j dependence of polarization effects in nucleon capture reactions

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Polarized proton capture data indicate a sensitivity to the j value $(l\pm\frac{1}{2})$ of the single particle final state for a given l value. The direct capture model is examined and a simple relationship between the analyzing powers for capture to the two j values is derived for the case of spin independent distortions. Calculations showing the effects of spin dependent distortion are also presented. The results are compared to the available data.

INTRODUCTION

The value of nucleon capture measurements using polarized beams has been well established. Many of the features of the measured analyzing powers for a number of nuclei can be accounted for by the direct-semidirect (DSD) model. In fact, the simplest form of this model, a renormalized direct capture calculation, can account for the main features of the angular dependence of the measured cross sections and analyzing powers.[1]

The presently available data suggest that polarized capture measurements display a j dependence for a given l transfer reminiscent of that observed in direct stripping reactions. Namely, the sign of the b_2 coefficient extracted from the data (see the following) is correlated with the $\vec{l},\vec{s}$ alignment in the final single particle state. It is this j dependence which will be investigated in this paper. It will be shown that the direct capture model leads to a simple relationship between the b_2 coefficients for the cases of direct capture to final states having single particle states with $j=l-\frac{1}{2}$ and $j=l+\frac{1}{2}$. This correlation should be a useful tool in assigning j values to final states populated in capture reactions.

SUMMARY OF EXPERIMENTAL RESULTS

The technique of polarized proton (or neutron) capture involves measurement of the analyzing power $A(\theta)$,

$$A(\theta)=\frac{1}{P}\frac{N_{+}-N_{-}}{N_{+}+N_{-}}\ , \tag{1}$$

where N_{+} and N_{-} are the yields obtained for spin-up and spin-down beams, respectively, and P is the beam polarization. The analyzing power data are expanded according to

$$\sigma(\theta)A(\theta)=A_0\sum_{k=1}^{n}b_k p_k^1(\cos\theta)\ . \tag{2}$$

If the reaction involves only $E1$ radiation, then there will be only one term in Eq. (2), namely $b_2P_2^1(\cos\theta)$. Since $E1$ radiation normally dominates the capture reactions for proton energies below 30 MeV, we will concentrate on the experimentally deduced b_2 coefficients in comparing the present results with experiment. However, we will see that our results should also be applicable to other b_k coefficients.

A list of experimentally determined b_2 coefficients, for selected nuclei and selected energies, is presented in Table I. Values reported here were chosen to be representative in that, when possible, energies were chosen for which b_2 was stable at nearby energies (within a few MeV). This procedure should help us to avoid complications due to effects arising from interfering resonance states such as secondary doorway states.[14,15]

The values shown indicate that the sign of the b_2 coefficient is correlated with the j of the single particle in the final state being either $l+\frac{1}{2}$ or $l-\frac{1}{2}$. For example, the b_2 values in ^{11}B are negative for capture to the $p_{3/2}$ single particle ground state of ^{12}C, and positive for the first excited state of ^{12}C, which should be largely a $p_{1/2}$ single particle state. Notice that the ratio of $b_2(l-\frac{1}{2})$ to $b_2(l+\frac{1}{2})$ is $0.266/-0.128=-2.08$ in this case. This pattern of negative b_2 values for $(l+\frac{1}{2})$ persists throughout the table, not only for $l=1$ capture, but for $l=0$, 2, 3, and 4 as well. Notice also that both ^{12}C(p,γ_3), leading to the $d_{5/2}$ single particle state in ^{13}N at 3.55 MeV and ^{11}B(p,γ_{19}), presumably leading to the $d_{5/2}$ single particle strength in ^{12}C near 19 MeV, have negative b_2 coefficients, a result consistent with the strength in the 19 MeV region being predominantly $d_{5/2}$ rather than $d_{3/2}$.

THEORY

Results such as those discussed previously have been analyzed and calculated in the past by means of the direct-semidirect reaction theory. In fact, many of the qualitative features of the angular dependence of $\sigma(\theta)$ and $A(\theta)$ have been successfully described with the direct capture formulation which is actually equivalent to the DSD model in its simplest form.[16] Exceptions to this may occur in the vicinity of "narrow" resonance structures

TABLE I. Experimentally determined b_2 coefficients for selected nuclei and selected nucleon energies.

Reaction	E_p (MeV)	b_2	Dominant single particle in final state	Reference
^{7}Li(p,γ_0)	14.0	-0.029 ± 0.008	$p_{3/2}$	2
^{7}Li(p,γ_1)	14.0	$+0.033 \pm 0.004$	$p_{3/2}+p_{1/2}$	2
^{7}Li(p,γ_{16})	14.0	-0.049 ± 0.006	$p_{1/2}+p_{3/2}$	2
^{11}B(p,γ_0)	13.3	-0.10 ± 0.01	$p_{3/2}$	3
^{11}B(p,γ_0)	28.5	-0.128 ± 0.054	$p_{3/2}$	4
^{11}B(p,γ_1)	28.5	$+0.266 \pm 0.01$	$p_{1/2}$	4
^{11}B(p,γ_{19})	28.5	-0.025 ± 0.02	$d_{5/2}$	4
^{12}C(p,γ_0)	28.5	$+0.0914 \pm 0.051$	$p_{1/2}$	4
^{12}C(p,γ_3)	28.5	-0.073 ± 0.028	$d_{5/2}$	4
^{13}C(p,γ_0)	15.0	$+0.23 \pm 0.01$	$p_{1/2}$	5
^{13}C(p,γ_1)	15.0	$+0.25 \pm 0.02$	$p_{1/2}$	5
^{30}Si(p,γ_0)	14.95	-0.05 ± 0.02	$s_{1/2}$	6
^{30}Si(p,γ_1)	14.95	$+0.23 \pm 0.01$	$d_{3/2}$	6
^{59}Co(p,γ_0)	11.8	-0.23 ± 0.02	$f_{7/2}$	7
^{59}Co(p,γ_1)	11.8	-0.10 ± 0.02	$p_{3/2}$	8
^{88}Sr(p,γ_0)	15.0	$+0.182 \pm 0.02$	$p_{1/2}$	9
^{88}Sr(p,γ_1)	15.0	-0.163 ± 0.013	$g_{9/2}$	9
^{15}N(p,γ_0)	15.7	$+0.247 \pm 0.011$	$p_{1/2}$	10
	15.74	$+0.250 \pm 0.006$		11
^{13}C(n,γ_0)	$E_n = 13$ MeV	$+0.200 \pm 0.040$	$p_{1/2}$	12
^{40}Ca(n,γ_0)	$E_n = 11$ MeV	-0.117 ± 0.034	$f_{7/2}$	13

such as secondary doorway states[14,15] and isobaric analog resonances.[17] This is why we choose to examine the experimental b_2 coefficients at energies where they are slowly varying as a function of energy.

The ability of the "direct" capture model to account for the observed b_2 coefficients will be demonstrated by examining the case of ^{7}Li(p,γ)^{8}Be. The experimentally determined values of the quantity $A(\theta)\sigma(\theta)/A_0$ for the reactions ^{7}Li(p,γ_0)^{8}Be and ^{7}Li(p,γ_1)^{8}Be at $E_p = 14$ MeV are shown in Fig. 1. Direct capture calculations were performed for both cases. The optical model potential used to generate the incident distorted waves was taken from Watson *et al.*[18] Woods-Saxon wells were used to generate the final bound state wave functions by varying the well depth in order to obtain the proper binding energy. The shell model results of Cohen and Kurath[19] were used to describe the single-particle nature of the final states of ^{8}Be. Their results describe the ground state of ^{8}Be as a $p_{3/2}$ single particle state having a spectroscopic factor of 2.89. The first excited state at 2.94 MeV is described as a mixed $p_{3/2}$ and $p_{1/2}$ single particle state having $S = 1.12$ for the $p_{3/2}$ component and $S = 0.751$ for the $p_{1/2}$ component, giving a total of $S = 1.87$.

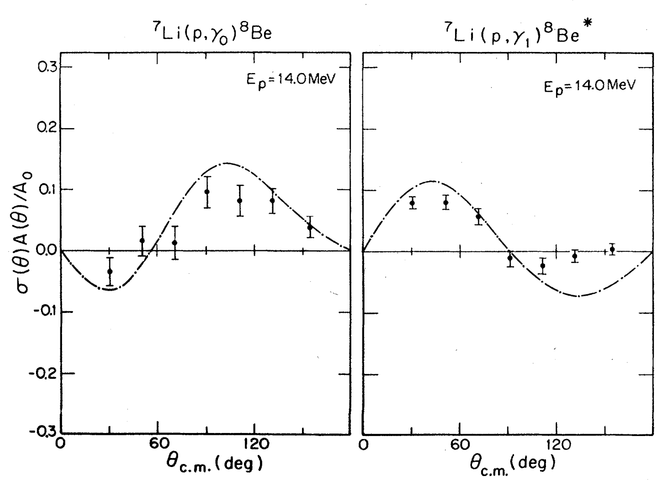

FIG. 1. The experimental data for the product of the cross section times the analyzing power are shown for the ^{7}Li(p,γ_0)^{8}Be and the ^{7}Li(p,γ_1)^{8}Be reactions at $E_p = 14$ MeV. The error bars represent the statistical uncertainties in the data points. The curves are the result of the direct capture model calculations.

The results of the direct capture calculations including both $E1$ and $E2$ radiation are shown in Fig. 1. The b_k coefficients are presented in Table II.

The direct capture model is able to reproduce the signs of the b_1, b_2, and b_3 coefficients even in the case where,

TABLE II. Comparison of experimentally determined and calculated b_k coefficients for the reactions ^{7}Li(p,γ_0) and ^{7}Li(p,γ_1) at $E_p = 14$ MeV.

		Experiment	Calculation
(p,γ_0)	b_1	0.061 ± 0.012	0.089
	b_2	-0.029 ± 0.008	-0.043
	b_3	-0.011 ± 0.006	-0.025
	b_4	-0.000 ± 0.005	-0.001
(p,γ_1)	b_1	0.023 ± 0.006	0.014
	b_2	0.033 ± 0.004	0.061
	b_3	0.015 ± 0.003	0.007
	b_4	-0.002 ± 0.003	0.001

as in (p,γ_1), the final state is an admixture of $l+\frac{1}{2}$ and $l-\frac{1}{2}$ single particle states.

These observations suggest the possibility of using the polarized nucleon capture reaction as a tool for making j assignments of single particle final states, especially when the direct or direct-semidirect model is applicable. This has motivated us to examine the capture formalism to see what, if any, conclusions can be drawn regarding the j dependence of the b_2 coefficients.

Following Seyler and Weller[20] we write, for the spin $\frac{1}{2}$ capture cross section on a spin a target leading to a residual state spin c and the emission of a photon of mode p and multipolarity L,

$$\sigma(\theta,\phi)=(\tfrac{1}{2}\lambda\!\!\!^{-})^2\frac{1}{2}\frac{1}{\hat{a}^2}\sum_k(\bar{a}_k P_k+\bar{b}_k P_k^1 p_y)\ , \tag{3}$$

where the $\bar{a}_k$ and $\bar{b}_k$ are given by

$$\bar{a}_k=\sum_{tt'}(-)^{a-c+1-(1/2)+k+j-j'}\hat{j}\hat{j}'\hat{l}\hat{l}'\hat{L}\hat{L}'\hat{b}^2\hat{b}'^2(l0,l'0\,|\,k0)W(ljl'j';\tfrac{1}{2}k)[\ \ ]$$
$$\times(L1,L'-1\,|\,k0)W(jbj'b';ak)W(LbL'b';ck)\mathrm{Re}(RR'^*) \tag{4}$$

and

$$\bar{b}_k=\frac{\sqrt{6}\hat{k}}{\sqrt{k(k+1)}}\sum_{tt'}(-)^{a-c+1+l'-j'}\hat{j}\hat{j}'\hat{l}\hat{l}'\hat{L}\hat{L}'\hat{b}^2\hat{b}'^2(l0,l'0\,|\,k0)(L1,L'-1\,|\,k0)[\ \ ]$$
$$\times W(jbj'b';ak)W(LbL'b';ck)X(l\tfrac{1}{2}j;l'\tfrac{1}{2}j';k1k)\mathrm{Re}(iRR'^*)\ . \tag{5}$$

R is the reduced matrix element $(pLcb\pi||R||l\frac{1}{2}jab\pi)$, t is an abbreviation for the five variables $pLljb$, and [ ] is the parity restriction factor $\frac{1}{2}[1+(-)^{k+L+p+L'+p'}]$.

We first consider the case of zero target spin ($a=0$). The electric multipole direct capture cross section for this case ($a=0$) may be obtained from Eqs. (3)–(5) by choosing for R

$$R=e\epsilon_L\left[\frac{8k_\gamma}{\hbar v_i}\right]^{1/2}B_L\frac{\hat{j}_f}{\hat{j}}T\ , \tag{6}$$

where from Ref. 1,

$$B_L=k_\gamma^L\left[\frac{L+1}{L}\right]^{1/2}\hat{L}/(2L+1)!! \tag{7}$$

and

$$T=(C^2S)^{1/2}_{l_fj_f}i^{l-l_f-L}(j_f\tfrac{1}{2},L0\,|\,j\tfrac{1}{2})I\ , \tag{8}$$

and where the radial integral I is given by

$$I=\langle u_{l_fj_f}\,|\,O(EL)\,|\,\chi_{lj}^{+}\rangle\ . \tag{9}$$

Equation (6) may be used in the total cross section expression of Seyler and Weller[20]

$$\sigma_T=\frac{\pi}{2}\lambda\!\!\!^{-2}\sum\hat{j}^2\,|\,R\,|^2$$

to verify its equivalence to the corresponding expression of Weller and Roberson[1]

$$\sigma_T=2\pi k_\gamma\epsilon_L^2\frac{e^2}{\hbar c}\hbar c\frac{\hat{j}_f^2B_L^2}{Ek}\sum|\,T\,|^2\ .$$

We shall use an alternative (more informative) expression for T, namely

$$T=(C^2S)^{1/2}i^{l-l_f-L}\hat{l}\hat{l}_f\hat{j}(-)^{L+l}$$
$$\times(l_f0,l0\,|\,L0)W(Ll_fj\tfrac{1}{2};lj_f)I/\hat{L}\ . \tag{10}$$

The T values of Eqs. (8) and (10) are numerically equal when the l and l_f values in Eq. (10) are selected to satisfy the parity restriction and the various vector triangle conditions inherent in Eq. (10). The use of Eq. (10) imposes these conditions automatically whereas the use of Eq. (8) requires the supplementary implementation of these conditions. Thus the use of Eq. (10) is preferable in Racah algebra applications as in the present work.

We want to examine the consequences of the assumption of spin-independent distortion (SID). This assumption is equivalent to assuming that the radial matrix element of Eq. (9) is independent of j and j_f. That is, I is assumed to have the same value for both $j=l+\frac{1}{2}$ and $j=l-\frac{1}{2}$ and similarly for $j_f=l_f\pm\frac{1}{2}$. This would be expected in the SID limit where the strengths of the spin orbit part of the bound state and continuum state optical potentials are set equal to zero.

Making the SID assumption, we neglect any j or j_f dependence in Eq. (9) and substitute Eqs. (6) and (10) into Eq. (4) with $a=0$, $b=j$, and $c=j_f$. We can then perform the sums over j and j',

$$\sum_{jj'}(-)^{l_f-j_f-(1/2)+k+j-j'}\hat{j}^3\hat{j}'^3W(LjL'j';j_fk)W(Ljl_f\tfrac{1}{2};j_fl)\,W(ljl'j';\tfrac{1}{2}k)W(L'l_fj'\tfrac{1}{2};l'j_f)W(jjj'j';0k)\equiv 1\ , \tag{11}$$

to find that, for a spin zero target, SID implies

$$\bar{a}_k(0,j_f)=\hat{j}_f^2(C^2S)_{l_fj_f}\hat{l}_f^2\sum_{ll'LL'}\hat{l}^2\hat{l}'^2(l0,l'0\,|\,k0)[\quad](L\,1,L'-1\,|\,k0)e^2\epsilon_L^2\frac{8k_\gamma}{\hbar v_i}$$
$$\times B_LB_L'(l_f0,l0\,|\,L0)(l_f0,l'0\,|\,L'0)(-)^{l_f+1}(i)^{l-l'+L'-L}\mathrm{Re}(II'^*)\ . \tag{12}$$

Similarly, by performing the following sums over j and j',

$$\sum_{jj'}\hat{j}^2\hat{j}'^2W(kLj'j_f;L'j)W(lL\tfrac{1}{2}j_f;l_fj)W(l'L'\tfrac{1}{2}j_f;l_fj')X(kll';kjj';1\tfrac{1}{2}\tfrac{1}{2})=W(1l_f\tfrac{1}{2}j_f;l_f\tfrac{1}{2})X(kll';kLL';1l_fl_f)\ , \tag{13}$$

we find from Eq. (5), for the same assumption,

$$\bar{b}_k(0,j_f)=\hat{j}_f^2W(1l_f\tfrac{1}{2}j_f;l_f\tfrac{1}{2})(C^2S)_{l_fj_f}\hat{l}_f^2\frac{\sqrt{6}k^2}{\sqrt{k(k+1)}}$$
$$\times\sum_{ll'LL'}\hat{l}^2\hat{l}'^2(l0,l'0\,|\,k0)[\quad](L\,1,L'-1\,|\,k0)e^2\epsilon_L^2\frac{8k_\gamma}{\hbar v_i}B_LB_{L'}(-)^{l_f}(i)^{l-l'+L'-L}$$
$$\times(l_f0,l0\,|\,L0)(l_f0,l'0\,|\,L'0)X(kll';kLL';1l_fl_f)\mathrm{Re}(iII'^*)\ . \tag{14}$$

In order to conveniently summarize the above results we rewrite Eq. (3) in the form

$$\sigma(\theta,\phi)=A_0\left[1+\sum_{k\neq0}(a_kP_k+b_kP_k^1p_y)\right]\ , \tag{15}$$

where

$$a_k=\bar{a}_k/\bar{a}_0\ ,$$
$$b_k=\bar{b}_k/\bar{a}_0\ , \tag{16}$$

and

$$A_0(\tfrac{1}{2}\lambda\!\!\!^{-})^2\tfrac{1}{2}\hat{a}^{-2}\bar{a}_0\ .$$

(The definitions of barred and unbarred a_k and b_k are here reversed from those of Ref. 20 to minimize the number of $\bar{b}$ symbols here.) For the case of electric multipole radiation in connection with SID proton capture by a target of zero spin ($a=0$), we can conclude from Eqs. (12) and (14) that

$$a_k \text{ is independent of } j_f\ , \tag{17}$$

$$b_k \text{ is proportional to } W(1l_f\tfrac{1}{2}j_f;l_f\tfrac{1}{2})\ , \tag{18}$$

which is proportional to $j_f(j_f+1)-l_f(l_f+1)-\frac{3}{4}$, and

$$A_0 \text{ or the total cross section is proportional to } \hat{j}_f^2(C^2S)_{l_fj_f}\ . \tag{19}$$

From Eqs. (17) and (15) it follows that the angular distribution of the unpolarized cross section is independent of j_f and from Eq. (18) it follows that

$$\frac{b_k(j_f=l_f-\frac{1}{2})}{b_k(j_f=l_f+\frac{1}{2})}=-\frac{l_f+1}{l_f}\ , \tag{20}$$

which implies that for SID the analyzing powers for the two j_f values $l_f\pm\frac{1}{2}$ will be in the ratio given in Eq. (20). These SID conclusions for radiative capture are the same as the familiar SID conclusions[21] for the DWBA treatment of nucleon transfer reactions.

The discussion is easily extended to targets of nonzero spin. Here we can make use of certain results of Ref. 22, where we showed for DSD capture that

$$\bar{a}_k(a,c)=\hat{c}^2\sum_{j_fl_f}\hat{j}_f^{-2}\bar{a}_k(0,j_f)\ , \tag{21}$$

$$\bar{b}_k(a,c)=\hat{c}^2\sum_{j_fl_f}\hat{j}_f^{-2}\bar{b}_k(0,j_f)\ , \tag{22}$$

and for the total cross section or A_0

$$A_0(a,c)=\hat{a}^{-2}\hat{c}^2\sum_{j_fl_f}\hat{j}_f^{-2}A_0(0,j_f)\ . \tag{23}$$

The notation is intended to indicate that the quantities $\bar{a}_k$, $\bar{b}_k$, and A_0 appearing on the left-hand sides are for arbitrary target and residual state spins, a and c, respectively, whereas those on the right-hand sides are for the special case $a=0$ (and therefore $c=j_f$) and are given by Eqs. (12) and (14). The summation over l_f is trivial since parity considerations will allow only a single l_f value for each j_f value. To illustrate the j_f dependence of the quantities on the left-hand sides of Eqs. (21)–(23), we consider a case where, for the residual state c, l_f is unique but where both values of j_f ($=l_f\pm\frac{1}{2}$) contribute. We distinguish these values by appending the coefficient of the $\frac{1}{2}$ term as a subscript to the spectroscopic factor. We find for the j_f dependences

$$A_0 \text{ is proportional to } (C^2S)_++(C^2S)_-\ , \tag{24}$$

$$a_k \text{ is independent of } j_f \text{ and } (C^2S)_\pm\ , \tag{25}$$

and

$$b_k \text{ is proportional to } [l_f(C^2S)_+-(l_f+1)(C^2S)_-]/[(C^2S_++(C^2S)_-]\ . \tag{26}$$

From Eq. (26) we find for SID that two final states (of nearly the same energy), each dominated by a single j_f value and having the same l_f value, will have the ratio of their b_k values given by

$$\frac{b_k(a,c)_-}{b_k(a,c)_+}=-\frac{l_f+1}{l_f}\ , \tag{27}$$

as was the case in Eq. (20) for a spin zero target. Notice that this result, -2 for $l_f=1$, is in agreement with the b_2 coefficient ratio (-2.08) observed in the ^{11}B(p,γ_0) and ^{11}B(p,γ_1) reactions previously discussed (cf. Table II).

The foregoing discussion has been based [see, e.g., Eqs. (6) and (8)] on the assumption that the mode of the radiation was electric only, and involved, in accord with the SID hypothesis, only the spin independent part of the electric multipole operator Q_{LM}. We would like to extend the development to include the possibility of magnetic radiation involving the spin independent part of the magnetic multipole operator M_{LM}. Rose and Brink[23] give the forms of these operators in the long wavelength approximation as

$$Q_{LM}=-2ig_L\beta\sqrt{4\pi}(\hbar k_\gamma\hat{L})^{-1}\nabla(r^LY_{LM})\cdot\vec{p} \tag{28}$$

and

$$M_{LM}=2g_L\beta\sqrt{4\pi}(L+1)^{-1}\hat{L}^{-1}\nabla(r^LY_{LM})\cdot\vec{L}\ . \tag{29}$$

The similarity of these operators suggests that the reduced matrix elements R, as given by Eqs. (6) and (10) for the electric case, have exactly the same j and j_f dependence for the magnetic (spin-independent) case. The explicit results of p. 95 of Ref. 24, after slight rearrangement, verify this conclusion. More generally, it can be shown that for the exact (spin-independent) electric and magnetic multipole operators, rather than their long wavelength forms, the j and j_f dependences are identical. This fact allows us to conclude that *all our above results* [Eqs. (17)—(27)] *apply for SID, independent of the modes and/or multipolarities* (possibly mixed) of the emitted radiation.

The fact that the photon has spin 1 like the deuteron permits one to repeat for the present case the SID and first order spin-dependent distortion (SDD) arguments presented in Ref. 21 for stripping reactions and draw the same conclusions reached there. The SID conclusion is exactly that found here [Eqs. (24)—(27)]. The (first order) SDD conclusion is that the product A_0b_k is altered from the SID result by the addition of a term independent of j_f. (There is also a small j_f independent correction term to A_0.) As a result of these corrections, we can modify Eq. (27) to include some SDD by writing

$$[A_0b_k(a,c)]_-/[A_0b_k(a,c)]_+$$
$$=[-(l_f+1)+\Delta]/[l_f+\Delta]\ , \tag{30}$$

where Δ is then a measure of the SDD. This result should be applicable in the case of relatively small SDD.

In order to verify the result of Eq. (27), direct capture calculations of b_2 coefficients were performed with all spin-orbit forces turned off. Both $E1$ and $E2$ radiation were included. The first case chosen was ^{11}B(p,γ_{19}), where the final state was taken to be an $l=2$ single particle state, either $d_{5/2}$ or $d_{3/2}$. Although this state is unbound, the present calculation was, for simplicity, performed using wave functions generated by assuming a binding energy of 1.0 MeV for both states. It has been previously shown that this assumption has very minor effects on the angle dependent observables.[1] According to Eq. (27), the SID ratio should be

$$R=\frac{b_2(d_{3/2})}{b_2(d_{5/2})}=-1.5\ . \tag{31}$$

The plot in Fig. 2 indicates that this ratio of -1.5 is indeed obtained from the detailed calculation in the SID limit.

In order to investigate the effects of spin-orbit forces on this ratio, we performed calculations in which the spin-orbit potential in the incident channel (V^I_{so}) was slowly increased, first with the bound state spin-orbit potential (V^B_{so}) set to zero, and then with V^B_{so} set to 7.0 MeV. The results shown in Fig. 2 show that the ratio departs smoothly from the value of -1.5 as V^I_{so} is raised, but keeps the sign relationship. The effect of the bound state spin orbit potential, on the other hand, is seen to be minimal.

Two additional calculations have been performed to verify Eq. (27) and to examine the deviation from the SID case. The reaction this time was ^{12}C(p,γ) at $E_p=28.5$ MeV. Direct capture was calculated for a single particle ground state having $j=l-\frac{1}{2}$ or $j=l+\frac{1}{2}$. The SID limits [Eq. (27)] for b_2 in this case are

$$R\equiv\frac{b_2(l-\frac{1}{2})}{b_2(l+\frac{1}{2})}=\begin{cases}-2 & l=1\\ -\frac{3}{2} & l=2\end{cases}\ . \tag{32}$$

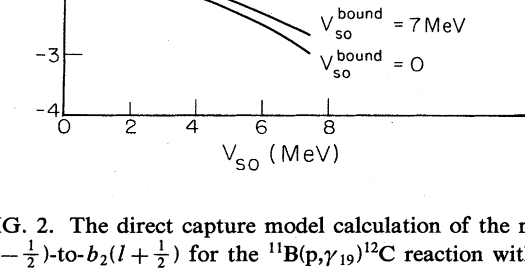

FIG. 2. The direct capture model calculation of the ratio of $b_2(l-\frac{1}{2})$-to-$b_2(l+\frac{1}{2})$ for the ^{11}B(p,γ_{19})^{12}C reaction with $l=2$ and $E_p=28.5$ MeV as a function of the spin-orbit potential in the incident channel. The value of V_{so} in the bound state was set equal to 0.0 and 7.0 MeV, respectively, for the two curves shown.

The results of these calculations for $l=1$ are presented in Fig. 3(b). The optical potential was again taken from Ref. 18. The Woods-Saxon well describing the ground state was adjusted to give the correct binding energy of 1.94 MeV. No spin-orbit potential was included in the bound state potential. These results [see Fig. 3(b)] indicate that $R=-2$ for $V_{so}^{I}=V_{so}^{B}=0$ in agreement with Eq. (27) for $l=1$. As the incident spin-orbit potential is turned on we see that R remains negative, but varies in magnitude between -1 and -3.

Finally, we computed the ratio of the b_2 coefficients as a function of V_{so}^{I} for the case of transitions to the $\frac{5}{2}^{+}$ state in ^{13}N as 3.55 MeV. In this case the final state is unbound by 1.6 MeV. As in the case of the 19 MeV states in ^{12}C, we assumed that we could represent this barely unbound state by a bound state wave function. If a binding energy of 1.0 MeV is assumed, the results shown in Fig. 3(a) are obtained. Again, the ratio of -1.5 is obtained in the SID limit in agreement with Eq. (27) for $l=2$. As the entrance channel spin-orbit strength is increased, we see that the ratio again remains negative, although the ratio begins to depart rather rapidly from the SID limit for $V_{so}^{I}>5$ MeV.

It is interesting to note that not only do the ratios behave as predicted by the SID limit, but, in fact, the correlation observed in Table I is also reproduced by the direct capture model. That is, the computed values of b_2 are always negative in the case of $j=l+\frac{1}{2}$ and positive in the case of $j=l-\frac{1}{2}$. This, in fact, is the sign which would be predicted by the Racah coefficient alone as shown in Eq. (18). However, there does not appear to be any model independent reason for the remaining factors in b_k to be positive. We are currently investigating this point. The fact that it appears to be so may be largely a result of systematics associated with the optical model parameters used in the calculations.

In looking for evidence that these relationships among the b_k coefficients are observed in the experimental results, we have found only one case where b_k coefficients (for more than one k value) are well determined for cases having $l+\frac{1}{2}$ and $l-\frac{1}{2}$. This is the case of ^{7}Li(p,γ_0) and ^{7}Li(p,γ_1) where, as previously discussed, the (p,γ_1) case leads in fact to a state which is an admixture of $l+\frac{1}{2}$ and $l-\frac{1}{2}$. As seen Table II, the $E1+E2$ direct capture model reproduces the signs of the b_1, b_2, and b_3 coefficients, the b_4 coefficient being too small and uncertain to be definitive. In this case the SID result [Eq. (26)] implies that the ratio of the b_k for the (p, γ_1) case to the b_k for the (p,γ_0) case has the value -0.2. As obtained from Table II, the experimental ratios for $k=1$, 2, and 3 are 2.7, -0.88, and -0.73, respectively. The SID sign relationship is observed to hold for b_2 and b_3, but not for b_1. The additional sensitivity to SDD arising from the large single particle mixing in the first excited state of ^{8}Be has destroyed the SID sign relationship for b_1 in this case. While it is remarkable that the direct capture calculations predict the *proper* sign relationships, it should be noted that further investigations of these results have indicated a substantial sensitivity of the sign of b_1 to the choice of optical model parameters.

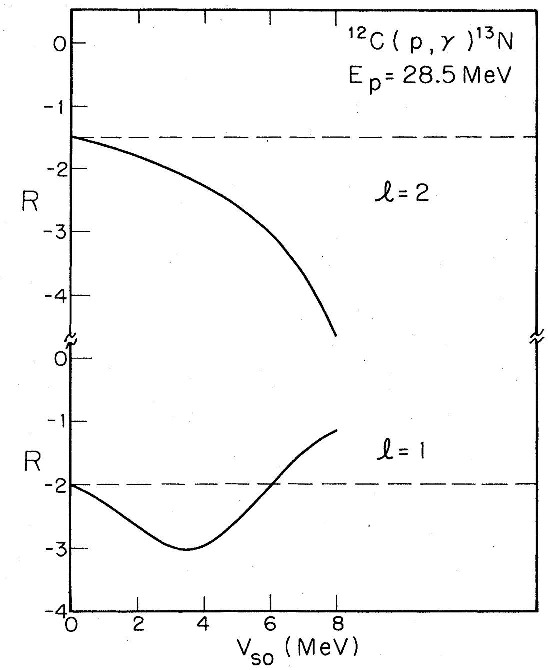

FIG. 3. The lower curve (b) is the direct capture model prediction for the quantity R [Eq. (32)] calculated for the reaction ^{12}C(p,γ_0)^{13}N at $E_p=28.5$ MeV. The final state was taken to have $l=1$ with $j=1+\frac{1}{2}$ or $j=1-\frac{1}{2}$. The incident channel spin-orbit potential strength was varied from 0.0 to 8.0 MeV. The upper curve (a) is the result of a similar direct capture calculation for the ^{12}C(p,γ_3)^{13}N reaction at $E_p=28.5$ MeV where the final state was taken to have $l=2$ with $j=2+\frac{1}{2}$ or $j=2-\frac{1}{2}$. Both curves are for $V_{so}^{B}=0$.

CONCLUSIONS

A review of the b_2 coefficients obtained in polarized nucleon capture experiments indicates a sensitivity of the b_2 coefficient to the j value of the single particle in the final state that is reminiscent of the j dependence of the analyzing power observed in direct stripping reactions for a given l transfer.[25] In the absence of spin distortions we have shown that if the transition operator does not alter the angular momentum of the target as the incident particle is captured to form the final state, then the b_k coefficients obey the relationship

$$\frac{b_k(j_f=l_f-\frac{1}{2})}{b_k(j_f=l_f+\frac{1}{2})}=-\frac{l_f+1}{l_f} .$$

This SID relationship holds for all k and for all multipolarities, both separately and together. It is true for direct and direct-semidirect model formulations as long as the condition above regarding the role of the target spin is valid. The available data are consistent with this simple relationship in that the signs of the b_2 coefficients for capture to $l+\frac{1}{2}$ and $l-\frac{1}{2}$ single particle states are opposite. In the cases where quantitative comparisons can be made, e.g., ^{11}B(p,γ) and ^{7}Li(p,γ), a reasonable agreement is found for the b_2 coefficients. The surprising feature of the data, however, is the fact that all b_2 coefficients for

$j=l-\frac{1}{2}$ are positive and all those for $j=l+\frac{1}{2}$ are negative. This result is not derivable in a model independent manner. It is not a rigorous result (we have generated violations with the model), but rather it appears to be valid for reasonable choices of potential parameters.

The application of our simple SID rule seems useful in circumstances where a number of final states at nearby energies are populated. In that case it should be reasonably reliable to make j assignments on the basis of the signs of the b_2 coefficients.

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