One-pion-exchange potential deuteron

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The one-pion-exchange potential which depends on a single (cutoff) parameter is shown to provide an adequate description of most of the deuteron observables, provided that the parameter is adjusted to produce the correct binding energy. In particular, the tensor observables η and Q are within a few percent of their experimental values. Substituting this potential for the $^3S_1$-$^3D_1$ potential of a super-soft-core model in a Faddeev calculation of the ^{3}H ground state increases the binding energy by only 0.1 MeV.

Early work on the two-nucleon problem emphasized the one-pion-exchange (OPE) aspect;[1-3] it was the only component of the potential with a sound theoretical basis. Singling out the OPE parts of operators was partially successful, but foundered on the fact that the OPE potential (OPEP) was too singular and required arbitrary cutoffs. Nevertheless, all modern, realistic potential models contain a (cutoff) OPEP as an integral part; this longest range part of the potential is well-founded experimentally.[4] The isovector OPE electromagnetic current[5] enhances isovector magnetic dipole processes and has been shown to bring certain experiments and theory into agreement for the few-nucleon systems.[6,7]

Chiral bag models of the nucleon[8] attempt to incorporate both the long-range fields associated with the light pion and the short-range dynamics associated with the nucleon quark substructure. Thus, such models for the deuteron will incorporate some of the OPE features discussed above. A natural question arises: what measures should be used to judge the success (or failure) of such a calculation? It is well-known that many of the deuteron's properties are merely a reflection of its small binding energy, while others are a reflection of details of the binding, such as the tensor force. Recently, considerable attention[9,10] has been directed at processes which are particularly sensitive to the OPEP. How important are the non-OPEP components of the force in such calculations? In an attempt to offer (at best) a partial answer to these questions, we explore below the extent to which a suitably defined "pure" one-pion-exchange potential can explicate the deuteron's properties. In addition this potential, although quantitatively cruder than "realistic" potential models, does provide a useful and simple gauge for examining processes involving the deuteron. We use this procedure in an examination of deuteron forward photodisintegration. Although none of what we do is particularly new, we wish to update, correlate, and extend what has been done before.

Blatt and Weisskopf[11] divide the deuteron's properties into two convenient categories: "inside" quantities and "outside" quantities. The former are sensitive to the interior portion of the deuteron and hence to the short-range parts of the nuclear force. Conversely, the latter are primarily determined by the long-range part of the potential and wave function. It has become increasingly evident that most of the familiar quantities fall in the outside category, although it must be borne in mind that sometimes these can also reflect the interior dynamics. One example of this caveat is the binding energy, E_B, which can be determined precisely from a knowledge of the tail of the *exact* wave function but which clearly depends upon the potential in the interior. The tail of the wave function plays a predominant role in the determination of the following quantities: A_S, the asymptotic S-wave normalization; η , the asymptotic D- to S-wave normalization ratio; Q, the quadrupole moment; α_E, the deuteron electric polarizability; and $\langle r^2\rangle^{1/2}$, the rms charge radius (without *nucleon* form factors). The deuteron magnetic moment is insensitive to virtually everything but the size of the D-state, which makes it an inside quantity. The various two-nucleon effective ranges, one of which $[\rho(-E_B,-E_B)]$ is based on an extrapolation to the deuteron pole, are also inside quantities. The asymptotic S-wave normalization, A_S, is moderately sensitive to the latter quantity.[12]

Low-energy electric dipole deuteron photodisintegration also is determined essentially by the tail of the deuteron wave function. The 90° (outgoing proton) cross section and the total cross section depend primarily upon A_S and the tail of the S-wave function.[13] The 0° cross section depends primarily on η and the tail of the D-wave function.[14] Both A_S and η are outside quantities. The latter cross section has proved to be difficult to understand theoretically. Finally, we note that the electric polarizability of the deuteron can be represented as an integral over the low-energy electric dipole total photoabsorption cross section.[15] In the zero-range limit, this can be shown to be proportional to A_S^2/E_B^2. Thus, it is exceptionally sensitive to the deuteron binding energy, as are all of the outside quantities.

We write the conventional one-pion-exchange potential as

$$V_\pi = \mu_\pi f_0^2(\vec{\tau}_1\cdot\vec{\tau}_2)(S_{12}V_T+\vec{\sigma}_1\cdot\vec{\sigma}_2V_C)/3 \ , \tag{1a}$$

$$V_T = h_0'' - h_0'/x \ , \tag{1b}$$

$$V_C = h_0'' + 2h_0'/x \ , \tag{1c}$$

where μ_π is the pion mass, $f_0^2 = 0.079$, S_{12} is the usual tensor operator, $x=\mu_\pi r$, and $\vec{\sigma}_i$ and $\vec{\tau}_i$ are the usual (Pauli) spin and isospin operators for nucleon i. Moreover, for $x \neq 0$

$$h_0(r) = \frac{4\pi}{(2\pi)^3\mu_\pi}\int\frac{d^3q e^{i\vec{q}\cdot\vec{r}}}{(\vec{q}^2+\mu_\pi^2)}F_{\pi NN}^2(\vec{q}^2) \to \frac{e^{-x}}{x} \ , \tag{2a}$$

$$V_T \to e^{-x}(1/x+3/x^2+3/x^3) \ , \tag{2b}$$

$$V_C \to e^{-x}/x \ , \tag{2c}$$

where the limiting relationships hold if $F_{\pi NN}$, the pion-nucleon form factor, is replaced by 1. Without the form factor the tensor potential overwhelms the angular momentum barrier and becomes too singular for a conventional solution. In this case there are severe problems of definition and interpretation discussed in Refs. 16 and 17.

The necessity of a pion-nucleon form factor is indicated by the Goldberger-Treiman (GT) discrepancy.[18] Assuming a conventional n-pole form factor

$$F_{\pi NN}(\vec{q}^2)=\left[\frac{\Lambda^2-\mu_\pi^2}{\vec{q}^2+\Lambda^2}\right]^n , \tag{3}$$

The discrepancy is given by $F_{\pi NN}(0)=(1-1/\beta^2)^n\cong 1-n/\beta^2$, where $\beta\mu_\pi=\Lambda$ and β is large. Thus for a fixed GT discrepancy, $\beta^2/n=\beta_0^2$ relates various $n-$values. Most calculations assume $n=1$ (monopole form). If the entire (6%) discrepancy is assumed, we obtain $\beta\cong 4.1$, while assuming half (3%) as argued by Refs. 18 and 10 produces $\beta\cong 5.8$ and a corresponding mass Λ of 800 MeV.

Given the above form of $F_{\pi NN}$ and $\xi=(\beta^2-1)/2\beta^2$ one obtains for an n-pole form factor with $m=2n$,

$$h_0^{[m]}=\frac{e^{-x}}{x}-\beta e^{-\beta x}\sum_{i=0}^{m-1}\frac{\xi^i}{i!}\delta_i(\beta x) , \tag{4}$$

where

$$\delta_{n+1}(z)=(2n-1)\delta_n(z)+z^2\delta_{n-1}(z) , \tag{5}$$

with $\delta_0=1/z$ and $\delta_1=1$. Using

$$\frac{d}{dz}\delta_n(z)=\delta_n(z)-z\delta_{n-1}(z) , \tag{6}$$

we also obtain

$$V_C^{[m]}=\frac{e^{-x}}{x}-\beta^3 e^{-\beta x}\sum_{i=0}^{m-1}\frac{\xi^i}{i!}(\delta_i-2i\delta_{i-1}) , \tag{7}$$

and

$$V_T^{[m]}=\frac{e^{-x}}{x}\left(1+\frac{3}{x}+\frac{3}{x^2}\right)-\beta^3 e^{-\beta x}\sum_{i=0}^{m-1}\frac{\xi^i}{i!}[\delta_i-(2i-3)\delta_{i-1}] . \tag{8}$$

The structure of potential terms of range $\beta\mu_\pi$ makes it virtually impossible to distinguish these form factor terms in OPEP from short-range meson exchange contributions to the potential. Finally, an interesting limit is given by $m\to\infty$ and assuming a fixed GT discrepancy with $\beta_m^2=m\beta_0^2$; in this limit $F_{\pi NN}\to\exp[-(1+\vec{q}^2/\mu_\pi^2)/\beta_0^2]$ and $h_0^{[\infty]}$ can be expressed in terms of the error function.

Given the potential Eq. (1) we can vary the parameter β to produce a deuteron bound state with energy 2.2246 MeV. We assume an "average" pion mass $\mu_\pi=138.03$ MeV, $\hbar^2/m=41.47$ MeV fm^2, and $\hbar c=197.328\,58$ MeV fm, and by fitting the binding energy determine β for various m's. Table I presents various deuteron quantities calculated with this potential and the corresponding experimental values. The effective range $\rho(-E_B,-E_B)$ is in worst agreement and this is reflected is A_S, but in view of the fact that the repulsive inner part of the nucleon-nucleon potential is determined by a single simple cutoff, the agreement is not at all bad. Particularly impressive are η and Q which are generated by the tensor part of the potential. Higher values of m are favored, although the range of values is not great if we disregard the occasionally used, but unphysical, square root ($m=1$) form factor. The hard core (HC) result[2] is more dissimilar. Note that $\beta_0(\infty)$ corresponds to a GT discrepancy of 3%, an interesting accident. All values of $\beta_0=\beta/\sqrt{m}$ are similar except for $m=1$. For comparison we have also listed as (2′) the monopole result corresponding to $f_0^2=0.81$, a $2\frac{1}{2}$% increase. Other values of f_0^2 near 0.079 can be obtained by linear interpolation.

At first glance these results appear rather astonishing. Further reflection shows them to be somewhat less so. We reiterate that obtaining the correct binding energy is crucial; moreover, many of the quantities listed are known to be very sensitive to the long-range potential. Since our A_S is approximately 2% smaller than experiment, A_D is presumably smaller by a similar amount. Nevertheless, our purpose was to establish the primacy of the OPEP in determining many of the "outside" deuteron properties, and we have done that. As a final comment we note that if the cutoff OPEP in any of the realistic potentials is extracted and its expectation value is calculated using the appropriate complete deuteron wave function, the resulting number compares favorably with the expectation value of the complete potential! In this sense also, the OPEP part of the potential dominates the deuteron.

If OPEP works well in the deuteron ground state, what about its efficacy in unbound states? The $^1S_0$ OPEP potential is insufficiently attractive and other low-lying partial

TABLE I. Various deuteron observables and their corresponding values. Experimental values have been taken from the convenient summaries in Refs. 9 and 10. The rms radius comes from Ref. 23 (see also the discussion of its relationship to A_S). The value of m in β_m is indicated as OPEP(m). The last entry in this column is the hard core radius in fm. The entry label (2′) corresponds to $f_0^2=0.081$.

	$\beta/\sqrt{m}$	E_B	P_D	A_S	η	Q	$\langle r^2\rangle^{1/2}$	$\rho(-E_B,-E_B)$
OPEP(1)	4.432 949	2.2246 MeV	6.37%	0.854 fm$^{-1/2}$	0.0264	0.264 fm^2	1.902 fm	1.57 fm
OPEP(2)	5.166 245	2.2246 MeV	6.65%	0.864 fm$^{-1/2}$	0.0269	0.276 fm^2	1.925 fm	1.64 fm
OPEP(4)	5.495 301	2.2246 MeV	6.75%	0.868 fm$^{-1/2}$	0.0271	0.280 fm^2	1.934 fm	1.66 fm
OPEP(10)	5.682 068	2.2246 MeV	6.80%	0.870 fm$^{-1/2}$	0.0272	0.282 fm^2	1.939 fm	1.68 fm
OPEP(∞)	5.802 450	2.2246 MeV	6.83%	0.872 fm$^{-1/2}$	0.0272	0.283 fm^2	1.941 fm	1.68 fm
OPEP(2′)	5.071 227	2.2246 MeV	6.72%	0.867 fm$^{-1/2}$	0.0273	0.280 fm^2	1.931 fm	1.66 fm
OPEP(HC)	0.485 099	2.2246 MeV	7.46%	0.878 fm$^{-1/2}$	0.0275	0.291 fm^2	1.957 fm	1.72 fm
Expt.	· · ·	2.224 575(9) MeV	· · ·	0.8846(8) fm$^{-1/2}$	0.0271(4)	0.2859(3) fm^2	1.955(5) fm	1.764(5) fm

waves also are deficient. Higher waves are OPEP dominated, however. Therefore, for purposes of detailed comparison we shall use realistic potentials in partial waves other than $^3S_1-{}^3D_1$.

We have calculated the 0° deuteron photodisintegration, which is a subject of considerable current interest. The results of using the monopole ($m=2$) OPEP deuteron with deTourreil-Rouben-Sprung (dTRS)[19] higher partial wave potentials (dashed curve) and a corresponding dTRS calculation (solid curve) are shown in Fig. 1. Both potentials have nearly identical values of A_D and, indeed, almost identical cross sections. If the triplet partial waves of the excited states are replaced by OPEP, the long-short dashed curve results, and is clearly inferior. Using the OPEP potential for low partial waves far from threshold is dangerous.

The OPEP is also found to play an extremely important role in the trinucleon system. If the $^3S_1-{}^3D_1$ partial wave potentials of the SSC(C) [Ref. 20] are replaced by our monopole ($m=2$) OPEP $^3S_1-{}^3D_1$ potentials, corresponding five-channel Faddeev calculations show a change in the binding energy from 7.457 to 7.554 MeV, a net accrual of 0.1 MeV. The additional repulsion in the realistic force presumably accounts for this small difference. The same calculation for the RSC potential increases the binding by 0.4 MeV. Although somewhat larger, the latter accrual is less than 1% of the total potential energy. We note that the $^3S_1-{}^3D_1$ partial waves account for at least 75% of the total potential energy and therefore dominate the binding.[21,22]

The reason for this small change is the relative insensitivity of the three-nucleon bound states to the higher-energy phase shifts of the two-nucleon systems. The very low-energy properties of the $^3S_1-{}^3D_1$ partial waves were constructed to be as correct as possible, and this is reflected in the small change noted above. More importantly, it illustrates the dominance of the long-range OPEP and the relative unimportance of the shorter-range parts of the potential in determining the trinucleon binding energy.

In conclusion, we have shown that the one-pion-exchange potential with a single cutoff parameter adjusted to give the deuteron's binding energy provides a very good description of the deuteron ground state properties and the binding of the trinucleon. In particular, the tensor observables in the former system are well-reproduced at the few percent level, independent of the type of cutoff used. This demonstrates that the OPEP is a very good starting point for investigating these quantities, provided that the deuteron binding energy is correct. The necessity of the short-range cutoff and the difficulty in separating a short-range cutoff from a massive meson exchange was pointed out. *Ab initio* calculations of the deuteron that do not reproduce the deuteron binding cannot expect to reproduce many of the deuteron's observables. Such calculations which automatically include the OPEP have a good chance to reproduce quantitatively the tensor observables η and Q.

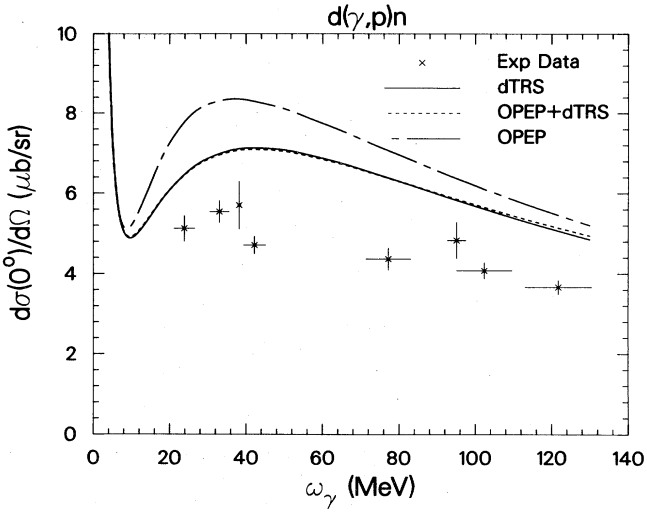

FIG. 1. Deuteron photodisintegration with a forwardgoing proton in the nonrelativistic impulse approximation. Data are from Refs. 24, 25, and 26.

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