Unusual optical potential in prior-form breakup theory

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The optical potential for the center of mass of two fragments in a three-body breakup channel does not resemble elastic optical potentials obtained in two-body channels. As a consequence the value of prior-form distorted wave Born approximation breakup theories based on center-of-mass optical potentials is questionable.

Since the exit channel for a breakup reaction contains three interacting unbound particles, the target nucleus, and the two fragments of the incident projectile, there are two distinct ways[1,2] to introduce products of two-body distorted wave functions in this channel. This is seen in the two equivalent exact forms for the one-step amplitude for a (d,pn) reaction

$$T_{\rm post}=\langle\chi_{\rm p}^{(-)}(\vec{r}_{\rm p})\chi_{\rm n}^{(-)}(\vec{r}_{\rm n})\,|\,V_{\rm pn}\,|\,\Psi^{(+)}(\vec{r},\vec{R})\rangle\ , \tag{1}$$

$$T_{\rm prior}=\langle\chi_{\rm d}^{(-)}(\vec{R})\phi^{(-)}(\vec{k},\vec{r})\,|\,U_{\rm p}(r_{\rm p})+U_{\rm n}(r_{\rm n})-U_{\rm d}(R)\,|\,\Psi^{(+)}(\vec{r},\vec{R})\ . \tag{2}$$

Here the target nucleus is assumed to be at rest at the origin, so that the usual coordinate relations are $\vec{R}=\frac{1}{2}(\vec{r}_{\rm p}+\vec{r}_{\rm n})$, $\vec{r}=\vec{r}_{\rm p}-\vec{r}_{\rm n}$. Internal coordinates of the target nucleus are suppressed, so the individual nucleon-nucleus interactions are potentials $U_{\rm p},U_{\rm n}$ with associated distorted waves $\chi_{\rm p}^{(-)}$, $\chi_{\rm n}^{(-)}$. A deuteron distorted wave $\chi_{\rm d}^{(-)}$ is defined in terms of an associated potential $U_{\rm d}$, which is subtracted in the interaction in (2). The relative wave function $\phi^{(-)}(\vec{k},\vec{r})$ is a p-n scattering state, with asymptotic relative momentum $\vec{k}$. The labels *post* and *prior* in the above expressions are conventional. Thus $V_{\rm pn}$ is the residual interaction left out of the *exit* channel wave function in $T_{\rm post}$. However, although $U_{\rm p}+U_{\rm n}-U_{\rm d}$ is again an exit channel residual interaction in $T_{\rm prior}$, it is also the *entrance* channel residual interaction in the distorted wave Born approximation (DWBA) version of $T_{\rm prior}$.

The DWBA versions of (1) and (2) are

$$T_{\rm post}^{\rm DW}=\langle\chi_{\rm p}^{(-)}(\vec{r}_{\rm p})\chi_{\rm n}^{(-)}(\vec{r}_{\rm n})\,|\,V_{\rm ph}\,|\,\phi_{\rm d}(\vec{r})\chi_{\rm d}^{(+)}(\vec{R})\rangle\ , \tag{3}$$

$$T_{\rm prior}^{\rm DW}=\langle\chi_{\rm d}^{(-)}(\vec{R})\phi^{(-)}(\vec{k},\vec{r})\,|\,U_{\rm p}+U_{\rm n}\,|\,\phi_{\rm d}(\vec{r})\chi_{\rm d}^{(+)}(\vec{R})\rangle\ , \tag{4}$$

where the explicit $U_{\rm d}(R)$ has dropped out of (4), because of orthogonality between $\phi^{(-)}(\vec{k},\vec{r})$ and $\phi_{\rm d}(\vec{r})$. [There is no post-prior equivalence between the approximate expressions (3) and (4), since these are based on different choices of product wave functions in the exit channel.] Most modern breakup calculations are based on the post-form DWBA,[3] using the contour rotation technique of Vincent and Fortune.[4] Good agreement with experiment is obtained. However, the prior-form DWBA has attractive features,[5] such as rapid convergence over the p-n relative angular momentum and a well-behaved integrand. Although the first application of $T_{\rm prior}^{\rm DW}$ to deuteron breakup disagreed with experiment,[5] the difficulty was attributed to special problems of Coulomb distortions, and there has been continued interest in the prior-form DWBA (Ref. 6) for applications in which Coulomb distortions are weak.

The purpose of the present paper is to develop a point already encountered in a preliminary, specialized calculation of a (d,pn) reaction,[2] that the customary choice of $U_{\rm d}$ for the calculation of $\chi_{\rm d}^{(-)}$ in $T_{\rm prior}^{\rm DW}$ has been grossly incorrect, irrespective of Coulomb distortions. The problem is fundamental, having to do with the kinds of distorted wave approximations that are appropriate in three-body channels.

Basic explanations of the DWBA method[7] point out that one hopes to choose an exit channel (for example) distorting potential in such a way as to minimize truncation errors in the associated entrance channel wave function. In the present case, one hopes to choose $U_{\rm d}(R)$ in (2) to minimize the effect of replacing $\Psi^{(+)}(\vec{r},\vec{R})$ by $\phi_{\rm d}(\vec{r})\chi_{\rm d}^{(+)}(\vec{R})$ in (4).

To investigate truncation errors of $\Psi^{(+)}(\vec{r},\vec{R})$, it is necessary to approximate the wave function in terms of some calculable model that goes beyond DWBA. The familiar $l=0$ three-body deuteron-nucleus model[8] has the wave function

$$\Psi^{(+)}(\vec{r},\vec{R})=\sum_{LM}a_{LM}R^{-1}Y_{LM}(\hat{R})[\phi_{\rm d}(r)f_L(R)+\int_0^\infty dk\,\phi(k,r)g_L(\lambda,R)]\ , \tag{5}$$

where f_L and g_L are coefficient functions of the bound deuteron state $\phi_{\rm d}(r)$ and of the p-n scattering states $\phi(k,r)$, respectively. The momentum λ is determined from k by energy conservation. Upon substitution of (5) in (2), contributions from the breakup part of (5) are eliminated if $U_{\rm d}$ is chosen to fulfill the relation

$$U_{\rm d}(R)g_L(\lambda,R)=\frac{2}{\pi}\int_0^\infty dk'\langle\phi(k,r)\,|\,U_{\rm p}+U_{\rm n}\,|\,\phi(k',r)\rangle g_L(\lambda',R)\ . \tag{6}$$

Thus U_d is determined by the continuum-continuum coupling term of the three-body model, as is indeed appropriate for an optical interaction in a three-body continuum. Such a U_d does not resemble the folding potentials that are familiar in two-body channels. For example, U_d in (6) is seen to vanish linearly as $k \to 0$, as the p-n pair loses overlap with the target nucleus. Even at higher k values the continuum-continuum interaction is much weaker than folding potentials. However, it is only under this choice of U_d that (4) is a justifiable approximation of (2). Previous applications of (4) used energy-independent exit-channel deuteron potentials fitted to deuteron elastic scattering; such procedures led to major errors in the breakup spectrum, especially in the region of low k values.[2,5,6] (Of course the *entrance-channel* deuteron potential must fit elastic scattering.)

Unfortunately, although (6) illustrates the properties of a suitable U_d, it does not suggest a practical way to determine this U_d for use in (4), independently of the three-body model. As a consequence there is probably no way to apply the prior-form DWBA breakup theory in practical calculations.

The exit-channel optical potentials of the post-form breakup theory are the underlying two-body interactions U_p, U_n, which are not adjustable, hence the above uncertainty of procedure is not present. Nevertheless, the breakup part of $\Psi^{(+)}$ produces some corrections when (5) is inserted in (1). Although these corrections have not been studied, they are likely to be small: at most values of R, within the range of V_{pn} the breakup term of (5) is small[9] compared to the direct term.

Despite its present success, it is not clear that post-form DWBA is the inevitable practical theory for breakup reactions. It requires lengthy partial wave sums. Moreover, the corrections mentioned above may not be negligible. Under these circumstances we can note that the three-body model contains all the physical effects mentioned above, and it uses the same convenient $\vec{r},\vec{R}$ coordinates as the prior-form DWBA. Either the full coupled-channels three-body model,[8] or adiabatic[2,10] or quasiadabatic[9] simplifications might provide acceptable practical theories of breakup.

Of course, the (d,pn) reaction is discussed here only as one example of a breakup reaction. Similar ideas apply to the breakup of other projectiles.

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