Projected Hartree-Fock and Core-Particle Coupling for $A=29$ and 30*

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A Projected Hartree-Fock calculation of the properties of ^{28}Si is used as the basis for a core-quasiparticle coupling treatment of ^{29}Si, ^{29}P, and ^{30}Si. Very good agreement with the known properties of these nuclei is achieved in this calculation with a variation of essentially one parameter, the coupling strength.

I. INTRODUCTION AND METHOD

A recent Hartree-Fock calculation,[1] based on the techniques of projection before variation and inclusion of vibrational correlations, has shown that the properties of low-lying levels of ^{28}Si can be much better accounted for by this projected Hartree-Fock (PHF) method than by projection after variation (HFP). In the adjacent nuclei ^{29,30}Si, however, rotational properties are less in evidence. As a result, calculations based on the Nilsson model[2,3] have not been able to account for the observed transition rates. In this note we wish to examine these nuclei in a core-quasiparticle coupling model, in which the properties of the core are taken from the PHF calculation on ^{28}Si.

The Hamiltonian of the coupled system can be written as

$$H = H_c + H_{sp} + H_{int} + H_{12}, \tag{1}$$

where H_c is the core Hamiltonian, defined in Eq. (3) of Ref. 1, H_{sp} is the single-quasiparticle Hamiltonian, and H_{12} is the residual interaction between the quasiparticles in the case where two quasiparticles are coupled to the core. The core-particle interaction is the quadrupole-quadrupole interaction

$$H_{int} = -\xi E_2 \sqrt{\tfrac{1}{5}\pi} \sum_\mu Q_{2\mu} Y_{2\mu}(\theta, \varphi), \tag{2}$$

where Q is the quadrupole operator for the core and $Y_{2\mu}(\theta, \phi)$ is the angular part of the quadrupole operator for the quasiparticle. E_2 is the energy of the first excited state of the core. Recent studies by Lee[4] for heavier nuclei suggest that such an interaction should be the dominant part of the core-particle Hamiltonian. In the basis in which the angular momenta of the core ($\vec{R}$) and of the quasiparticle ($\vec{j}$) are coupled to $\vec{I}$, with z-component M, the matrix elements of H_{int} are given by

$$\langle j' : R' : IM | H_{int} | j : R : IM \rangle = (-1)^{I+R'+\frac{1}{2}} \xi E_2 [\tfrac{1}{4}(2j+1)(2j'+1)]^{1/2} \begin{Bmatrix} j & 2 & j' \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{Bmatrix} \begin{Bmatrix} I & R & j \\ 2 & j' & R' \end{Bmatrix} (R' \| Q \| R)(u_j u_{j'} - v_j v_{j'}), \tag{3}$$

where u_j and v_j are the usual quasiparticle and quasihole amplitudes for state j and may be derived from the occupation numbers in Table I of Ref. 1. These results are, of course, those of the standard intermediate coupling vibrational model (I.C.V.M.)[5,6] except for the choices of H_c, which here gives rise to a projected rotational-type spectrum rather than a harmonic vibrational spectrum, and of the matrix elements of Q. In the present calculation, these matrix elements are taken from the calculated core $E2$ transition rates and quadrupole moments (Q_R), the relations being

$$(R' \| Q \| R) \propto (2R+1)^{1/2} [B(E2 : R \to R')]^{1/2} \tag{4}$$

and

$$(R \| Q \| R) \propto \frac{5}{4} \left(\frac{7}{2\pi} \right)^{1/2} Q_R,$$

with the normalization that $(2 \| Q \| 0) = \sqrt{5}$, so that the coupling parameter ξ has the same meaning as in the I.C.V.M. We do not here set out the corresponding results for the two quasiparticle case and for the transition rates, as they are simple modifications of the I.C.V.M. results given, for example, in works of Heyde and Brussaard[7] and Thankappan and Pandya[8] and laid out in detail in the work of Castel, Stewart, and Harvey.[9]

The core states taken into account in this calculation are the 0^+, 2^+, 4^+, and 6^+ states of the "oblate band," these being the only low-lying states which have non-negligible matrix elements of Q among themselves. The inclusion or not of the 6^+, indeed, is found to have very little effect on the results. The results of Ref. 1 are used to define the matrix elements of Q and H_c.

II. RESULTS AND DISCUSSION

The nuclei ^{29}Si and ^{29}P have previously been discussed in both the rotational[2,3] and vibrational[10] models, in both cases with only partial success.

TABLE I. $E2$ transition rates in $A=29$ nuclei.

Transition J_i	Transition J_f	$B(E2)$ (W.u.) ^{29}Si Calculation	^{29}Si Experiment [a]	^{29}P Calculation	^{29}P Experiment [b]
$\frac{3}{2}$	$\frac{1}{2}$	7.3	6.4	10.0	10.0 ± 1.1
$\frac{5}{2}$	$\frac{1}{2}$	9.9	11.9	10.4	14 ± 3
	$\frac{3}{2}$	1.5		1.8	
$\frac{3}{2}^*$	$\frac{1}{2}$	2.8	5.4	1.4	
	$\frac{3}{2}$	0.3		0.1	
	$\frac{5}{2}$	0.8		0.7	
$\frac{5}{2}^*$	$\frac{1}{2}$	0.001	<0.8	0.00	
	$\frac{3}{2}$	0.004	(21)	0.01	
	$\frac{5}{2}$	0.004	(2)	0.01	
$\frac{7}{2}$	$\frac{3}{2}$	10.5	7.5	12.9	31 ± 9 or 1.5 ± 0.8
	$\frac{5}{2}$	0.7		1.2	3.5 ± 2.3

[a] Reference 11.
[b] C. F. Monahan, H. C. Evans, J. Montague, W. R. Paulson, and W. M. Zuk, Can. J. Phys. 48, 2683 (1970).

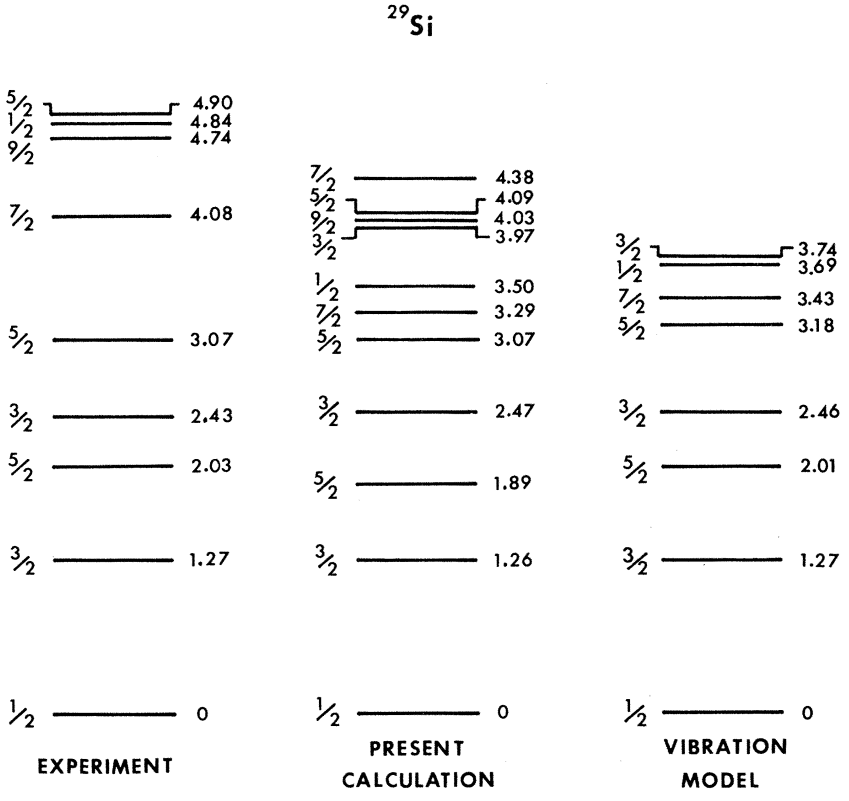

FIG. 1. Energy levels of ^{29}Si. The results of the present calculation (column 2) are compared with those of a calculation using a vibrational core (column 3) and with experiment.

In the present case, the properties of the core are all defined as in Sec. I, and there remains only the single quasiparticle energies and the coupling strength ξ to be determined. The quasiparticle energies might be taken from the single-particle energies used in the PHF calculation [modified by the factor $(u_j^2 - v_j^2)$], but it is not clear that the single-particle energies to use when coupling additional particles in the center of the shell are necessarily the same as those derived from the ^{17}O spectrum. Indeed, the single-particle energies derived from the ^{39}K spectrum lead to quasiparticle energy separations which may differ by about 1.5 MeV from the ^{17}O spacings. Accordingly, we have permitted the spacings to vary within this range, and use the quasiparticle energy spacings $\epsilon_{3/2} - \epsilon_{1/2} = 1.78$ MeV and $\epsilon_{5/2} - \epsilon_{1/2} = 3.05$ MeV, which are in fact quite close to the ^{39}K values. The coupling parameter ξ is chosen ($\xi = 1.8$) to give a best fit to the experimental spectrum, and the results (for positive-parity states) of the present calculation are compared, for ^{29}Si, with experiment[11,12] and with our previous I.C.V.M. calculation[10] in Fig. 1. The two calculations, despite involving different values of the parameters, clearly give very similar results, both of which agree well with experiment. The $\frac{7}{2}^+$ state appears at an energy lower than is observed, which is associated with the fact that the calculated energy of the 4^+ state in the core is also too low.

The structure of the wave functions may be investigated by considering the $E2$ transition rates shown in Table I. The observed pattern of strong and weak transitions is well reproduced (those for which no experimental value is given are small), and good agreement in the magnitude of the strong transitions is obtained. The only exception appears to be the $\frac{5}{2}^+$ state, whose wave function has almost entirely $d_{5/2}$ single-particle character and is "cut off" from the other states by the accident that $(u_{5/2}u_j - v_{5/2}v_j)$ is almost zero for both $j = \frac{1}{2}$ and $\frac{3}{2}$. The $E2$ rates have been calculated for an effective neutron charge of $0.5e$ and a deformation "stiffness parameter" $C = 37.5$ MeV, the latter being deduced from the PHF calculation. The $M1$ transition rates, however, are generally not so well reproduced.

The nucleus ^{30}Si has been the subject of studies using both the shell model[13,14] and the I.C.V.M.[8] In the present case we have restricted our single quasiparticle states to the $d_{3/2}$ and $s_{1/2}$ orbits. Inclusion of $d_{5/2}$ states may be necessary in the case of higher-lying states, but most of the features of the spectrum below 6 MeV appear to be described without them. The core properties are exactly as before, with the exception that it is necessary to multiply H_c by a scale factor 1.2 to obtain the correct density of states. For the residual interaction H_{12} we have used the matrix elements of Kuo.[15] These, like the single-particle energies discussed above, are based on the properties of the beginning of the sd shell, and study of the end of the shell indicates that the matrix elements relevant to this calculation (involving the $s_{1/2}$ and $d_{3/2}$ states) should increase towards the end of the shell. A scale factor of 1.4 has been introduced to account for this effect. The single quasiparticle energy spacing used is $\epsilon_{3/2} - \epsilon_{1/2} = 0.77$ MeV, and the coupling parameter for best fit is $\xi = 1.5$. This parameter is related to the core-particle-coupling matrix element $k = \langle jl|k(r)|j'l'\rangle$ by $\xi = k[5/(2\pi E_2 C)]^{1/2}$ and corresponds to the values $k = 17$ and 18 MeV in ^{29}Si and ^{30}Si, respectively. This is consistent with the typical value $k = 20$ MeV

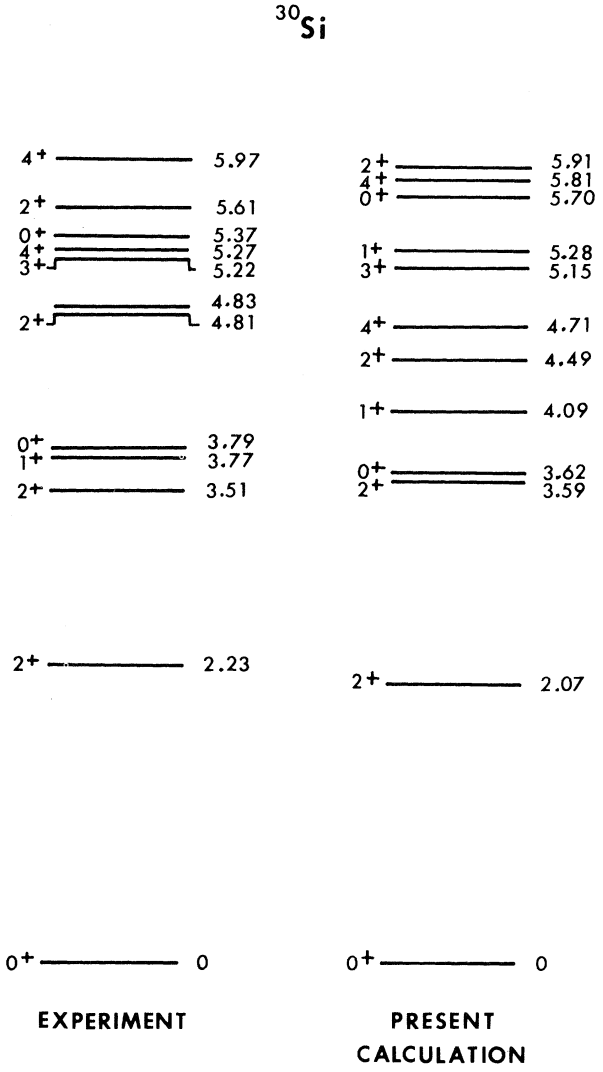

FIG. 2. Comparison of the observed and calculated spectra of ^{30}Si.

TABLE II. $E2$ and $M1$ transition rates in ^{30}Si.

Transition J_i	Transition J_f	$B(E2)$ (W.u.) Calculation	$B(E2)$ (W.u.) Experiment	$B(M1)$ (W.u.$\times 10^{-2}$) Calculation	$B(M1)$ (W.u.$\times 10^{-2}$) Experiment
2	0	7.9	6.9 ± 0.5 [a]		
2*	0	0.3	1.2 ± 0.3 [a]		
	2	0.2	6.4 ± 4.0 [a]	4.8	7.7 ± 3.0 [a]
1	0			0.001	0.3 [a]
	2	0.1	2.5 ± 1.0 [b]	9.5	9.0 ± 2.0 [b]
	2*	2.0		2.7	
0*	2	0.2	<2.7 [a]		
	2*	0.6			
	1			0.14	
2**	0	0.02	0.09 ± 0.03 [b]		
	2	0.2	0.29 ± 0.15 [b]	0.01	0.15 ± 0.07 [b]
	2*	2.7	2.3 ± 2.0 [b]	0.9	3.6 ± 0.9 [b]
3	2	0.2	$0.01^{+0.04}_{-0.01}$ or 1.2 ± 0.3 [b]	0.0001	0.25 or 0.002 [b]
	2*	7.6	1.0 ± 0.5 [b]	1.6	4.5 [b]
4	2	8.1	4.0 ± 0.8 [a]		
	2*	0.1			

[a]Reference 16 and 18.

[b]Reference 18; R. D. Symes, B. E. Crossfield, N. Dawson, G. D. Jones, P. J. Mulhern, I. G. Main, M. F. Thomas, and P. J. Twin, private communication and to be published.

for the *sd* shell in the I.C.V.M., but is half the value of k (40 MeV) found elsewhere. This is probably related to the fact that the neutrons and protons are filling the same subshells in the *sd* shell, unlike the situation elsewhere.

In Fig. 2 we compare the calculated and experimental[16-18] spectra of ^{30}Si. Our results this time differ substantially from those of the I.C.V.M.,[8] probably because of the differences in core properties and in the residual interaction H_{12} used. Our calculation reproduces all the positive-parity states observed up to 6 MeV, with the exception of the (2^+) state at 4.83 MeV, and the ordering of levels is generally good. Again, the depression of the energy of the 4_1^+ state corresponds to the depression of the 4^+ state of the core.

The calculated $E2$ and $M1$ transition rates are compared with experiment in Table II. Experimental detail is not yet as extensive as might be wished, but we find general agreement for those cases where comparison is possible. It may be that the strong $4^+ \rightarrow 2_1^+$ transition found for the 4_1^+ state should belong to the 4_2^+ state, which has an 8-W.u. transition to the 2_1^+ state, and it has been suggested[17] that the 4_1^+ state may contain a large $d_{5/2}^{-1} d_{3/2} s_{1/2}^2$ component. Although the transition rates are not known, the calculated decays of the 2_3^+, 3_1^+, and 0_3^+ states seem to be in good agreement with the observed pattern of decays of these states.

In conclusion, we have here attempted a description of the nuclei with $A = 29$ and 30 in terms of a model with essentially one variation parameter. Small variations of the single-particle energies were allowed, however, and the values used found to be consistent with the usual estimates for this region. One should point out that the derivation of the u's and v's from the occupation numbers calculated in Ref. 1, lead to an ambiguity in the sign of these parameters. One could argue, therefore, that the signs chosen correspond to additional parameters. We have assumed positive signs for these amplitudes throughout the calculation, so as to be consistent with previous calculations[19,20] which determined both sizes and signs of the quasiparticle amplitudes by solving a BCS-type equation.

Altogether, the simplicity of this approach, in which a rotational character is assumed for the core nucleus, indicates that this method can be used readily in regions where rotational rather than vibrational structure is apparent.

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Optical-Potential Scattering of Nucleons by Carbon at Medium Energies

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The elastic scattering of nucleons by carbon, at incident energies of 142, 210, and 330 MeV, is calculated from an optical potential retaining all terms to second order in the N-N amplitudes. The amplitudes are calculated using recent N-N phase-parameter sets obtained by the Yale, Livermore, and Michigan State Groups. Nuclear correlation functions of both the Fermi and Brueckner-Gammel types are used to represent nuclear structure. Comparison with experiment is made in order to distinguish among different N-N phase-parameter sets, and among different types of nuclear correlation functions.

In earlier work,[1] the elastic scattering of nucleons by light nuclei, at energies of 140 to 310 MeV, was calculated from an optical potential retaining all terms to second order in the two-nucleon amplitudes. The amplitudes were calculated using the YLAN3M-YLAM and L-IV phase-parameter sets of the Yale[2] and Livermore[3] groups, respectively. Since that time, these phase-parameter sets have been superseded by more recent searches.[4-7] It is the purpose of this work to determine the effect of these new phase-parameter sets on the agreement of the calculated nucleon-nuclear observables with experiment and to investigate the degree to which the calculated observables depend on the shape of the nuclear correlation function.

The potential used in this calculation is derived from the Watson[8] formalism. The double-scattering term and an impulse-approximation correction term, both of which are binary in the two-nucleon amplitudes, are retained. The resulting optical potential is used in the numerical solution of the partial-wave integral-scattering equation in momentum space. Details of the calculation are given in Ref. 1 and elsewhere.[9]

The two-nucleon amplitudes are calculated using the recent Yale[4] and Livermore[5,6] phase-parameter sets at 142, 210, and 330 MeV, and the Michigan State[7] phase-parameter set at 210 MeV.

In Ref. 1, nuclear structure was represented by both Fermi (F) and Brueckner-Gammel (BG) type correlations. For the Fermi type, correlation functions were derived by assuming a Fermi gas of nucleons. For the Brueckner-Gammel case, however, only the correlation length was known, and a form for the correlation function was chosen for convenience. The BG-type function was taken to be

$$G_B = -(1 - \delta r^2/b^2)e^{-r^2/b^2}, \tag{1}$$

subject to the conditions

$$\int G_B d\tau = 0 \tag{2}$$

and

$$\int_0^\infty G_B dr = R_c\ , \tag{3}$$