that isospin mixing is small in the evolution process towards equilibrium. This is because the density of $T_{<}$ levels in which an intermediate $T_{>}$ state can be damped is increasing rapidly as a function of the complexity of the intermediate states.⁹ As a result, isospin mixing will be generated most likely in the equilibrium stage where the involved compound states do possess a high density.

However, the isospin mixing ϵ^2 as defined above represents the total mixing generated in the decay of the $T_>$ dipole state and can be written in terms of the isospin mixing μ^2 produced after spreading alone [when assumption (a) holds] as

$$\epsilon^2 = \mu^2 (1 - \alpha^2) ,$$

where $\alpha^2 = \Gamma^{\dagger} / \Gamma$ is the probability for direct decay of the $T_{>}$ state (Γ^{\dagger} is the escape width). This relation expresses the fact that the isospin mixing can be approximated by the mixing of the underlying states, only when spreading dominates. The parameter μ^2 may well be taken as the isospin mixing between compound states.

If the conditions mentioned above are fulfilled, it is possible to determine the mixing parameter μ^2 by means of a Hauser-Feshbach calculation¹⁰⁻¹² (which takes into account isospin mixing) provided that the statistical part of both the (γ ,p) and the (γ ,n) cross section is known. The statistical cross section $\sigma_i(E)$ for decay of the $T_{<}$ and $T_{>}$ dipole doorway states to a specific residual state (channel *i*) is then given as a function of the statistical parts of both the $T_{<}$ and the $T_{>}$ photoabsorption cross sections, $\sigma_{st}(T_{<})$ and $\sigma_{st}(T_{>})$, respectively, by

$$\sigma_i = \sigma_i(T_{<}) + \sigma_i(T_{>}) , \qquad (1)$$

$$\sigma_i(T_{<}) = \sigma_{\rm st}(T_{<}) [\eta^2 g_i(T_{>}) + (1 - \eta^2) g_i(T_{<})], \qquad (2)$$

$$\sigma_i(T_>) = \sigma_{\rm st}(T_>) [(1 - \mu^2)g_i(T_>) + \mu^2 g_i(T_<)], \qquad (3)$$

$$g_{i}(T) = \frac{\langle T_{i}t_{i}; T_{3i}t_{3i} | TT_{3} \rangle^{2} \sum_{l_{i}S_{i}} T_{l_{i}}\delta_{\pi\pi_{i}}}{\sum_{j} \langle T_{j}t_{j}; T_{3j}t_{3j} | TT_{3} \rangle^{2} \sum_{l_{j}S_{j}} T_{l_{j}}\delta_{\pi\pi_{j}}}$$
(4)

The quantity with brackets in expression (4) denotes the isospin vector coupling coefficient for the specific channel *i*, while T_{l_i} , S_i , and π_i represent the isospin independent transmission coefficient, the channel spin, and parity, respectively. The quantity η^2 can be defined for $T_{<}$ compound states in the same way as μ^2 for $T_{>}$ states, and represents the isospin mixing of $T_{<}$ compound states into $T_{>}$ ones. Their relation can be expressed as

$$\eta^2 = \mu^2 \frac{N_2}{N_1} , (5)$$

in which N_m represents the effective number of decay channels for levels with isospin $T_<$ (m=1) or $T_>$ (m=2) and is given by the denominator of Eq. (4). For the energy and mass region under study, the number of decay channels for $T_{<}$ states is much larger than the number of channels for $T_{>}$ states $(N_1 \gg N_2)$, and consequently, the $T_{<}$ to $T_{>}$ mixing η^2 is negligible. This means that the statistical decay of the $T_{<}$ dipole state can be described without taking mixing into account.

As a consequence of relation (3), a good estimate for the isospin mixing parameter μ^2 can already be obtained from

$$\mu^2 \simeq \frac{\sigma_{\rm st}(T_>,n)}{\sigma_{\rm st}(T_>)}$$
,

in which $\sigma_{\rm st}(T_>,n)$ denotes the statistical part of the $T_>$ (γ ,n) cross section. This is based on the fact that, for the nuclei considered here, the statistical decay of a $T_<$ or a $T_>$ compound state nearly exclusively populates neutron or proton channels, respectively,

$$\sum_{n} g_i(T_{<}) \simeq \sum_{p} g_i(T_{>}) \simeq 1$$

and

$$\sum_{n} g_i(T_{>}) \simeq \sum_{p} g_i(T_{<}) \simeq 0.$$

Nevertheless, the mixing μ^2 can more correctly be determined from a fit of Eq. (3) to both the experimentally measured statistical parts of the (γ, p) and (γ, n) cross sections. However, in general, these statistical parts are not known since the direct decay mechanism also contributes to the observed cross section.

In the case of the ${}^{89}Y(\gamma,p){}^{88}Sr$ reaction we have measured the total cross section as well as the cross sections for various reaction channels.⁵ The total (γ,p) cross section, depicted in Fig. 1, definitely shows the existence of



FIG. 1. The data points show the experimentally determined total ⁸⁹Y(γ , p)⁸⁸Sr cross section while the full line represents the sum of the two Lorentzians (shown as dashed lines) that were fitted to the $T_{<}$ and $T_{>}$ coherent dipole states, respectively (Ref. 5). The dot-dashed line is the result of a statistical calculation using a Hauser-Feshbach formalism, representing the contribution to the statistical cross section of low-energy photoprotons ($T_{p} \leq 4$ MeV) that are lost in the background region.

the $T_{>}$ resonance in ⁸⁹Y, located at 21.8 MeV; a $T_{>}$ to $T_{<}$ strength ratio $S_{>}/S_{<}\simeq 0.13-0.14$ was deduced. From the results of this same measurement, the proton decay leading to simple proton-hole states in ⁸⁸Sr could be separated from the decay leaving the residual nucleus in more complicated states above 4.5 MeV. Because the first process originates predominantly from a direct decay mechanism and the second one from a nondirect (mostly statistical) mechanism, the separation of direct and statistical parts could be made for the (γ, p) cross sections of ⁸⁹Y; both are shown in Fig. 2. It is clear that the statistical part has its maximum around $E_R \simeq 21.8$ MeV, where the $T_{>}$ GDR in ⁸⁹Y is located, while the direct part peaks around 16.8 MeV, i.e., at the position of the $T_{<}$ resonance. Since the existence of the $T_{>}$ state at 21.8 MeV is also reflected in the direct decay cross section, we have fitted a sum of two Lorentz lines to this cross section in



FIG. 2. The (γ, p) cross sections for the statistical and the direct decay of the GDR in ⁸⁹Y are shown in the upper and the lower part, respectively, of the figure. To the direct decay cross section a sum of two Lorentzians (shown as dashed lines, while the sum is depicted as a full line) was fitted to the experimental data, using the parameters (energy, width) from Ref. 5. The full line in the statistical cross section represents the result of a Hauser-Feshbach statistical calculation, using expression (1), and taking into account an isospin mixing $\mu^2 = 0.63$; the dashed line shows the $T_{>}$ part only of this statistical cross section.

order to determine the respective strengths of the $T_{<}$ and $T_{>}$ components in the direct proton channel.⁵ For the lower-energy Lorentz line, the well-known parameters^{6,13} of the $T_{<}$ GDR were taken, while for the high-energy one, its width was determined⁵ to be 4 MeV.

On the other hand, due to the high separation energy $S_n(T+\frac{1}{2}) \simeq 19.5$ MeV for the $T+\frac{1}{2}$ neutron channels, with respect to the position E_R of the $T_>$ resonance, direct neutron decay will be strongly suppressed (see Fig. 3). This means that neutron decay of the $T_>$ resonance will originate from a statistical mechanism via isospin mixing.

In the case of ⁸⁹Y this statistical neutron $T_{>}$ part can be deduced from the (γ, xn) measurements of Refs. 6 and 13 by a subtraction of the $T_{<}$ Lorentz line which fits the lower part of the cross section very well. In order to extract the $T_{>}$ neutron strength explicitly, a Lorentz line possessing a width of 4 MeV [derived from our (γ, p) experiments⁵] is fitted to the excess (γ , xn) cross section. Unfortunately, the discrepancy in magnitude between the data of Refs. 6 and 13 is responsible for the major source of uncertainty on the total $T_{>}$ strength. However, using the (γ, n) data of Ref. 13 and our (γ, p) cross sections, we could extract⁵ all parameters of the Lorentzians describing both the $T_{<}$ and $T_{>}$ giant dipole resonances; these are summarized in Table I, together with the deduced⁵ values of the decay widths. From these data one can derive the peak values of the statistical $T_{<}$ and $T_{>}$ resonances in ⁸⁹Y:

$$\sigma_{\rm st}(T_{\sim}) = 147 \text{ mb}, \ \sigma_{\rm st}(T_{\sim}) = 30 \text{ mb}.$$

Taking into account the direct decay cross section $\sigma_D(T_>)=6.3$ mb (see Fig. 2 and Table I), the direct decay probability $\alpha^2=0.17\pm0.03$ was derived for the $T_>$ resonance. Furthermore, we can now deduce both (γ,n) and (γ,p) components to $\sigma_{\rm st}(T_>)$:

$$\sigma_{\rm st}(T_>,n) = 18 \text{ mb}, \ \sigma_{\rm st}(T_>,p) = 12 \text{ mb}$$



FIG. 3. Decay scheme of the $T_>$ GDR, including the squared isospin vector coupling coefficients.

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Resonance	Resonance	Peak	Peak	Escape	Spreading	Total
	energy	cross section	cross section	width	width	width
	E_R (MeV)	$\sigma_{0,n}$ (mb)	$\sigma_{0,p}$ (mb)	Γ [†] (MeV)	Γ [↓] (MeV)	Γ (MeV
$T_{<}$	16.8	185±5	20.8±1.5	1.15	2.85	4.0
$T_{>}$	21.8	18±2	18.3±1.5	0.70	3.30	4.0

TABLE I. Parameters of the $T_{<}$ and $T_{>}$ coherent dipole states in ⁸⁹Y.

To illustrate the procedure for extracting μ^2 , the calculated [using relation (1)] statistical cross section in the photoproton channel of ⁸⁹Y is shown in the upper part of Fig. 2. For the evaluation of expression (4), the known lowlying residual states in ⁸⁸Y and ⁸⁸Sr are summed, while the higher-lying states are taken into account by an integration using a level density formula.^{16,17} The transmission coefficients were calculated using the optical potential parameters of Rosen *et al.*¹⁴ Note that in this derivation of μ^2 allowance was made for those photoprotons with a kinetic energy T_p smaller than 4 MeV that not could be detected due to the presence of the background in the measured proton spectra.¹⁵

A constant (as a function of excitation energy) isospin mixing parameter $\mu^2 = 0.63$ describes the (γ, p) cross section fairly well. However, a much better fit to each data point could be obtained by letting μ^2 be energy dependent; however, the limited statistical accuracy of both the $T_>$ (γ, p) and (γ, n) cross sections may not lead to a meaningful energy dependence for μ^2 .

Led by the rather small value of α^2 for ⁸⁹Y, it is assumed that spreading of the $T_>$ strength dominates as well for the other spherical nuclei around A=90 under study. The $T_>$ cross sections for these nuclei, taken from Refs. 4 and 6–8, are then considered as being solely due to the statistical process ($\alpha^2=0$ and $\epsilon^2=\mu^2$); this assumption, however, may lead to an underestimation of the isospin mixing parameter μ^2 .

Because of the rather large uncertainties on the absolute values of the $T_{>}$ strengths and due to our approximations, the obtained mixing parameters, though energy dependent, have only been derived at the $T_{>}$ resonance energy E_R . They are shown in Table II, where the relative uncertainties are estimated to be no larger than 30%.

Although our values for μ^2 are somewhat higher [which may well be due to the limited accuracy of the $T_>$ (γ ,n) cross sections^{6,13,18}], they show a fair correspondence with the existing mixing data for overlapping $T_>$ and $T_<$ compound states. Most of these have been deduced by means of other probes such as (p,p'), (p,α) , (α,p) , and (α,α') reactions.¹⁹ From these experiments a rather constant mixing with a mean value of about 45% in the neighborhood of 20 MeV was derived.¹⁰ However, this result concerns the mixing between compound states of various angular momenta and parities, whereas our μ^2 represents mixing between 1⁻ states $(\frac{1}{2}^+ \text{ and } \frac{3}{2}^+ \text{ states})$ for ⁸⁹Y) only. This correspondence between our μ^2 values and the existing mixing data justifies assumptions (b) and (c), and consequently, the isospin mixing mechanism in the $T_{>}$ resonance essentially consists in the coupling of overlapping $T_{>}$ and $T_{<}$ compound states. This mechanism leads to large values of μ^2 and thus to a large isospin forbidden cross section in the neutron channel. However, such high μ^2 values still imply that isospin remains a meaningful quantum number, even for the statistical decay of the $T_{>}$ resonance, since the mixing is not complete [see further, expression (9)]. As a consequence, the isospin selection rules hold to some extent and favor the proton channel for the observation of the $T_{>}$ resonance.

Finally, using this mixing mechanism, and since external isospin mixing seems to be small for compound states,²⁰ it is possible to relate the observed μ^2 to the mean isospin breaking Coulomb matrix element between $T_>$ and $T_<$ compound states, i.e.,

$$H_{c} = |\langle T_{<} | H_{\text{Coul}} | T_{>} \rangle|$$

The mean mixing width Γ_m^{\downarrow} of a $T_{<}$ or $T_{>}$ compound level (expressed as a function of H_c) and the mean escape width Γ_m^{\uparrow} are given by

$$\Gamma_m^{\downarrow} = 2\pi H_c^2 \frac{\rho_1 \rho_2}{\rho_m} , \qquad (6)$$
$$\Gamma_m^{\dagger} = \frac{1}{2\pi \rho_m} N_m ,$$

wherein ρ_m is the density of levels of isospin $T_{<}$ (m=1)

TABLE II. Mixing coefficients μ^2 and ϵ^2 and the mean Coulomb matrix elements H_c , calculated using the level densities of Ref. 16.

Nucleus	$S_{\rm n}(T+{1\over 2})$ (MeV)	E_R (MeV)	J^{π}	α^2	μ^2 (%)	ϵ^2 (%)	H_c (eV)
⁶⁰ Ni	19.7	21.0	1-		86	•	1000
⁸⁸ Sr	22.8	21.6	1-		73		134
⁸⁹ Y	19.5	21.8	$\frac{1}{2}^+, \frac{3}{2}^+$	0.17±0.03	63 ± 10^{a}	52 ± 8	21ª
⁹⁰ Zr	21.1	20.4	1-		62		123
⁹² Mo	20.8	20.5	1-		48		75

^aAn experimentally determined level density parameter was used, taken from Ref. 17.

or $T_{>}$ (m=2). The relation between the isospin mixing μ^2 and the mean escape and mixing widths is given by

$$\frac{\Gamma_2^4}{\Gamma_2^{\dagger}} = \frac{\mu^2}{1-\mu^2} \left[1 - \frac{N_2}{N_1} \frac{\mu^2}{1-\mu^2} \right]^{-1}.$$
 (7)

Combining (6) and (7) leads to the relation from which H_c can be calculated¹⁰:

$$H_c^2 = \frac{\mu^2}{1 - \mu^2 \left[1 + \frac{N_2}{N_1}\right]} \frac{1}{2\pi} \frac{\Gamma_2}{\rho_1} .$$
(8)

For the system containing the two classes of compound states only $(T_{<} \text{ or } T_{>})$, μ^2 can be defined in the same way as ϵ^2 ; this means that the occupation probability for the $T_{>}$ compound state can be set equal to 1 at $t=t_1$. Expression (7) and also (5) can then easily be obtained using a master equation²¹ which governs the occupation probability flow in time. This equation is represented by a set of two coupled differential equations from which the occupation probabilities for both the $T_{<}$ and $T_{>}$ compound states can be calculated. The transition probabilities per unit time, entering in this equation, are given by Γ_m^{\perp}/\hbar and Γ_m^{\perp}/\hbar when external isospin mixing is negligible. From relation (8) the maximum or complete isospin mixing is given by

$$\mu_c^2 = \frac{N_1}{N_1 + N_2} \tag{9}$$

and is generated by an infinitely large mean Coulomb matrix element H_c . For the nuclei under study this complete mixing μ_c^2 is about equal to 1, since $N_1 \gg N_2$.

The mean mixing Coulomb matrix elements calculated using (8) are also shown in Table II. While μ^2 is nearly independent of the level density, H_c does strongly depend on it, so it is difficult to estimate uncertainties on the derived H_c values (in general, level densities are not very well known). In Fig. 4 the matrix elements, calculated with the level density of Ref. 16, are plotted vs \sqrt{aU} , where a and U stand for the level density parameter and the effective excitation energy, respectively. On the same figure the straight line represents a fit to the known data.²² Our values show the same exponential dependence, $H_c \sim \exp(-\sqrt{aU})$, which means that relation (8) is dominated by the $T_{<}$ level density. This behavior of H_c and the near constancy of μ^2 , regardless of the nucleus, has been interpreted²² as being due to the fact that an analog $T_{>}$ state can only couple to its respective antia-nalog $T_{<}$. The deviation by a factor of about 5 between our matrix elements and the straight line may well be explained by the use of different level densities in the analysis (which, unfortunately, are not always specified),





³K. Shoda, Phys. Rep. **53**, 341 (1979).

FIG. 4. Mean isospin mixing Coulomb matrix elements between the underlying $T_{>}$ and $T_{<}$ compound states of the $T_{>}$ GDR. The full line is a fit through the previously published data (Ref. 22). The ⁸⁹Y point denoted by an open square ($H_c = 67 \text{ eV}$) was obtained using the same level density parameters (Ref. 16) as for the other nuclei.

and by the fact that in our case the compound states are restricted to be $1^{-}(\frac{1}{2}^{+},\frac{3}{2}^{+}$ for ⁸⁹Y) states.

III. CONCLUSIONS

The main result of this paper consists in the determination, for medium-heavy nuclei, of the amount of isospin mixing generated during the decay of the $T_{>}$ coherent dipole state using both (γ, n) and (γ, p) cross sections. It can be concluded that this mixing is large (since the spreading of the $T_{>}$ state dominates) but otherwise not complete, i.e., $\mu^2 < N_1/(N_1+N_2)$, and that it may be attributed to the coupling of overlapping $T_{>}$ and $T_{<}$ compound states. Mean mixing Coulomb matrix elements for compound states could be derived, although the absolute values must be regarded with some caution since they are strongly dependent on the specific level density used in the analysis.

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