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Study of the Neutron-Proton Interaction

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Systematics of n-p interaction energies I_{np}, obtained from a linear combination of nuclear binding energies, are discussed. The calculated I_{np} values show evidence of shell effects. A closed expression, connecting n-p interaction energies with nuclear size and spin, is given for even-even nuclei.

1. INTRODUCTION

The presence of residual forces follows in part from the pairing energy. A study of the residual n-p interaction is of great importance. Considerable success has been achieved in the description of nuclear properties by using a pairing and P_2 force as a residual interaction between identical nucleons.[1,2] The neglect of the n-p interaction is a major weakness of the pairing-force theory. However, the residual n-p pairing force is comparable with that between two identical nucleons. The basic difficulties in trying to extend the pairing-force theory to include both neutrons and protons have been clearly discussed by Lane.[3] Several authors[4,5] have considered the problem in light nuclei. It is also important to solve this problem in heavier nuclei. Mitra and Pal[6] and Silverberg[7] studied heavy nuclei. They treated neutrons and protons separately with pairing-force theory and then took into account the n-p interaction as a perturbation. One may question this method because of the fact that the interaction of like nuclei cannot be treated by standard perturbation theory. The work of Goswami and collaborators[8-11] must also be mentioned in this connection. Hamamoto[12] studied the effect of a short-range n-p interaction on the pairing model. Some investigations[13,14] in which a more realistic residual interaction is used, have been carried out.

There are different methods to obtain n-p interaction parameters. One method is to use the energy-level differences between two levels in the same configurations. It requires at least two levels for the same nucleus and thus greatly reduces the number of data available for the analysis. One can also determine the n-p interaction parameter I_{np} using the "center of gravity" of all levels of a configuration for determining the zeroth Slater integral. Again, in many cases, too few data are available to perform this kind of analysis. Another approach is to estimate I_{np} from the analysis of nuclear binding energies. We have computed the n-p interaction parameters from a certain linear combination of nuclear binding energies. I_{np}, thus obtained, will represent the matrix elements of the effective two-body n-p interaction in the nucleus. We have used the 1964 mass table.[15] The observed systematics of I_{np} are explained on the basis of the shell model for nuclei. We also discuss magic- and submagic-number effects on n-p interaction parameters. A closed expression for the I_{np} values in even-even nuclei is given which correlates them with nuclear size and the resultant angular momentum J. The value of J is obtained on the basis of a simple coupling scheme.

2. SYSTEMATICS OF n-p INTERACTION ENERGIES

In this section we give a graph (Fig. 1) of n-p interaction energies I_{np} for the most stable even-odd-mass nuclei. The general trends and features of I_{np} are discussed. Evidence of magic- and submagic-number effects are also discussed in the light of Figs. 2 and 3.

Towards the derivation of the n-p interaction energy relation, two imaginary processes are carried out. In the first process a neutron and a proton are simultaneously struck off a nucleus (N, Z). In the second process a neutron (proton) is first struck off the nucleus (N, Z), and then a proton (neutron) is struck off the same nucleus (N, Z). The difference between the energies needed in the second and in the first process will give the n-p interaction energy between the last neutron and the last proton of the nucleus (N, Z). Thus,

$$I_{np}(N, Z) = E(N, Z) + E(N-1, Z-1) - E(N-1, Z) - E(N, Z-1), \tag{1}$$

where $E(N, Z)$ denotes the binding energy of the nucleus (N, Z).

The relation (1) gives the correct n-p interaction energy inside the deuteron and is equal to its binding energy. Another check on the relation (1) is that one gets the correct neutron (proton) pairing energies from it when one replaces the proton (neutron) by a neutron (proton) in the imaginary processes considered in the derivation of the relation for $I_{np}(N, Z)$. Hebach and Kümmel[16] studied the n-p interaction effect on nuclear binding energies. However, the expression according to which they have calculated I_{np} for the nucleus (N, Z), in actuality, gives I_{np} for the nucleus $(N+1, Z+1)$.

Using relation (1), we have calculated the n-p interaction energies for all strongly bound odd- and even-mass nuclei with the help of the 1964 mass table.[15] Figure 1 shows the plot of I_{np} values against mass number A. Points for light nuclei ($A<30$) are not shown in the figure, because of the large fluctuations in the values. Points having the same value of Z are connected by separate lines for even- and odd-mass numbers Numbers in the figure are proton numbers. Kravtsov[17] and Gupta[18] also discussed some properties of I_{np}, using older data. The 1964 mass table reveals a great improvement of the experimental binding energies. Plots of local values of n-p interaction energies defined in different ways have also been given and discussed by Ghosal and Saxena,[19,20] Ghose and Sen,[21] Nilsson and Prior,[22] and Nemirovsky.[23]

A close study of Fig. 1 reveals the following features of the n-p interactions:

For all even-A nuclei, with a few exceptions, the n-p interaction energies are larger than those for odd-A nuclei. For nuclei with $N=Z$, I_{np} is comparatively large. There are cases where it is even zero or negative. The difference in I_{np} values for even- and odd-mass nuclei decreases with mass number. From the study of I_{np} values for isotopic series, it follows that the effect of increase of neutrons on n-p interaction energies does not follow a general nature as pointed out by Gupta.[18]

The irregular character of n-p interaction energies cannot be explained on the basis of the liquid-drop model for nuclei. The trends are attributed to the detailed behavior of the interaction of the extracore nucleons.

If we consider the energy relation [Eq. (1)][24]

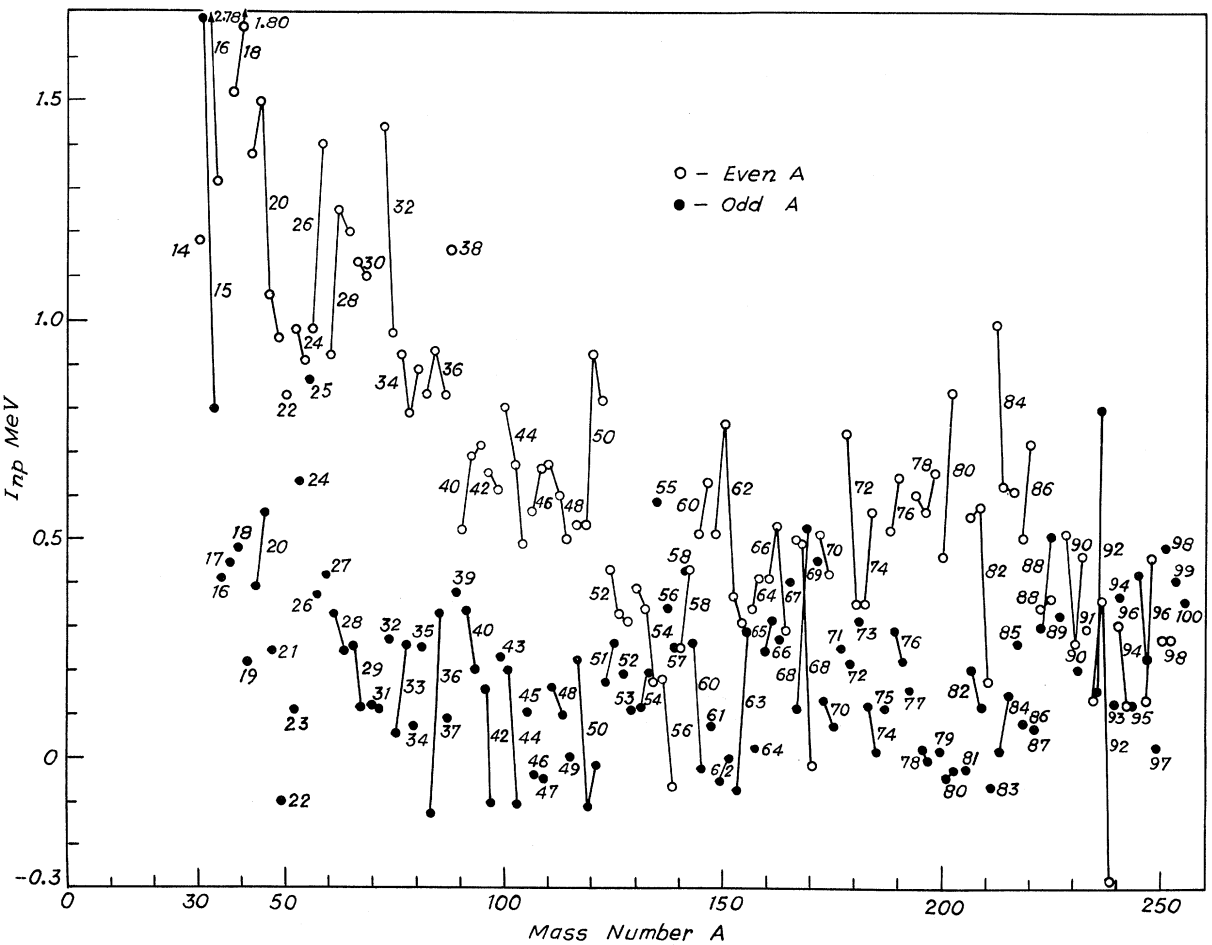

FIG. 1. Plot of I_{np} vs mass number A. Numbers in the figure indicate proton numbers.

based on the single-particle approximation, we find

$$I_{np}(\text{even } A) = I_0 + I' , \tag{2a}$$

$$I_{np}(\text{odd } A) = I_0 - I' , \tag{2b}$$

where I_0 signifies the orientation-independent part of the average interaction of a neutron-proton pair, and I' is the so-called neutron-proton pairing energies in the unfilled shells. In the general single-particle approximation, I_0 and I' are both positive.[25] This clearly explains the difference of I_{np} values for even- and odd-mass nuclei. I' depends on J, the resultant angular momentum. The systematics of I_0 and I' have been discussed by Zeldes, Gronau, and Lev.[24] In fact, I' can be negative (probably as a result of the configuration interaction), and sometimes it becomes comparable to I_0 in heavy nuclei. Thus, the negative values of I_{np} for some even-even nuclei are not unexpected.

Figures 2 and 3 show plots of I_{np} values against mass number A for particular values of $N-Z$ for even-even and odd-odd nuclei. Plots clearly indicate the magic- and submagic-number effect on I_{np} values. From the curves it is clear that for a particular value of $N-Z$, the behavior of I_{np} values is uniform except when the last nucleon closes or crosses a major or minor shell, in which case I_{np} suffers a depression from its uniform behavior. Besides shell or subshell closure, an abnormal change in j values of a neutron (proton) state also causes a depression in the I_{np} values (${}_{26}\mathrm{Cr}^{50}_{24}$ in Fig. 2, ${}_{43}\mathrm{Br}^{78}_{35}$ in Fig. 3). This is expected from the jj-coupling shell model for nuclei. Plots for higher N-Z have not been shown, since the magic and submagic effects become obscured by an over-all fall in the I_{np} values for large A.

3. DEPENDENCE OF n-p INTERACTION ENERGIES ON NUCLEAR SIZE AND SPIN

To correlate n-p interaction energies with nuclear size and spin for a given configuration, we have considered only strongly bound even-even nuclei. One expects, from the liquid-drop model, an A^{-1} dependence of the second-order binding-energy differences which determine n-p interaction energies inside nuclei. Wigner's supermultiplet theory,[26] which explains the odd-even effect, also requires an A^{-1} dependence of I_{np}.

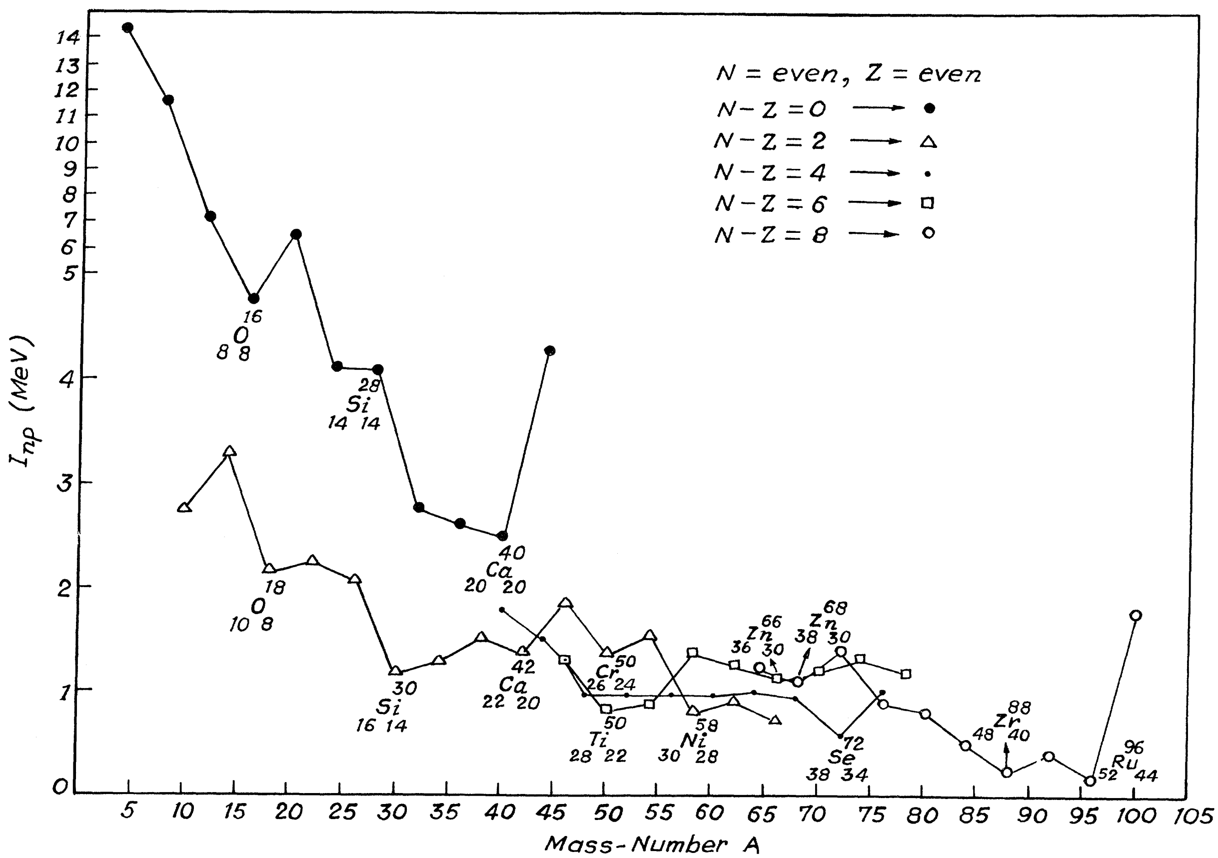

FIG. 2. Plot of I_{np} vs mass number A (even even). Points with same N-Z are connected by lines.

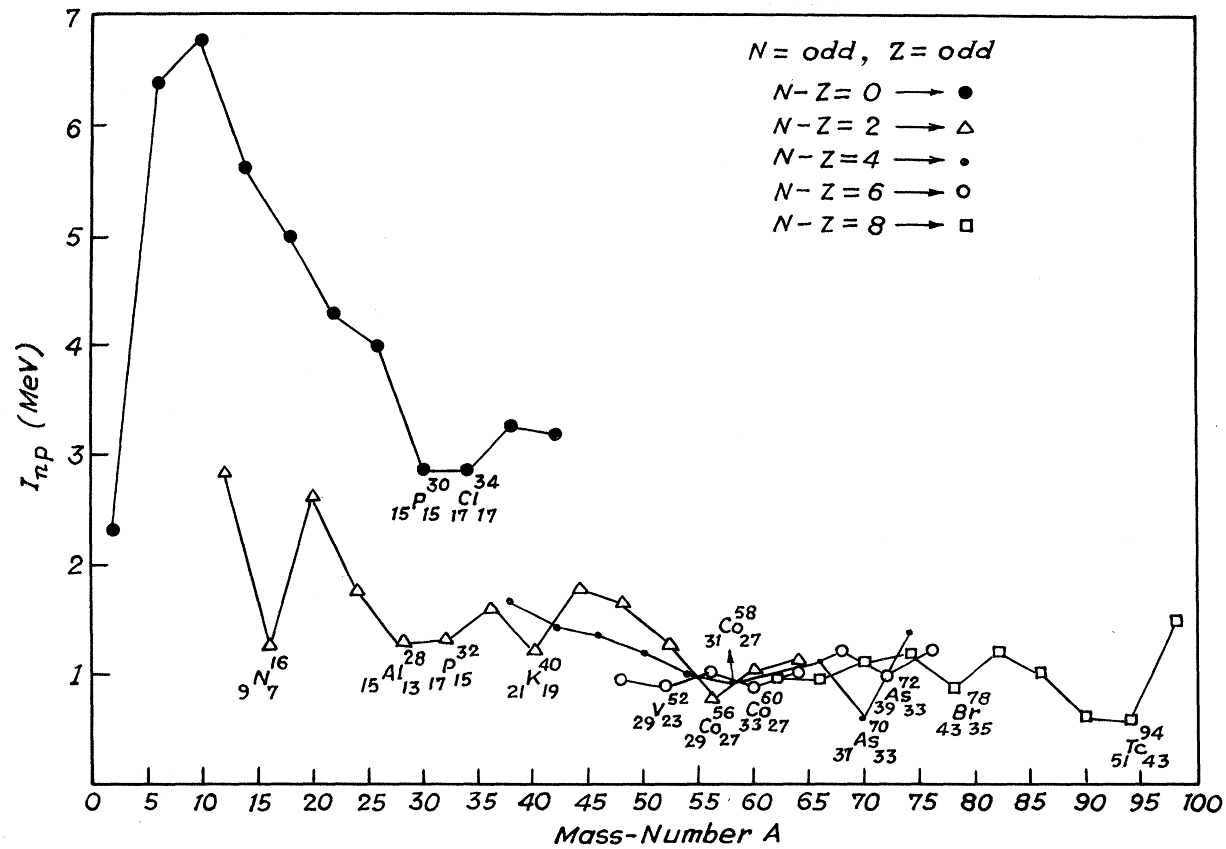

FIG. 3. Plot of I_{np} vs mass number A (odd odd). Points with same N-Z are connected by lines.

To obtain the dependence of I_{np} on mass number A, we have calculated the products $I_{np}A$ and $I_{np}A^{1/2}$. A smaller spread has been found in the product $I_{np}A^{1/2}$. Parameters in both the cases are determined by separate least-squares fits of the formulas to the experimental values. The mean deviation is found less in the second case. It gives an $A^{-1/2}$ dependence of n-p interaction energies.

Since I_{np} is a linear combination of nuclear binding-energy differences, it determines the effective n-p interaction energy of the last neutron and proton. Thus

$$I_{np}=\langle j_n j_p, J|V_{np}|j_n j_p, J\rangle\,, \tag{3}$$

where V_{np} is the effective interaction between the neutron and proton, j_n and j_p are the spin states of the last neutron and proton orbit, and j_n and j_p couple together to give J, the resultant spin.

In the shell-model calculations the radial wave functions for the single-particle states are usually taken as harmonic-oscillator wave functions with parameters adjusted to fit the actual nucleus. The n-p force enters in the form of a Slater integral. The two-body interaction matrix elements of different nuclei with different sizes can be compared by relating the harmonic-oscillator constant to the nuclear radius. This can be done in various ways.[27] If we make the assumption that the expectation value of the potential energy of a particle is $\frac{1}{2}$ the oscillator energy, we get an A^{-1} dependence of I_{np}. However, another connection between the oscillator frequency, the depth of the

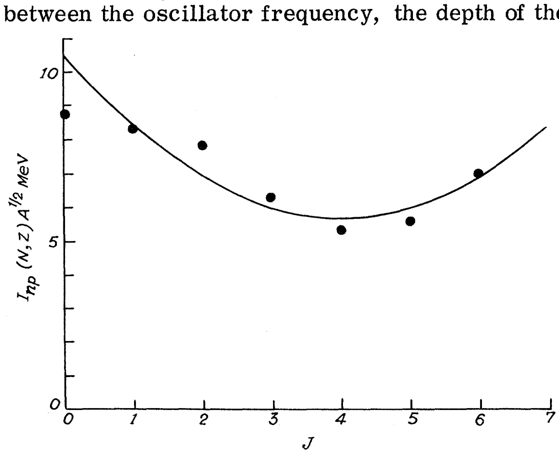

FIG. 4. Dependence of n-p interaction energy on J, the resultant angular momentum.

potential well, and the nuclear radius is obtained from the consideration that the energy of the highest filled level is approximately equal to the well depth. This assumption[28] leads to an $A^{-1/2}$ dependence of I_{np}, which agrees with our findings.

For a given configuration $j_n j_p$, I_{np} depends on J, the resultant angular momentum. To obtain the dependence of I_{np} on J, we assume a simple coupling scheme. Since an even-even nucleus (N, Z) has zero angular momentum, we assume that j_n is the ground-state spin of the $(N-1, Z)$ nucleus and j_p is that of the $(N, Z-1)$ nucleus. We propose that j_n couples with j_p to give J, which is the ground-state spin of the $(N-1, Z-1)$ nucleus. The values of J are taken from the table of nuclear constants.[29,30] Figure 4 shows the plot of $I_{np}(N, Z)A^{1/2}$ against J. Each point is the mean of all values with the same J. The over-all fit of I_{np} for $A > 32$ is given by the relation

$$I_{np}(N, Z) = [10.5 + 0.3J(J-8)]A^{-1/2}. \tag{4}$$

The mean deviation is of the order of ± 0.20 MeV. Ferguson[28] did not find any correlation between n-p interaction parameters and J. He, however, approximated the effective n-p interaction in nuclei by an extreme short-range force with an ordinary spin-dependent component. We have made no attempt to obtain the two-body matrix elements of the n-p interaction in terms of any

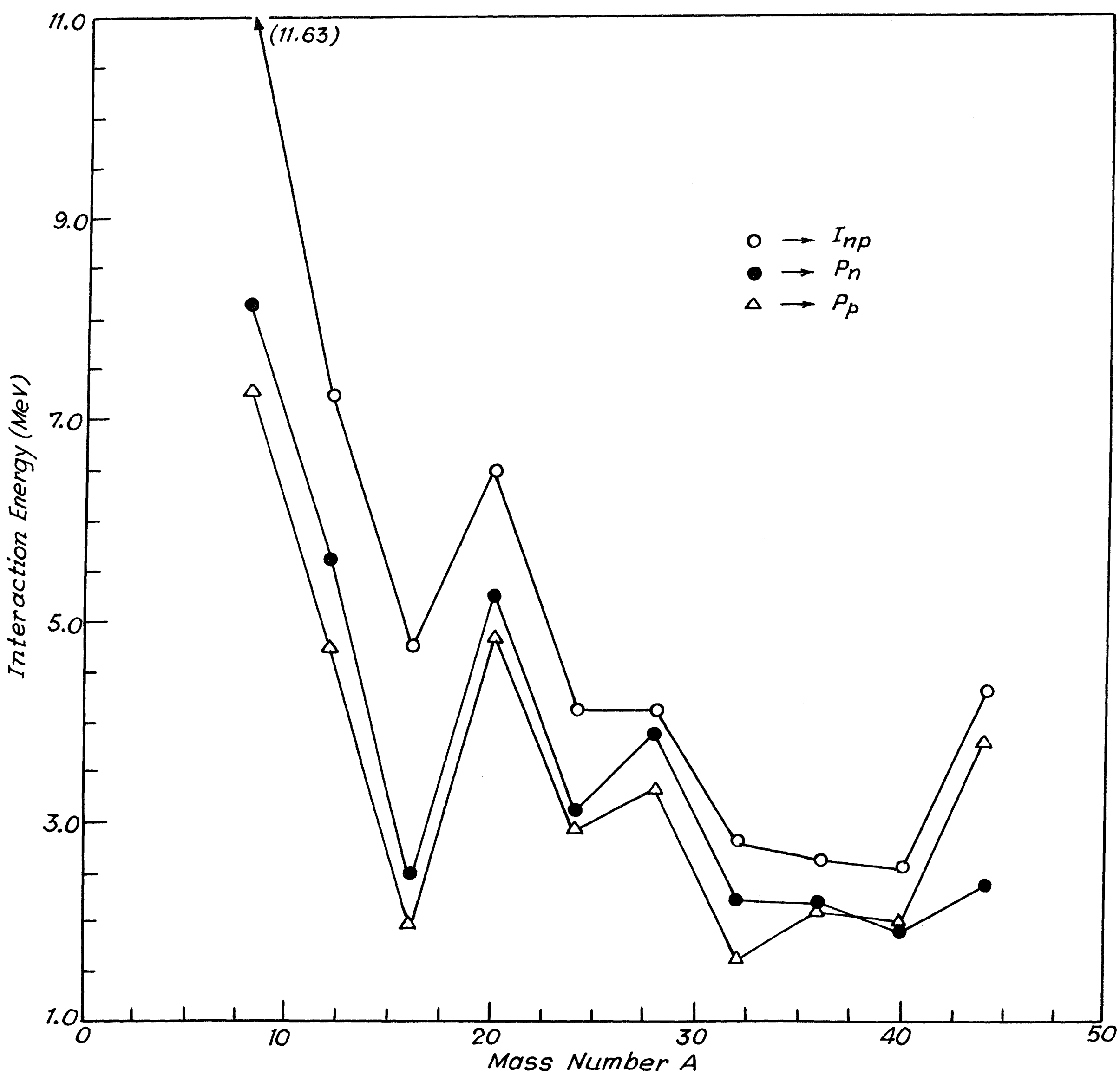

FIG. 5. Comparison of n-n, p-p, and n-p interaction energies for self-conjugate even-even nuclei.

basic two-body interaction potential.

In previous communications[31,32] we have dealt with neutron (P_n) and proton (P_p) pairing energies of the strongly bound even-even nuclei. We find that $P_n(N, Z)$ and $P_p(N, Z)$ are greater than $I_{np}(N, Z)$ for heavy nuclei. This is qualitatively very plausible. Neutrons and protons fill different shells in heavy nuclei. The matrix elements become small because of the poor overlap between neutron and proton orbitals. Thus, the effective n-p potential within a nucleus is essentially the long-range part of the actual two-nucleon potential.

One can get some idea about the degree of validity of charge symmetry and charge independence of nuclear forces from a comparative study of P_n, P_p, and I_{np} of even-even nuclei with $N=Z$. In even-even self-conjugate nuclei, the pairing nucleons, being in the same angular momentum state, should have the same interaction value if the charge-independence hypothesis is valid.[33] Figure 5 is the plot of P_n, and P_p, and I_{np} against A for all even-even self-conjugate nuclei. It is clear from the figure that the plots have the same qualitative behavior. But quantitatively, I_{np} is always greater than the corresponding P_n and $P_p \cdot P_p$ is always less than the corresponding P_n except at $A=44$, where the spin of the last pair of neutrons is not known. It may be different from that of the last pair of protons. The difference between proton and neutron pairing energies can be well accounted for, within the limit of experimental error, by taking into consideration the repulsive Coulomb energy between the interacting protons. This is in conformity with the charge symmetry of nuclear forces.

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