Mass differences between mirror nuclei in a hybrid quark-nucleon model

J. M. Greben*
Department of Physics, University of Alberta, Edmonton, Alberta, Canada T6G 2J1

A. W. Thomas†
CERN, Geneva, Switzerland
and Physics Department, University of Adelaide, Adelaide, South Australia
(Received 22 February 1984)

A hybrid quark-nucleon model of nuclei is developed in which nucleons merge into multiquark bags at short distances. This model is applied to calculate mass differences between ^{3}He and ^{3}H and a number of other mirror nuclei. For light nuclei we obtain a reduction of the discrepancy between experiment and conventional theory. Probabilities for the formation of six-quark bags and nine-quark bags in these nuclei are evaluated, and the consequences of our results are discussed. In particular we comment on the compatibility of conventional and the hybrid quark-nucleon results.

I. INTRODUCTION

In the last few years considerable excitement has been generated by the idea that we may one day achieve a truly unified description of nucleon and nuclear structure.[1–4] For the present most attempts at such a unification are necessarily based on quantum chromodynamic (QCD) motivated phenomenology, rather than QCD itself. Given the present diversity of models of hadron structure, it is hardly surprising that there is no agreement on how to deal with nuclei. Nevertheless, a number of attempts have already been made to describe the short-distance N-N force at the quark level.

Undoubtedly the most sophisticated calculations of the N-N force using the quark model have been based on the nonrelativistic (constituent) quark model.[5–8] There, one has the tremendous technical advantage that one can draw on long experience in the application of resonating group methods to light ion reactions. With the addition of a long range interaction associated with pion exchange, this approach has even achieved semiquantitative agreement with *S*-wave N-N data.

The first application of the bag model (which has the advantage of avoiding color van der Waals forces) was similar in spirit to the nonrelativistic quark model.[9] But recently the *P*-matrix method[10] has been more commonly used—again with considerable success.[11–16] The essential idea is that inside some boundary radius the N-N system should be described as a six-quark ($6q$) bag. By analogy with the old *R*-matrix theory, one demands that the exterior wave function vanishes at the boundary when the total energy of the system matches the mass of the internal $6q$ state.

There is no doubt that in order to be credible in nuclear physics, any quark model must eventually provide a fit to N-N elastic scattering data at least as good as that provided by the Paris potential.[17] However, that day may be some years away. In the meantime it is tempting to assume that eventually such a fit will be achieved, and to ask whether this new description of the short distance physics may have other consequences.[4,18] Examples of such applications include the electrodisintegration of the deuteron[19] and ^{3}He,[20] parity violation in the N-N system,[21,22] the EMC effect,[23–25] and so on.

In this paper we address the theoretical question of how to make consistent calculations in finite nuclei using such a hybrid quark-nucleon model. As an example we investigate the consequences for the systematic discrepancy in the energy differences of mirror nuclei—the Nolen-Schiffer anomaly.[26–28] In particular, it has already been observed that the mass difference between $6q$ bags formed from two protons and two neutrons is not equal to twice the proton-neutron mass difference.[4] Similar arguments apply to $9q$ bags if the overlap of more than two nucleons is important. These mass differences amount to a somewhat different model of the charge-symmetry violating N-N force, and therefore could contribute to reducing the size of the Nolen-Schiffer anomaly.

The plan of the paper is as follows. In Sec. II we discuss the mass differences of $6q$ bags. In Sec. III we develop the formalism for a hybrid description of nuclear systems. For the present problem, where we deal only with energy differences, the probability that two nucleons are within the critical radius b is the essential quantity. This probability is defined and calculated for several values of b in Sec. IV. Because it is amenable to exact treatment the 3N system is dealt with separately in Secs. V and VI. The results for larger nuclei are presented in Sec. VII, and a brief discussion follows in Sec. VIII.

II. MASS DIFFERENCES IN THE BAG MODEL

At the quark level the n-p mass difference must have at least two sources.[29] In fact, the Coulomb interaction would make the proton heavier than the neutron—typically by $\sim$0.5 MeV.[29,30] Within about 10% this result can be represented by

$$\Delta M_{\rm em} = 1.44 \sum_{i<j} \frac{Q_i Q_j}{R} , \tag{2.1}$$

which yields $m_{\rm p} - m_{\rm n} = 0.48$ MeV with $R = 1$ fm. In order to explain the observed mass difference one needs to assign different masses to the quarks themselves. Within the bag model the energy of a quark is $E=\omega/R$, with ω the eigenfrequency implied by the nonlinear boundary condition.[4,31] By numerical solution of the Dirac equation for a quark of mass m, in a cavity of radius R, one finds[32] for $R\approx 1$ fm:

$$\frac{d\omega}{d(mR)}\approx 0.49 \ . \qquad (2.2)$$

A quark mass difference of $(m_d-m_u)\sim 4$ MeV (Refs. 32 and 33) then gives the correct n-p mass difference, provided the difference in the color hyperfine interaction for a d and u quark[32] is taken into account as well. The hyperfine interaction term can approximately be accounted for by using the value 0.42 instead of 0.49 in the left-hand side of Eq. (2.2).

Let us now consider the region where two nucleons overlap sufficiently to be considered a $6q$ bag—in the boundary condition model this is when $r<b$. Clearly the Coulomb interaction (2.1) will now involve a sum over $i\in(1,6)$, which will, e.g., not be simply twice the value obtained in the proton for a bag containing $4u$ and $2d$. A further correction arises because the radius of a $6q$ bag is about 20% bigger than a $3q$ bag. For the Coulomb force this is trivial to include, but for the quark mass effect it is much more unclear.

If the quark mass m really was a scalar number independent of the environment, $(\omega_u-\omega_d)/R$ would not change between a $3q$ and a $6q$ bag, because of Eq. (2.2). However, we know that the light quark masses are still a mystery. They are presumably the residual effect of renormalization due to interactions with a much larger energy scale, and in a cavity they may depend on the size. For dimensional reasons it would be natural to set $m\propto R^{-1}$ (the weaker assumption $m_d-m_u\sim R^{-1}$ is in fact sufficient), in which case we find

$$(E_d-E_u)_6=(E_d-E_u)_3\left[\frac{R_3}{R_6}\right] . \qquad (2.3)$$

[Here $(E_d-E_u)_i$ is the difference in the total energy of a d and u quark in a bag of i quarks, and (R_3,R_6) are the corresponding bag radii.] It is interesting that an R-dependent mass was phenomenologically necessary for Deshpande *et al.* to reproduce the mass differences for the strange members of the nucleon octet.[30] We shall adopt Eq. (2.3) here as a working hypothesis. However, we stress that it is no more than that in the absence of a deeper theoretical understanding of light quark masses.

Even worse, from the point of view of serious quantitative predictions we note that there are other mass dependent corrections to the mass of the MIT bag, for which there is as yet no theoretical consensus on the sign.[34,35] Clearly we are at an early stage of understanding quark dynamics, and one cannot expect high precision in the predictions. Nevertheless, it is our belief that Eqs. (2.1) and (2.3) should provide at least an indication of the magnitude of the charge-symmetry violation to be expected in a quark bag model.

III. HYBRID MODEL OF QUARKS IN NUCLEI

Symbolically we represent the N-N system as follows:

$$\Psi=\mathscr{A}\Psi_1\Psi_2\phi_{12}(\vec{r}),\ \ r_{12}>b\ ,$$
$$\Psi=C\Phi_6(\xi_1,\ldots,\xi_6),\ \ r_{12}<b\ . \qquad (3.1)$$

In these equations Ψ_1 and Ψ_2 represent normalized nucleon wave functions, i.e., nonrelativistic Pauli spinors, while $\phi_{12}(\vec{r})$ is the relative two-nucleon wave function. The six-quark wave function is written as a product of a normalized wave function ϕ_6 and a probability amplitude C. Larger nuclei are then described using conventional models modified to account for the short-range behavior implied by Eqs. (3.1).

Clearly this does not provide for a complete description of the strong dynamics in nuclei; however, before developing the model in further detail we want to consider what information is needed to calculate the mass differences according to our prescriptions in Sec. II. Essential for these calculations will be the six quark probability $|C|^2$, the transition radius b, the bag radii R_3 and R_6, and to a smaller extent the radius R_9. Obviously these five quantities are not independent, although their exact relationship depends on the details of the model or theory. If we assume that the quark density in the $(3n)$-quark bags is constant, then $R_{3n}=n^{1/3}R_3$. On the other hand, if we consider a multiquark MIT bag with just a volume and a mass term $(\sim 1/R)$, and assume that all quarks are in an S state, then the nonlinear boundary condition leads to $R_{3n}=n^{1/4}R_3$. We have used a conservative value for the exponent between these two extremes, namely 0.27. In a more detailed description one would also expect to find a relation between b and the bag radii; however, we treat b as a free parameter (within reasonable limits). The six-quark probability $|C|^2$ depends strongly on b and will in general not be treated as a free parameter. In the following we discuss various different approaches, all of which give a unique determination of $|C|^2$ for a specific b.

If $\phi_{12}(\vec{r})$ in Eq. (3.1) is taken to be identical to the conventional nuclear wave function, then $|C|^2$ automatically equals the probability defect of this wave function for $r<b$. This is the simplest prescription for the six-quark probability. We also consider the following modifications. First, because of the different strong dynamics for $r<b$, the probability to find six quarks with $r<b$ does not have to be the same as that of finding two nucleons with $r<b$ in the conventional picture. To accommodate this change we could allow for a different normalization of the external wave function, even though its shape remains the same. Second, the effective potential for $r>b$ may have to be modified to accommodate the different dynamics for $r<b$. This would even lead to a different shape of the external wave function.

In order to decide which approximations are most appropriate for calculating $|C|^2$, we look for guidance in the nonrelativistic quark potential calculations.[5-8] All of these calculations indicate that there is no sudden decrease in the six-quark probability for small r. The sign change of the S-wave phase shifts, which is usually explained by short-range repulsion or equivalently by the vanishing of the short-range N-N wave function, can then be interpreted as the absence of N-N components in the short-range six-quark wave function or as a node in the conventional wave function for small r. Therefore, if we want to determine the six-quark probability from the conventional wave function defect, we should not use strongly repulsive N-N potentials for the short-distance behavior. In nuclei we can, therefore, use uncorrelated shell-model wave functions rather than correlated ones, unless the correlation function only represents a modest short range repulsion. We thus see that the use of a hybrid quark-nucleon model can even lead to a simpler description of nuclei. In most of our calculations we have employed the uncorrelated wave functions; however, to check the stability of our results against this particular assumption we have also performed some calculations with correlation functions, and some calculations in which $|C|^2$ is basically treated as a free parameter.

For the three-body system, where exact conventional wave functions are available, we have also opted for a simpler uncorrelated wave function. Because of the quark-potential argument above it does not seem appropriate to base the wave function on conventional potentials; in addition the three-body calculations have been rather unsuccessful in reproducing the major physical properties of the three-body system (the three-body binding energy and the charge form factor), so that the arguments for using the "exact" wave functions are not particularly strong. We note in this connection that exact three-body wave functions have been used for the study of quark effects in deep inelastic scattering by Vary.[20]

In a recent study,[36] where a similar boundary condition model was used for the description of continuum wave functions in the two-body system, it was shown that current conservation guarantees the identity of the six-quark probability and the conventional wave function defect for $r<b$ as long as we do not change the interaction for $r>b$. Thus it may seem that the six-quark probability is independent of the internal dynamics. This is of course not the case. It is simply that in Ref. 36 only those descriptions for $r<b$ are allowed which together with the conventional potential for $r>b$ lead back to the original phase shifts. Whether there exists a model of the internal dynamics which can satisfy such a constraint is still an open question. While this identity was derived for the continuum case, it has subsequently also been stated to hold in the bound state case.[36,37] This conclusion is clearly subject to the same caution expressed for the continuum case. If true, it guarantees automatically the correctness of our first prescription for calculating $|C|^2$.

For larger nuclei, where we have little information on the normalization of the conventional wave function, we do not have any strong constraints on the six-quark probability and we have to rely on our physical intuition to decide which of the possible options for determining $|C|^2$ are most reasonable.

Let us now describe in some more detail our hybrid model of nuclei. Using the uncorrelated shell model wave function we can easily write the conventional part of the wave function of the $(A+1)$-nucleon system as the following:

$$\Psi_N(1,2,\ldots,A+1)=\mathscr{A}\left\{\prod_{i<j}^{A+1}[1+f_{ij}(r_{ij})]\times\prod_{i=1}^{A+1}\phi_{\alpha_i}(i)\right\}, \quad (3.2)$$

where

$$f_{ij}(r_{ij})=-\theta(b-r_{ij}), \quad (3.3)$$

and the $\phi_{\alpha_i}(i)$ are normalized single-particle states with quantum numbers α_i. For the correlated shell-model wave function, f_{ij} should be nonzero for $r_{ij}>b$ and the wave function should be renormalized. The third possible description is to change the normalization of Ψ_N arbitrarily, and to maintain the correct overall normalization by adjusting the six-quark probability.

Since the valence particle, in which we are interested mostly, is characterized by its single-particle quantum numbers α_v, we prefer to represent ψ_N as follows:

$$\Psi_N(1,2,\ldots,A)=\mathscr{A}\left\{\prod_{\alpha_i<\alpha_j}^{\alpha_v}[1-\theta(b-r_{\alpha_i\alpha_j})]\times\prod_{i'=1}^{A+1}\phi_{\alpha_{i'}}(i')\right\}. \quad (3.4)$$

The radius $r_{\alpha_i\alpha_j}$ should now be considered as an operator defined by the following:

$$r_{\alpha_i\alpha_j}\phi_{\alpha_i}(m)\phi_{\alpha_j}(n)=r_{mn}\phi_{\alpha_i}(m)\phi_{\alpha_j}(n). \quad (3.5)$$

This notation has the advantage that $\mathscr{A}$ can operate directly on the single-particle wave functions as it commutes with $r_{\alpha_i\alpha_j}$. Since we are mainly concerned with the state of the valence particle and do not care whether the core particles form six-quark bags between themselves, we define the new "conventional" wave function

$$\psi_N^v(1,2,\ldots,A)=\mathscr{A}\left\{\prod_{\alpha_i<\alpha_v}[1-\theta(b-r_{\alpha_i\alpha_v})]\times\prod_{i=1}^{A+1}\phi_{\alpha_i}(i)\right\}, \quad (3.6)$$

which is constructed to guarantee that the valence particle is not in a six-quark bag. In Eq. (3.6) we have represented the core wave function by a single determinant, so that it also includes the six-quark configurations for $r_{\alpha_i\alpha_j}<b$ with $\alpha_i<\alpha_j<\alpha_v$. This is why we put our "conventional" in quotation marks. If we now define the full Slater determinant by

$$\Psi^0(1,2,\ldots,A)=\mathscr{A}\left[\prod_{i=1}^{A+1}\phi_{\alpha_i}(i)\right], \quad (3.7)$$

then we can interpret

$$P_Q^v = \langle \Psi^0 | \Psi^0 \rangle - \langle \Psi_N^v | \Psi_N^v \rangle = 1 - \langle \psi_N^v | \Psi_N^v \rangle \quad (3.8)$$

as the probability of the valence particle being part of one or more six-quark bags. Since we have only discussed the situation that two nucleons merge into a single six-quark bag, it is convenient to define

$$| \psi_{Q_1}^v \rangle = \mathscr{A} \left\{ \sum_{\alpha_i} \theta(b - r_{\alpha_i \alpha_v}) \prod_{\alpha_j \neq \alpha_i}^{\alpha_A} [1 - \theta(b - r_{\alpha_j \alpha_v})] \times \prod_{i=1}^{A+1} \phi_{\alpha_i}(i) \right\} , \quad (3.9)$$

which represents that part of the full wave function for which the valence particle forms a six-quark bag with a single core nucleon. The corresponding probability

$$P_{Q_1}^v = \langle \psi_{Q_1}^v | \psi_{Q_1}^v \rangle \quad (3.10)$$

can therefore be used in connection with our model of mass differences between six-quark bags. Calculating $P_{Q_1}^v$ implies calculating the expectation value of an $(A+1)$-body operator for $|\psi_0\rangle$. This is only feasible for the three-body case ($A=2$), as we will demonstrate in Sec. V. For now we deal with the large A case, and we rely on an expansion in the correlation function, observing that matrix elements of $\theta(b - r_{ij})$ will be small if b is small. The lowest order result in $\theta(b - r_{ij})$, which will be denoted by P_b^v, is a sum of single particle terms:

$$P_b^v = \sum_{\alpha_m = \alpha_1}^{\alpha_A} P_{\alpha_m}(b) = \sum_{nlji_z} (2j+1) P_{nlji_z}(b) , \quad (3.11)$$

where

$$P_{\alpha_m}(b) = \langle \phi_{\alpha_v}(1) \phi_{\alpha_m}(2) | \theta(b - r_{12}) | \phi_{\alpha_v}(1) \phi_{\alpha_m}(2) - \phi_{\alpha_v}(2) \phi_{\alpha_m}(1) \rangle . \quad (3.12)$$

Notice that despite the fact that $|\psi_{Q_1}\rangle$ is first order in $\theta(b - r_{ij})$, the quadratic expression (3.10) is still first order in $\theta(b - r_{ij})$ because of the identity

$$\theta(b - r_{ij}) \theta(b - r_{ij}) = \theta(b - r_{ij}) . \quad (3.13)$$

The lowest order result (3.12) is identical to what we would have obtained from Eq. (3.8) in lowest order. Although it looks remarkably similar to the matrix element of a residual short-range interaction, it would be wrong to identify the operator $\sum_{i<j} \theta(b - r_{ij})$ this way, since for the higher-order terms such an interpretation breaks down.

The calculation of higher order terms becomes more and more difficult. We can avoid these complications by giving a classical interpretation to (3.11), namely by interpreting $P_{nlji_z}(b)$ as the probability for the valence particle to be within a distance b of a *specified* core particle with quantum numbers $nlji_z$. Then, by assuming that the chance for the valence particle to overlap with a core particle does not depend on whether it already overlaps with other core particles, we can calculate all required probabilities in a straightforward fashion. For example, the chance for the valence nucleon to form exactly one and only one pair with a core nucleon is given by the following:

$$P_{Q_1}^v = \sum_{nlji_z} (2j+1) P_{nlji_z}(b) / [1 - P_{nlji_z}(b)] \times \prod_{n'l'j'i'_z} [1 - P_{n'l'j'i'_z}(b)]^{2j'+1} . \quad (3.14)$$

In practice the dependence on single-particle quantum numbers is completely insignificant in calculating these average properties, and one might just as well use the average probability P_b^v / A. With this simplification we can write

$$P_{Q_1}^v = P_b^v (1 - P_b^v / A)^{A-1}$$

and

$$P_{Q_2}^v = \binom{A}{2} \left[\frac{P_b^v}{A} \right]^2 \left[1 - \frac{P_b^v}{A} \right]^{A-2} . \quad (3.15)$$

Finally, the chance for the valence particle to form at least one six-quark bag is

$$P_Q^v = 1 - \left[1 - \frac{P_b^v}{A} \right]^A . \quad (3.16)$$

In the three-body case we can evaluate all of these quantities explicitly using the quantum mechanical expressions and therefore we can test our semiclassical model explicitly. Not unexpectedly, it will appear that the model does not work very well in the three-body case. However, the suggested modifications for the three-body case, when applied to the many-body case, do not change the results significantly, so that it appears that the semiclassical model can be used with some confidence in the many-body case.

IV. THE PROBABILITY FOR OVERLAP OF THE VALENCE NUCLEON WITH A NUCLEON IN THE CORE

In Eqs. (3.11) and (3.12) we defined the overlap probability $P_{nlji_z}(b)$, which can also be written as follows:

$$P_{nlji_z}(b) = \frac{1}{2j_v + 1} \frac{1}{2j+1} \sum_{\alpha_v \alpha_m} \langle \phi_{\alpha_m}(1) \phi_{\alpha_v}(2) | \theta(b - r_{12}) | \phi_{\alpha_m}(1) \phi_{\alpha_v}(2) - \phi_{\alpha_m}(2) \phi_{\alpha_v}(1) \rangle , \quad (4.1)$$

where the sum is over the magnetic substates. We assume that the differences between neutron and proton orbits can be

TABLE I. Probabilities for the valence nucleon to form six-quark bags with one ($P^v_{Q_1}$) or two core nucleons ($P^v_{Q_2}$). The transition radius is $b=0.95$ fm. The values in parentheses are defined using higher order correlations as described in Sec. V.

Core	^{12}C	^{16}O	^{28}Si	^{32}S	^{40}Ca
$P^v_{Q_1}$	0.151	0.117	0.168	0.167	0.137
$P^v_{Q_2}$	0.013	0.0074	0.017	0.017	0.011
	(0.014)	(0.0081)	(0.018)	(0.018)	(0.011)

ignored, and will from now on suppress the isospin index where possible. For the direct term we obtain after some standard angular momentum algebra:

$$P^d_{nlj}(b)=\int_0^\infty dr_1 r_1^2 \int_0^\infty dr_2 r_2^2 \, |\phi_{nlj}(r_1)\phi_{n_v l_v j_v}(r_2)|^2 \tfrac{1}{2}\int_{-1}^{1} d\cos\theta \; \theta(b-r_{12}) \; . \tag{4.2}$$

The exchange term is found to be the following:

$$P^e_{nlji_z}(b)=\delta_{i_z i_v}\int_0^\infty dr_1 r_1^2 \int_0^\infty dr_2 r_2^2 e_{ljl_vj_v}(r_1,r_2)\phi^*_{nlj}(r_1)\phi^*_{nlj}(r_2)\phi_{n_vl_vj_v}(r_1)\phi_{n_vl_vj_v}(r_2) \; , \tag{4.3}$$

where

$$e_{ljl_vj_v}(r_1,r_2)=(2l+1)(2l_v+1)\sum_\lambda (2\lambda+1)\left[\begin{pmatrix} l & l_v & \lambda \\ 0 & 0 & 0 \end{pmatrix}\begin{Bmatrix} \lambda & l & l_v \\ \frac{1}{2} & j_v & j \end{Bmatrix}\right]^2 \tfrac{1}{2}\int_{-1}^{1} d\cos\theta\;\theta(b-r_{12})P_\lambda(\cos\theta) \; . \tag{4.4}$$

If $b\to\infty$ the angular integral reduces to a constant ($\delta_{\lambda 0}$), and the exchange integral vanishes because of the orthogonality of the single particle states. In summary we have the following:

$$P^v_b=\sum_{nlj}(2j+1)[2P^d_{nlj}(b)-P^e_{nlj}(b)] \; , \tag{4.5}$$

where the factor 2 stems from the identical proton and neutron direct contributions. Using this average probability, or the individual probabilities $P_{nlj}(b)$, we can evaluate the probability for the valence nucleon to overlap with any one ($P^v_{Q_1}$), or any two ($P^v_{Q_2}$) core nucleons according to Eqs. (3.14) and (3.15).

In Table I we show results for the nuclei ^{12}C, ^{16}O, ^{28}Si, ^{32}S, and ^{40}Ca, which in the present investigation are considered as ideal magic nuclei. The results were obtained for one particular transition radius ($b=0.95$ fm). However, as P^v_b behaves very nearly like b^3, we can easily obtain the results for other values of b. We have also done calculations of $P^v_{Q_1}$ and $P^v_{Q_2}$ using the simpler Fermi gas model. These latter results are somewhat larger (between 2% and 10%) than the shell-model results, but are otherwise similar. In particular, comparison of both calculations shows that the A dependence of the six-quark probabilities is due to the single particle nature of the valence particle, rather than to the single-particle nature of the core nucleons, as the Fermi gas model, which does not take account of the single particle nature of different core nucleons, leads to the same A dependence as the microscopic calculations. Although not shown in the table, it is worth noting that the Pauli exchange term (4.3) and (4.4) is about 21% of the direct term, and therefore leads to a sizable reduction of the six-quark probability.

V. A HYBRID DESCRIPTION OF THE THREE-NUCLEON SYSTEM

Given a simple three-nucleon wave function we can calculate all six-quark probabilities exactly in the three-body system. We can also exactly include the effects of the core (a proton plus neutron in this case) and thereby assess the consequences of our neglect of core effects in the preceding section. For our calculations we use a simple wave function without short-range correlations, namely the wave function given by Wildermuth and Tang:[38]

$$\phi=C\sum_{i=1}^{3}A_i\exp\left[-\tfrac{1}{2}\alpha_i\sum_{j=1}^{3}(\vec{r}_j-\vec{R})^2\right] \; , \tag{5.1}$$

where $\vec{R}$ is the center of mass vector. Since this is a symmetric S-state wave function, the exchange terms in the quark probabilities vanish. We define the basic matrix element

$$p=\langle\phi\,|\,\theta(b-r_{12})\,|\,\phi\rangle \; , \tag{5.2}$$

so that $P^v_b=2p$. Using P^v_b or p we can evaluate all other probabilities with the approximations suggested in Sec. III. We can also calculate these exactly, using the definitions

$$P^v_{Q_1}=2\langle\phi\,|\,\theta(b-r_{12})\theta(r_{13}-b)\,|\,\phi\rangle \; , \tag{5.3}$$

$$P^v_{Q_2}=\langle\phi\,|\,\theta(b-r_{12})\theta(b-r_{13})\,|\,\phi\rangle \; , \tag{5.4}$$

and

$$P^v_N=\langle\phi\,|\,\theta(r_{12}-b)\theta(r_{13}-b)\,|\,\phi\rangle \; , \tag{5.5}$$

with $P^v_Q=1-P^v_N$. For all these probabilities the state of the core nucleons 2 and 3 is not specified. One easily checks that the total probability $P_N^v+P_{Q_1}^v+P_{Q_2}^v=1$.

In addition we now define exclusive probabilities, for which the state of the core is specified as well:

$$P_N=\langle\phi\,|\,\theta(r_{12}-b)\theta(r_{13}-b)\theta(r_{23}-b)\,|\,\phi\rangle\ , \tag{5.6}$$

$$P_{Q_1}^E=\langle\phi\,|\,\theta(b-r_{12})\theta(r_{13}-b)\theta(r_{23}-b)\,|\,\phi\rangle\ , \tag{5.7}$$

$$P_{Q_2}^E=\langle\phi\,|\,\theta(b-r_{12})\theta(b-r_{13})\theta(r_{23}-b)\,|\,\phi\rangle\ , \tag{5.8}$$

and

$$P_{Q_3}=\langle\phi\,|\,\theta(b-r_{12})\theta(b-r_{13})\theta(b-r_{23})\,|\,\phi\rangle\ . \tag{5.9}$$

The physical meaning of these probabilities is the following: $P_{Q_1}^E$ is the chance for finding a specific pair close but no other pairs close; $P_{Q_2}^E$ is the chance that two specific pairs are close but the third pair is not; and P_{Q_3} is the chance that all three nucleons are close. The completeness of the wave function is now embodied by the identity

$$P_N+3P_{Q_1}^E+3P_{Q_2}^E+P_{Q_3}=1\ .$$

The connection between these inclusive and exclusive probabilities is the following:

$$P_N^v=P_N+P_{Q_1}^E\ ,$$

$$P_{Q_1}^v=2(P_{Q_1}^E+P_{Q_2}^E)\ , \tag{5.10}$$

and

$$P_{Q_2}^v=P_{Q_2}^E+P_{Q_3}\ .$$

Another useful identity is $P_b^v=P_{Q_1}^v+2P_{Q_2}^v$, giving a complete breakup of the valence six-quark probability in one and two pair components.

In Table II we list these quantities for $b=0.95$ fm. Again results for other b values can easily be obtained. In this case $P_{Q_1}^E\sim b^2$ while $P_{Q_2}^E$ and $P_{Q_3}\sim b^5$.

Let us analyze the results in Table II in some detail. First compare the exact results with the classical approximations discussed in Sec. III. We see that the approximation for $P_{Q_2}^v$ is very poor. Clearly there is a very strong center-of-mass correlation in the $A=3$ system. If two particles are close together, then the chance for the third particle to be close to one of them is enhanced by as much as a factor of 2 (since the particles like to be close we could have expected an enhancement, but the factor 2 is somewhat of a surprise). To compensate for this effect in comparing with our classical calculation for heavy nuclei, the chance for two particles to be close should be reduced if we know that the third particle is far away from one of them. We may also expect that these correlations are A dependent, since the effect of one particle on another will be less if there are many other nucleons around. The following formulae give the correct description of the probabilities in the three-body system ($A=2$), and for larger A give roughly the expected A dependence:

$$p_>=\frac{A}{A-1}p\ , \tag{5.11}$$

$$p_<=\frac{1-\dfrac{A}{A-1}p}{1-p}p\ . \tag{5.12}$$

Here $p_<$ has been constructed to satisfy the requirement

$$pp_>+(1-p)p_<=p\ , \tag{5.13}$$

representing the fact that the average probability for finding a close pair should still be p. We could also consider higher order correlations in the three-body system, e.g., we could consider the chance $p_{\ll}$ to find two particles close together if they are both far away from the third one. For $A=2$ this $p_{\ll}$ is then determined by the following:

$$P_N=(1-p)(1-p_<)(1-p_{\ll})\ . \tag{5.14}$$

However, we find that $p_{\ll}$ equals $p_<$ to within 3%, so that we neglect these higher order correlations and set $p_{\ll}=p_<$.

We can now give the general expressions for the correlated probabilities $P_{Q_1}^v$, $P_{Q_2}^v$, and P_Q^v:

$$P_{Q_1}^v=Ap(1-p_>)(1-p)(1-p_<)^{A-3}\ , \tag{5.15}$$

$$P_{Q_2}^v=\frac{A(A-1)}{2}pp_>(1-p_>)^2(1-p)(1-p_<)^{A-5}\ , \tag{5.16}$$

and

$$P_N^v=(1-p)(1-p_<)^{A-1}\ . \tag{5.17}$$

It should be obvious that the products on the right-hand side (RHS) of (5.15) and (5.16) are truncated in the few-body case, e.g., for $A=2$ we find $P_{Q_2}^v=[A(A-1)/2]pp_>$ and $P_{Q_1}^v=Ap(1-p_>)$. Analogous expressions for other probabilities in the three-body system are easily written out as well. From Table II we see that we have succeeded in giving an excellent representation of the exact three-body results using these approximate correlated probabilities.

Results for the many-body systems with these correlated expressions were already shown in Table I (entries in

TABLE II. Probabilities for quark configurations in the three-nucleon system for $b=0.95$ fm. The uncorrelated results are based on the approximation in Sec. III. Correlated results are based on approximations suggested in this section.

	P_b^v	P_N^v	$P_{Q_1}^v$	$P_{Q_2}^v$	P_N	$P_{Q_1}^E$	$P_{Q_2}^E$	P_{Q_3}
Exact	0.179	0.836	0.148	0.0158	0.796	0.0672	0.0068	0.0091
Uncorrelated		0.829	0.163	0.0081	0.754	0.0743	0.0073	0.0007
Correlated		0.837	0.147	0.0161	0.769	0.0670	0.0068	0.0093

parentheses). The effect of the correlations on $P^{v}_{Q_1}$ is negligible and the enhancement of $P^{v}_{Q_2}$ is quite minor, amounting to only 8.7%, 6.5%, 3.7%, 3.2%, and 2.5% for the successive nuclei. Obviously, correlations play a much smaller role in the many-body system than in the three-body system, in accordance with the independent particle model of nuclei. Of course, we should keep in mind that this result was obtained under the natural, but *ad hoc*, many-body generalization of the three-body results. Notice also that the expressions for the correlated probabilities can only be valid in a limited range, since they will exceed unity if p becomes large (e.g., if $b \to \infty$).

VI. MASS DIFFERENCE IN THE THREE-BODY SYSTEM

According to the preceding discussion the three-baryon wave function can be decomposed in a conventional component P_N and the multiquark bag components $P^{E}_{Q_1}$, $P^{E}_{Q_2}$, and P_{Q_3}. In Sec. II we discussed the evaluation of mass differences for six-quark bags, i.e., for those parts of the wave function represented by $P^{E}_{Q_1}$. Since the components $P^{E}_{Q_2}$ and P_{Q_3} are not negligible, we now have to discuss how masses for these components are to be evaluated. If all three particles are close, as they are in P_{Q_3}, it is natural to assume that the three six-quark bags have merged into a nine-quark bag. But even in the case that only two of the three pairs are close (i.e., have merged into a six-quark bag), it seems natural to assume that the system is best described by a nine-quark bag. If quarks can move freely between bag one and two, and between two and three, then they can also move freely between bag one and bag three. This system of nine freely moving quarks is best represented by a nine-quark bag. Simple geometrical considerations also favor a nine-quark bag description of the $P^{E}_{Q_2}$ components; however, we do not want to elaborate on these arguments.

Our next problem is then how to describe mass differences between nine-quark bags. For six-quark bags the mass differences could not simply be expressed in terms of the masses of the "originating" nucleons. Likewise, we do not expect that nine-quark bag mass differences can be expressed in terms of the underlying six-quark bag mass differences. It is more natural to extend our model for the three- and six-quark bag to the general $(3n)$-quark bag by means of the following general equation (we omit terms which do not contribute to the mass differences):

$$E = 1.44 \sum_{i<j}^{3n} \frac{Q_i Q_j}{R_{3n}} + 0.42 \sum_{i=1}^{3n} \frac{c_i}{R_{3n}} , \tag{6.1}$$

where we wrote $m_i = c_i / R$ $(c_d - c_u = 4$ MeV fm). This leads to the following mass differences for $R_3 = 1$ fm, $R_6 = 1.2$ fm, and $R_9 = 1.35$ fm:

$$m_{\rm pp} - m_{\rm nn} = -0.94 \text{ MeV}; \quad m_{\rm pp} - m_{\rm pn} = 0.13 \text{ MeV} ;$$
$$m_{\rm ppp} - m_{\rm ppn} = 1.18 \text{ MeV}; \quad m_{\rm ppp} - m_{\rm pnn} = 1.30 \text{ MeV} ; \tag{6.2}$$
$$m_{\rm ppp} - m_{\rm nnn} = 0.35 \text{ MeV} .$$

All other mass differences follow trivially. One easily verifies that the nine-quark mass differences cannot easily be represented in terms of nucleon or six-quark mass differences.

We can now write the masses of the three-baryon nuclei. For ^{3}He we obtain the following:

$$M_{^3\rm He} = P_N(2m_{\rm p} + m_{\rm n}) + P^{E}_{Q_1}(2m_{\rm p} + 2m_{\rm np} + m_{\rm n} + m_{\rm pp}) + (3P^{E}_{Q_2} + P_{Q_3}) m_{\rm ppn} + V'_C , \tag{6.3}$$

where V'_C is the conventional Coulomb energy reduced by the exclusion of the short-distance contribution. In evaluating V'_C we assume that the Coulomb potential between two protons will not change if one of the protons forms a six-quark bag with the neutron. The Coulomb energy of two protons, when they are closer than b, should naturally be excluded, because it is already included in the six-quark bag mass. The components contributing to the conventional Coulomb energy are therefore P_N, $2 \times P^{E}_{Q_1}$, and perhaps $1 \times P^{E}_{Q_2}$. The sum of these components exactly represents that part of the three-body wave function for which the two protons are further apart than b. In calculating quark probabilities we assumed that we could represent the wave function for $r < b$ by the conventional one. If we now make the same assumption for the proton-neutron wave function when we do the Coulomb potential integration, then this integration can be trivially performed. This assumption is not unreasonable, since the neutron only plays a spectator role as far as the Coulomb integration is concerned.

Although convenient, this Coulomb energy determination suffers from one problem. The $P^{E}_{Q_2}$ component, with two protons not close, contributes both to the conventional Coulomb energy and to the nine-quark bag Coulomb energy, since we decided to treat the $P^{E}_{Q_2}$ components as nine-quark bag states. To correct for this one can subtract the Coulomb energy of four u and two d quarks in the nine-quark bag from the overall mass, or one can suppress the contribution to the conventional Coulomb energy which corresponds to two "distant" protons, both of which are close to the neutron. Since the product sum $\sum_{i<j} Q_i Q_j$ is unity, independent of whether the charges Q_i are proton or constituent quark charges, the numerical value of this correction is not particularly sensitive to the procedure chosen. We have calculated the correction using the former procedure, and included it in the Coulomb energy V'_C.

For the triton we obtain the following expression:

$$M_{^3\rm H} = P_N(2m_{\rm n} + m_{\rm p}) + P^{E}_{Q_1}(2m_{\rm n} + 2m_{\rm np} + m_{\rm p} + m_{\rm nn}) + (3P^{E}_{Q_1} + P_{Q_3}) m_{\rm pnn} . \tag{6.4}$$

By subtracting (6.3) and (6.4) we obtain the following:

$$m_{^3\rm He} - m_{^3\rm H} - (m_{\rm p} - m_{\rm n}) - V_C = \Delta_Q + \Delta_C , \tag{6.5}$$

where

$$\Delta_Q = P^{E}_{Q_1}(m_{\rm pp} - m_{\rm nn} + 2m_{\rm n} - 2m_{\rm p}) + (3P^{E}_{Q_2} + P_{Q_3})(m_{\rm ppn} - m_{\rm pnn} - m_{\rm p} + m_{\rm n}) \tag{6.6}$$

and

$$\Delta_C = V'_C - V_C \ . \tag{6.7}$$

The mass shift Δ_Q can now be calculated from the mass differences in Eq. (6.2), and the probabilities $P^E_{Q_1}$, $P^E_{Q_2}$, and P_{Q_3}, some of which were already listed in Table II. The mass shift Δ_C reflects the reduction of the Coulomb energy, due to the suppression of the interior, plus the small correction term to prevent double counting. If we use the three-body wave function (5.1), and account for the finite size of the proton using the Auerbach *et al.* form factor,[39] then we obtain a Coulomb energy of $V_C = 749$ keV. This value is substantially larger than the value obtained with conventional three-body wave functions, and would be larger yet, if we had considered the protons as pointlike particles. This confronts us with a certain problem in the presentation of our results: We would like to single out the effect of the multiquark bags in our description, however, our "conventional" results (i.e., those for $b=0$) are already substantially different from the usual ones, as they are based on a three-body wave function tailored towards our model of the short-range behavior. Therefore, we rather compare our calculation of V'_C to the best conventional value of V_C (including in this latter value other small conventional isospin breaking corrections). We have taken the value $V_C = 683 \pm 29$ keV from Ref. 40. Since V_C appears both on the left-hand and the right-hand sides of Eq. (6.6), its value is immaterial in determining the quality of our prediction, and only plays a role in the interpretation of our results for Δ_C. This choice for V_C implies that the left-hand side of Eq. (6.5) represents the current discrepancy between experiment and (conventional) theory; i.e., $764-683$ keV$=81$ keV. Our model, therefore, could claim a success if the sum of $\Delta_Q + \Delta_C$ lies in the neighborhood of 80 keV.

In Table III we show the results for the quark probabilities, the corresponding values for Δ_Q, and the Coulomb shifts Δ_C. Our three-body wave function does not have any short-range suppression, leading to the fairly large quark content of the wave function ($P_Q = 18\%$ for $b=0.85$ fm and 23% for $b=0.95$ fm). In order to have some results with smaller percentage quark states, and also to acquire some insight in the model dependence of our calculations, we have also performed calculations with a correlated three-body wave function. We follow the procedure of Hadjimichael *et al.*,[41] rescaling the short-range N-N wave function to the Reid soft core wave function with a correlation length of 1 fm. Obviously, we have to renormalize the three-body wave function if we introduce correlations; the results are given under the heading type 2 in Table III. We can also put the wave function defect, arising from the introduction of correlations, directly into the total six-quark probability. In this case our theory no longer dictates the division of the total quark probability over $P^E_{Q_1}$, $P^E_{Q_2}$, and P_{Q_3}; but lacking an alternative we still follow the formulae presented in Sec. V (type 3).

We see that in all cases considered $\Delta_Q + \Delta_C$ is positive and removes part of the original discrepancy of 81 keV. In our main calculation (type 1, $b=0.95$ fm) we obtain virtual agreement with experiment (the difference of 10 keV is not significant considering the uncertainties in the calculation). Notice that our results are not particularly sensitive to b, due to the strong cancellations between Δ_Q and Δ_C.

VII. MASS DIFFERENCES IN THE MANY-BODY CASE

In the many-body case we express the mass differences in terms of the valence probabilities P^v_N, $P^v_{Q_1}$, and $P^v_{Q_2}$, which were defined previously in Sec. III. The completeness of the wave function is now given by

$$P^v_N + P^v_{Q_1} + P^v_{Q_2} + \cdots = 1 \ , \tag{7.1}$$

but since $P^v_{Q_2}$ is already quite small, we have neglected higher order terms in (7.1) and use the completeness in approximate form. The components $P^v_{Q_1}$ and $P^v_{Q_2}$ can be further broken up into components for which the isospin nature of the core nucleon(s), participating in the multiquark bags, is specified. Assuming that the core proton and core neutron probabilities are identical—a reasonable assumption considering that we deal with $N=Z$ cores—we can write the mass of the core plus one additional nucleon as follows (we omit core contributions which are irrelevant for the mass difference):

TABLE III. Multiquark bag probabilities and their contribution (in keV) to the ^{3}He and ^{3}H mass difference (Δ_Q) and the reduction of the conventional Coulomb energy (Δ_C). Uncorrelated (type 1), renormalized correlated (type 2), and correlated calculations with high quark content (type 3) are shown. Agreement with experiment is obtained if the sum $\Delta_Q + \Delta_C$ equals 81 keV, cancelling the conventional discrepancy (Ref. 40).

Type	b (fm)	$P^E_{Q_1}$	$P^E_{Q_2}$	P_{Q_3}	Δ_Q	Δ_C	$\Delta_Q+\Delta_C$
1	0.85	0.0537	0.0041	0.0051	112	−44	68
2	0.85	0.0272	0.0008	0.0010	49	−12	36
3	0.85	0.0570	0.0045	0.0057	120	−94	26
1	0.95	0.0672	0.0068	0.0091	150	−78	73
2	0.95	0.0433	0.0022	0.0030	84	−42	42
3	0.95	0.0687	0.0073	0.0100	156	−122	34

$$m_{N,Z+1}=P_N^v m_p+\tfrac{1}{2}P_{Q_1}^v(m_{pp}-m_p)+\tfrac{1}{2}P_{Q_1}^v(m_{pn}-m_n)+\tfrac{1}{2}P_{Q_2}^v(m_{ppn}-m_p-m_n)+\tfrac{1}{4}P_{Q_2}^v(m_{ppp}-2m_p)+\tfrac{1}{4}P_{Q_2}^v(m_{pnn}-2m_n)+V_C' \quad (7.2)$$

and

$$m_{N+1,Z}=P_N^v m_n+\tfrac{1}{2}P_{Q_1}^v(m_{nn}-m_n)+\tfrac{1}{2}P_{Q_1}^v(m_{pn}-m_p)+\tfrac{1}{2}P_{Q_2}^v(m_{nnp}-m_p-m_n)+\tfrac{1}{4}P_{Q_2}^v(m_{nnn}-2m_n)+\tfrac{1}{4}P_{Q_2}^v(m_{npp}-2m_p)\ . \quad (7.3)$$

Note that we recover the conventional result if we replace the multiquark bag masses by their conventional values (e.g., $m_{pp}=2m_p$, etc.).

After subtracting these masses we obtain the following:

$$m_{N,Z+1}-m_{N+1,Z}-(m_p-m_n)-V_C=\Delta_Q+\Delta_C\ , \quad (7.4)$$

where

$$\Delta_Q=\tfrac{1}{2}P_{Q_1}^v(m_{pp}-2m_p-m_{nn}+2m_n)+\tfrac{1}{4}P_{Q_2}^v(m_{ppn}-m_{nnp}+m_{ppp}-m_{nnn}-4m_p+4m_n) \quad (7.5)$$

and

$$\Delta_C=V_C'-V_C\ . \quad (7.6)$$

To help the interpretation of our results, we will use values for V_C listed by Nolen and Schiffer[26] [these include various corrections and are called Δ(calc) in Ref. 26]. Some of the more recent conventional results for the Coulomb displacement energies are given in the discussion. With this choice for V_C, the left-hand side of (7.4) represents the original Nolen-Schiffer anomaly.

In Table IV we show our results for a transition radius of 0.85 and 0.95 fm. We include the Coulomb energies V_C for reference. The finite size of the protons in the calculation of V_C' is again implemented using the Auerbach *et al.* form factor.[39] In order to get agreement with experiment, $\Delta_Q+\Delta_C$ should equal the right-hand side of Eq. (7.4), which in Table IV is denoted by Δ. A positive $\Delta_Q+\Delta_C$ represents a reduction of the Nolen-Schiffer anomaly, whereas a negative value represents an increase thereof. For light nuclei there is an improvement, in particular in ^{12}C, where the discrepancy is cut in half. For the two larger nuclei the anomaly has increased. In every case the agreement with experiment improves if b increases; however, the sensitivity to b is not sufficient to get full agreement at some values of b, as the cancellation between Δ_Q and Δ_C is too large. These results will further be discussed in the next section.

VIII. SUMMARY AND DISCUSSION

In this paper we have developed a hybrid description of nuclei in terms of quarks and nucleons. The most detailed description was given of the three-body system, and some of the insights obtained for this system were used to improve the treatment of many-body systems. This new description of nuclei was applied to the calculation of mass differences between ^{3}He and ^{3}H, and between mirror nuclei, motivated by persistent problems in reproducing the experimental Coulomb displacement energies. Unfortunately, the assumptions made in Sec. II concerning the dynamical description of mass differences of bags are not (yet) on solid ground, and clearly require further QCD studies of few-nucleon systems. However, they should provide a reasonable indication of the charge-symmetry violations to be expected in the quark bag model.

Originally we had expected that the mass differences between protons and neutrons would reduce in the multiquark bag environment, thereby reducing the Nolen-Schiffer anomalies. Although this effect is certainly present ($\Delta_Q>0$ in all cases) it is cancelled to a large extent by the reduction of the (conventional) Coulomb energy, represented by the quantity Δ_C. Therefore, the sensitivity to the value of the transition radius in b is not as large as expected. Generally, our results improve if b is increased, however, this improvement is so slow that we cannot use the experimental mass difference to fix the value of b. A remarkable consequence of this result is that, as far as the mass differences are concerned, the conventional and quark-nucleon description are largely compatible. If this result survives further improvements of the model, and if it also has validity for other nuclear properties, then it would explain why conventional nuclear physics could have been so successful despite the presence of large quark components in the wave function.

TABLE IV. The shifts Δ_Q and Δ_C in the mass differences due to the presence of multiquark bags. The conventional discrepancy between experiment and theory (the Nolen-Schiffer anomaly) is represented by Δ; the discrepancy in the present theory is $\Delta-(\Delta_Q+\Delta_C)$. All energies are in MeV.

	b (fm)	^{12}C	^{16}O	^{28}Si	^{32}S	^{40}Ca
V_C		2.79	3.23	5.53	6.11	6.66
Δ_Q	0.85	0.106	0.076	0.118	0.114	0.095
Δ_C	0.85	−0.029	−0.060	−0.091	−0.282	−0.212
$\Delta_Q+\Delta_C$	0.85	0.077	0.016	0.027	−0.168	−0.117
Δ_Q	0.95	0.145	0.108	0.163	0.162	0.128
Δ_C	0.95	−0.055	−0.080	−0.128	−0.313	−0.235
$\Delta_Q+\Delta_C$	0.95	0.090	0.028	0.035	−0.151	−0.107
Δ		0.210	0.310	0.200	0.240	0.620

Our results are most encouraging for the light nuclei. For the three-nucleon system the anomaly is reduced from 80 to 10 keV, for ^{12}C from 210 to 125 keV. For ^{16}O and ^{28}Si there is still a small improvement; however, for the larger nuclei (^{32}S and ^{40}Ca) our simple description of the charge-symmetry breaking effects breaks down. In these latter cases we clearly need a more detailed description of the conventional wave function, and a better study of various other contributions to the mass difference. In Ref. 42 such a study was attempted for ^{40}Ca, and in one variant of their calculations the discrepancy was less than 150 eV.

A disturbing consequence of our present results is that the strong cancellation between Δ_Q and Δ_C, and the modest model dependence indicated in Table III, precludes the acquisition of accurate predictions. It also means that even the hybrid results will remain sensitive to the detailed treatment of the exterior wave function and to small corrections, a situation which became most obvious in the ^{40}Ca case. The uncertainty of these corrections (again there seem to be large cancellations between different effects[42]) adds to the uncertainty in the predictions. It therefore appears necessary to test such hybrid descriptions also in the context of other processes, a task which is presently actively pursued[19–21,36,37] using similar models. Ultimately we may be able to put sufficient constraints on the models to exclude some of them, although the present study also warns us that very different descriptions can lead to similar results, thereby precluding discrimination through experiment alone.

Finally we mention one spin-off of the present study which seems particularly relevant at this time. The announcement last year by the European muon collaboration[43] that the structure function of Fe was not simply 56 times that of an isolated, isoscalar nucleon, has led to a great deal of interest in the topic of quarks in nuclei.[25,44] Although there seems to be general agreement that this measurement indicates a change of scale for quarks in a many-body system, it is not yet clear whether one needs to invoke explicit multiquark configurations or not.[45] However, with respect to the recent calculations of Jaffe *et al.*,[46] it is interesting to note that they estimated the scale change by calculating the overlap of N-N pairs in finite nuclei, but neglected the possibility of multinucleon overlap. While not a rigorous proof of their approximation, our discussion in Secs. IV and V [especially near Eqs. (5.11)–(5.17)] indicates that it is probably a good approximation for nuclei heavier than He. Further studies along the lines introduced here seem worthwhile.

We would like to acknowledge financial support by the Natural Sciences and Engineering Research Council of Canada and the University Research Fund of the University of Alberta.

---

*Permanent address: Theoretical Physics Division, CSIR, P.O. Box 395, Pretoria 0001, South Africa.

†Permanent address: Physics Department, University of Adelaide, Adelaide, South Australia.

[1]D. Robson, Nucl. Phys. **A308**, 381 (1978).

[2]G. A. Miller, S. Théberge, and A. W. Thomas, Comments Nucl. Part. Phys. **10**, 101 (1981).

[3]G. E. Brown, in *Progress in Particle and Nuclear Physics*, edited by D. Wilkinson (Pergamon, Oxford, 1982), Vol. 8, p. 147.

[4]A. W. Thomas, Adv. Nucl. Phys. **13**, 1 (1983).

[5]M. Oka and Y. Yazaki, Phys. Lett. **90B**, 41 (1980); Nucl. Phys. **A402**, 477 (1983); S. Ohta *et al.*, Phys. Lett. **119B**, 35 (1982); M. H. Storm and A. Watt, Nucl. Phys. **A408**, 397 (1983); M. Cvetič *et al.*, *ibid.* **A395**, 349 (1983).

[6]M. Harvey, in *Proceedings of the International Symposium on Clustering Phenomena in Nuclei, Tübingen, 1981*, edited by P. Kramer and R. Schultheis (Attempo, Tübingen, 1981), p. 179; Nucl. Phys. **A352**, 301 (1981); **A352**, 326 (1981).

[7]A. Faessler, F. Fernandez, G. Lübeck, and K. Shimizu, Phys. Lett. **112B**, 201 (1982); A. Faessler and F. Fernandez, *ibid.* **124B**, 145 (1983).

[8]K. Maltman and N. Isgur, Phys. Rev. Lett. **50**, 1827 (1983).

[9]C. De Tar, Phys. Rev. D **17**, 323 (1978); **19**, 1028 (1979); **19**, 1451 (1979).

[10]R. L. Jaffe and F. Low, Phys. Rev. D **19**, 2105 (1979).

[11]Yu. A. Simonov, Phys. Lett. **107B**, 1 (1981).

[12]P. J. Mulders, Phys. Rev. D **25**, 3039 (1982); MIT Report CTP-1076, 1983.

[13]E. Lomon, in *The Interaction Between Medium Energy Nucleons in Nuclei—1982 (Bloomington)*, Proceedings of the Workshop on the Interaction Between Medium Energy Nucleons in Nuclei, Bloomington, 1982, AIP Conf. Proc. No. 97, edited by H. O. Meyer (AIP, New York, 1983), p. 78.

[14]R. L. Jaffe and M. P. Shatz, California Institute of Technology Report CALT-68-775, 1980.

[15]R. P. Bickerstaff, Phys. Rev. D **27**, 1178 (1983).

[16]B. L. G. Bakker and M. van der Velde, in *Proceedings of the Xth International Conference on Few-Body Problems, Karlsruhe, 1983*, edited by B. Zeitnitz (North-Holland, Amsterdam, 1984), p. 613.

[17]R. Vinh Mau, Nucl. Phys. **A374**, 3 (1982).

[18]G. A. Miller, in Proceedings of the IUCF Workshop, 1982, CERN Report CERN TH-3516, 1983.

[19]L. S. Kisslinger, Phys. Lett. **112B**, 307 (1982).

[20]H. J. Pirner and J. P. Vary, Phys. Rev. Lett. **46**, 1376 (1981); J. P. Vary, Proceedings of the Quark Matter 1983 Workshop, Brookhaven National Laboratory (unpublished).

[21]L. S. Kisslinger and G. A. Miller, Phys. Rev. C **27**, 1602 (1983).

[22]V. M. Dubovik and I. T. Obukhovsky, Z. Phys. C **10**, 123 (1981).

[23]R. Jaffe, Phys. Rev. Lett. **50**, 228 (1983).

[24]H. J. Pirner and J. P. Vary, Universität Heidelberg Report UNI-HD-83-02, 1983.

[25]A. W. Thomas, in *Progress in Particle and Nuclear Physics*, edited by D. Wilkinson (Pergamon, Oxford, in press).

[26]J. A. Nolen, Jr. and J. P. Schiffer, Annu. Rev. Nucl. Sci. **19**, 471 (1969).

[27]J. W. Negele, in *Proceedings of the International Conference on Nuclear Structure and Spectroscopy, Amsterdam, The Netherlands*, edited by H. E. Blok and A. E. L. Dieperink (Scholars, Amsterdam, 1974), Vol. 2, p. 618.

[28]S. Shlomo, Rep. Prog. Phys. **41**, 937 (1978).

[29]J. Gasser and H. Leutwyler, Phys. Rep. **87**, 77 (1982).

[30]N. G. Deshpande, D. A. Dicus, K. Johnson, and V. L. Teplitz, Phys. Rev. D **15**, 1885 (1977).

[31]C. De Tar and J. F. Donoghue, submitted to Annu. Rev. Nucl. Part. Sci.
[32]R. P. Bickerstaff and A. W. Thomas, Phys. Rev. D **25**, 1869 (1982).
[33]K. F. Liu and C. W. Wong, Phys. Rev. D **17**, 920 (1978).
[34]J. D. Breit, Columbia University Report CU-TP-229, 1982.
[35]J. Baacke, Y. Igarashi, G. Kasperidus, and H. Usler, CERN Report TH-3509, 1983 and erratum.
[36]E. M. Henley, L. S. Kisslinger, and G. A. Miller, Phys. Rev. C **28**, 1277 (1983).
[37]G. A. Miller, in *Proceedings of Manifestations of Hadron Substructure in Nuclear Physics, Bloomington, 1983,* AIP Conf. Proc. No. 110 (AIP, New York, 1984), p. 38.
[38]K. Wildermuth and Y. C. Tang, *A Unified Theory of the Nucleus* (Academic, New York, 1977), p. 76.
[39]N. Auerbach, J. Hüfner, A. K. Kerman, and C. M. Shakin, Rev. Mod. Phys. **44**, 48 (1972).
[40]R. A. Brandenburg, S. A. Coon, and P. U. Sauer, Nucl. Phys. **A294**, 305 (1978); H. Baier, W. Bentz, Ch. Hajduk, and P. U. Sauer, *ibid.* **A386**, 460 (1982).
[41]E. Hadjimichael, E. Harms, and V. Newton, Phys. Lett. **40B**, 61 (1972); E. Hadjimichael, S. N. Yang, and G. E. Brown, *ibid.* **39B**, 594 (1972).
[42]N. Auerbach, V. Bernard, and Nguyen Van Giai, Phys. Rev. C **21**, 744 (1980).
[43]J. J. Aubert *et al.*, Phys. Lett. **123B**, 275 (1983).
[44]R. L. Jaffe, Phys. Rev. Lett. **50**, 228 (1983).
[45]C. H. Llewellyn-Smith, Phys. Lett. **128B**, 107 (1983); M. Ericson and A. W. Thomas, *ibid.* **128B**, 112 (1983).
[46]R. J. Jaffe, F. E. Close, R. G. Roberts, and G. G. Ross, RAL Report RL-83-106, 1983.