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Shell-Model Structure of ^{208}Bi

W. P. Alford, J. P. Schiffer,* and J. J. Schwartz†
Nuclear Structure Research Laboratory, University of Rochester, Rochester, New York 14627
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The level structure of ^{208}Bi has been investigated using both stripping and pickup reactions. The ^{207}Pb(^{3}He,d) and ^{207}Pb(α,t) reactions permit the identification of a series of doublets arising from the coupling of a proton in the $h_{9/2}$, $f_{7/2}$, $i_{13/2}$, $f_{5/2}$, and $p_{3/2}$ single-particle states with a neutron hole in the $p_{1/2}$ shell. The ^{209}Bi(d,t) and ^{209}Bi(^{3}He, α) reactions populate the multiplets arising from coupling of a neutron hole in the $p_{1/2}$, $f_{5/2}$, $p_{3/2}$, $i_{13/2}$, $f_{7/2}$, and $h_{9/2}$ states with a proton in the $h_{9/2}$ shell. A detailed analysis shows that the total strength in each multiplet is close to that for the corresponding single-particle or single-hole transition on ^{208}Pb. In addition to the strong transitions leading to states of these multiplets, some relatively weak transitions are observed which provide a measure of the mixing of different configurations. The structure and purity of the particle-hole multiplets below about 3-MeV excitation is in good agreement with shell-model calculations.

INTRODUCTION

The nucleus ^{208}Bi is of special interest from a shell-model point of view, since the low-lying states are expected to arise from the coupling of a single proton with a neutron hole outside the ^{208}Pb spherical closed core. The proton single-particle states[1] in ^{209}Bi and the neutron single-hole states[2,3] in ^{207}Pb are well known and have been extensively studied. The spectrum of ^{208}Bi is then expected to consist of a number of multiplets, each centered near the appropriate particle-hole energy, with a splitting determined by the particle-hole residual interaction. Figure 1 shows the known single-particle states in ^{209}Bi and single-hole states in ^{207}Pb. The unperturbed centroids of the low-lying multiplets arising from particle-hole couplings in ^{208}Bi are also indicated on the figure, along with the range of spin values expected for the states of each multiplet. The multiplets identified in the present measurements are indicated in the figure.

Several calculations[4-7] using various reasonable residual interactions all predict that the energy spread within each multiplet is relatively small. In addition, calculated wave functions show that the low-lying states are rather well described as simple particle-hole states, i.e., configuration mixing is generally small.

The first experimental results which clearly demonstrated this simple structure were those of Erskine[8] on the ^{209}Bi(d, t)^{208}Bi reaction. These results provided identification of the members of the multiplets arising from the coupling of a neutron hole in the $p_{1/2}$, $f_{5/2}$, and $p_{3/2}$ orbits with the $h_{9/2}$ proton. Spins were assigned on the basis of relative intensities of transitions within each multiplet. A preliminary report of the present work[9] extended Erkine's results to higher excitations, identifying the $h_{9/2}\times i_{13/2}{}^{-1}$ and $h_{9/2}\times f_{7/2}{}^{-1}$ multiplets. In addition, measurements of the ^{207}Pb(^{3}He, d)^{208}Bi reaction showed a series of doublets which could be interpreted as arising from the coupling of protons in $h_{9/2}$, $f_{7/2}$, $i_{13/2}$, $f_{5/2}$, and $p_{3/2}$ with the $p_{1/2}$ neutron hole.

Additional information on the levels of ^{208}Bi has been provided by decay studies.[10] In particular, γ-ray measurements following electron capture[11] in ^{208}Po and the decay of an isomeric state[12] in ^{208}Bi provide spin determinations for several levels which agree with the assignments proposed by Erskine.

The present measurements confirm Erskine's results for the low-lying states observed in ^{209}Bi(d, t)^{208}Bi and extend them to higher excitations. A number of new multiplets have been identified, and quantitative estimates of configuration mixing effects have been obtained.

EXPERIMENTAL

All measurements were carried out using the beam from the Rochester MP tandem Van de Graaff accelerator. Reaction products were detected in nuclear emulsions in the focal surface of an Enge broad-range spectrograph. The spectrograph has been calibrated using α particles from a ThC' source. Subsequently, several comparisons between level energies measured with the spectrograph and with high-resolution Ge-Li detectors have been made. Even though these comparisons are made at fields considerably higher than those used in the original calibration, they indicate that the uncertainty in the calibration is less than 5 keV for the present measurements. In studies of each reaction, data at different angles were normalized relative to the elastic scattering

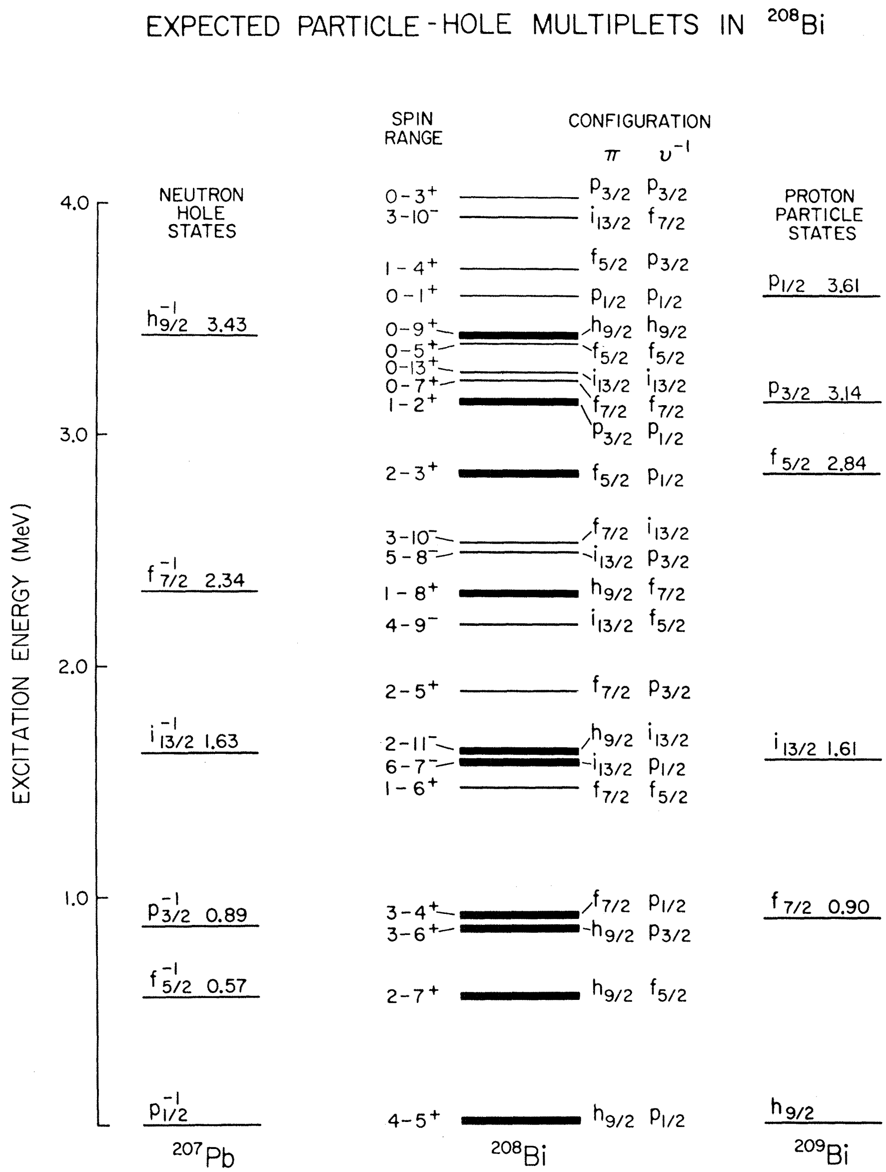

FIG. 1. Proton-particle and neutron-hole states on ^{208}Pb core, and expected particle-hole multiples. The multiplets identified in present measurements are indicated by heavy ions.

group observed in a fixed monitor counter. Details of each measurement are as follows.

A. ^{209}Bi$(d, t)^{208}$Bi

The target consisted of about 50 μg/cm^2 of bismuth metal evaporated on a carbon backing 20 μg/cm^2 in thickness. Spectra were recorded at angles of 10, 15, 20, 25, 30, 40, 50, 60, and 80° using a 20-MeV deuteron beam. Tritons were recorded in Kodak type NTB emulsions 50 μm in thickness. Plates were scanned in intervals of $\frac{1}{3}$ or $\frac{1}{4}$ mm. The energy resolution was typically about 12-keV full width at half maximum. A typical spectrum is shown in Fig. 2. Also shown in this figure are the relative locations of the known neutron single-hole states in ^{207}Pb with the ground-state coincident with the centroid of the ground-state doublet in ^{208}Bi.

B. ^{209}Bi$(^3$He$, \alpha)^{208}$Bi

α particles from this reaction were observed only at angles of 40 and 60°, with a beam energy of 28 MeV. The target was the same at that used in (d, t) measurements. α particles were recorded

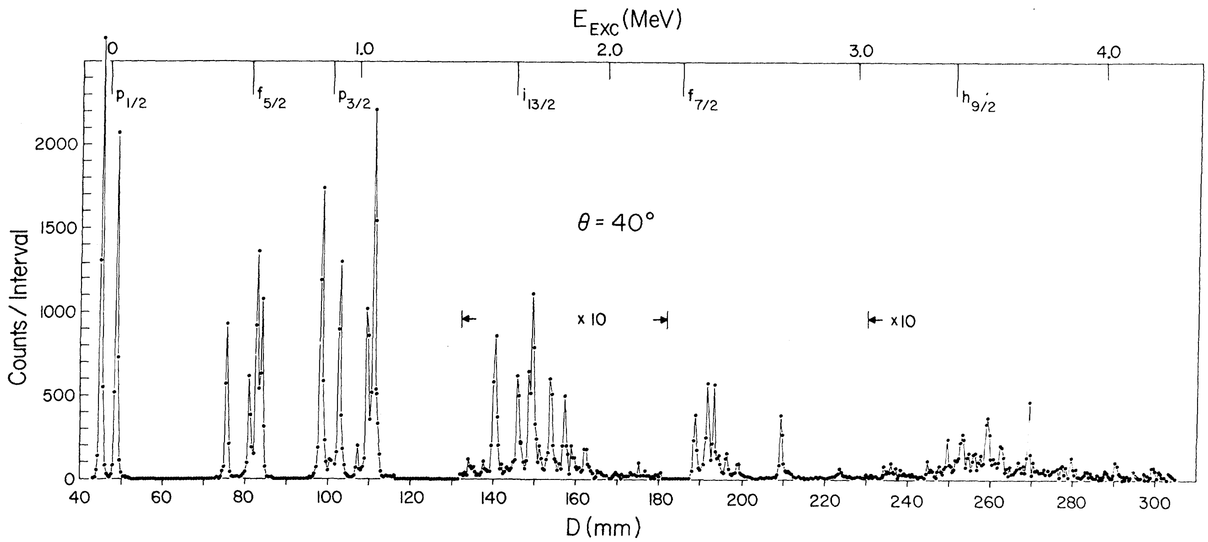

FIG. 2. Spectrum of tritons from the $^{209}Bi(d,t)^{208}Bi$ reaction at 20 MeV. Relative locations of neutron-hole states in ^{207}Pb are shown at the top of the figure.

in Ilford type KO plates which allowed easy discrimination of α tracks from those of hydrogen isotopes. The resolution was about 20 keV. A typical spectrum is shown in Fig. 3.

C. $^{207}Pb(^{3}He, d)^{208}Bi$

The target was prepared by evaporating metallic lead on a carbon backing about 20 $\mu g/cm^2$ in thickness. The target thickness was about 50 $\mu g/cm^2$. The target material, obtained from the Stable Isotopes Division, Oak Ridge National Laboratory, contained 2.2% ^{206}Pb, 92.4% ^{207}Pb, and 5.5% ^{208}Pb. Data were taken at angles of 10, 15, 20, 40; 25, 30, 35, 40; and 40, 50, 60, 70° in three different runs at an incident energy of 30 MeV. Reaction products were recorded in Kodak type NTB emulsions 100 μm in thickness.

Emulsions were covered by a $\frac{1}{32}$-in. Al absorber which stopped all particles focused through the spectrograph except the deuterons. Plates were

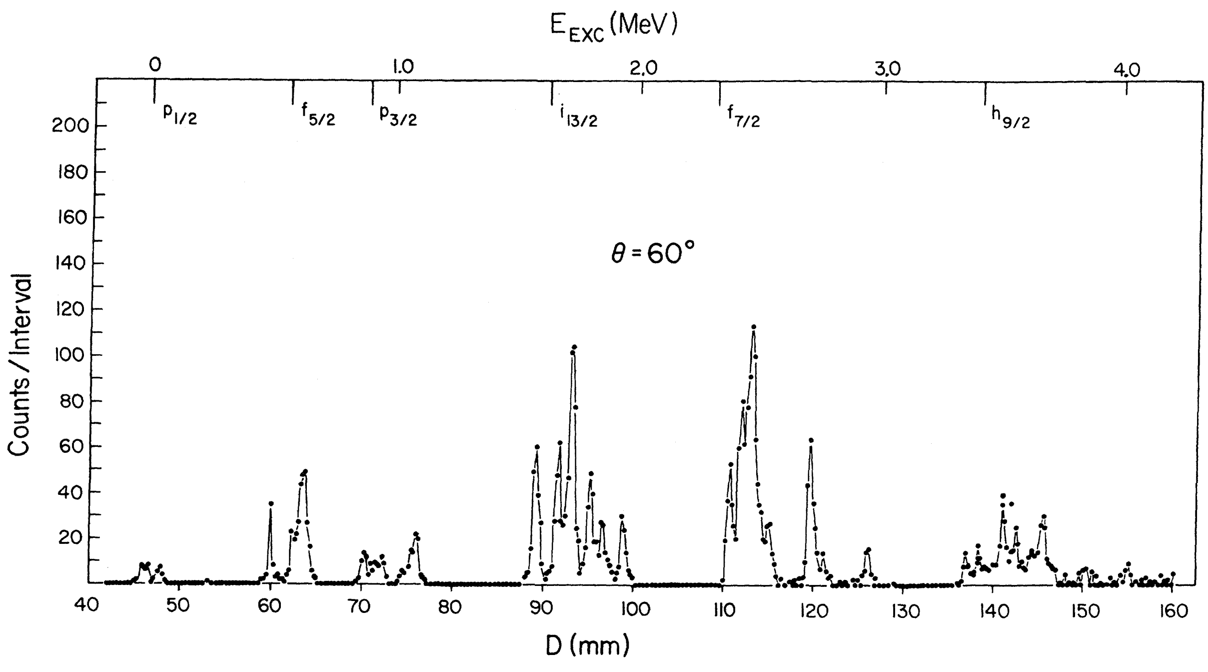

FIG. 3. Spectrum of α particles from the $^{209}Bi(^{3}He, \alpha)^{208}Bi$ reaction at 28 MeV.

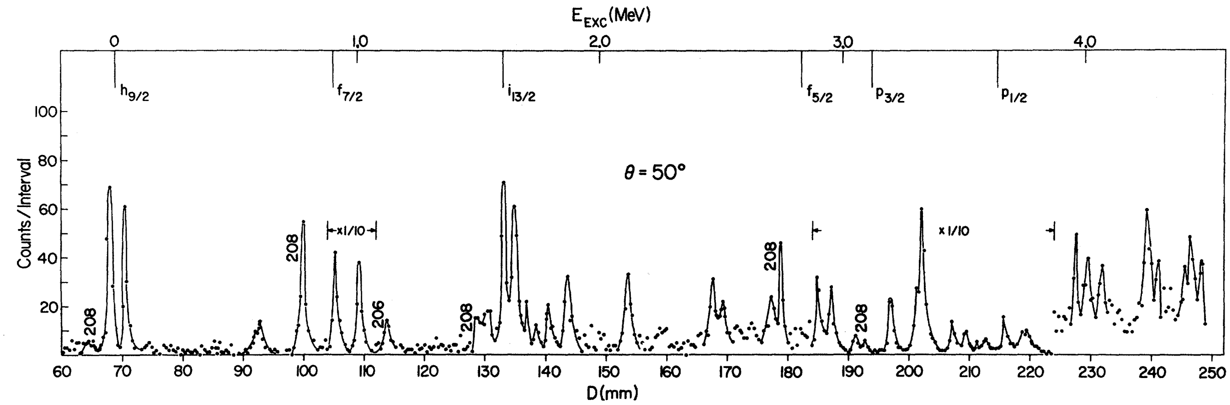

FIG. 4. Spectrum of deuterons from the ^{207}Pb(^{3}He, d)^{208}Bi reaction at 30 MeV. The relative locations of proton-particle states in ^{209}Bi are shown at the top of the figure.

scanned for the most part in 0.25-mm steps. A typical spectrum of deuterons is shown in Fig. 4. In addition to the groups from the reaction of interest, deuterons groups from the ^{206}Pb and ^{208}Pb in the target were observed. These are labeled in Fig. 4. At forward angles (<40°) interference from reaction groups originating in the carbon backing were very troublesome and obscured significant parts of the spectrum. The resolution obtained in these measurements was about 20 keV, arising mainly from target thickness and terminal voltage fluctuations. Also shown in this figure are the relative positions of the known single-particle states in ^{209}Bi with the ground state adjusted to be coincident with the centroid of the ground-state doublet in ^{208}Bi.

D. ^{207}Pb(α, t)^{208}Bi

Tritons from this reaction were recorded at angles of 20 and 50° only, at an incident energy of 30 MeV. The target used was the same as for the (^{3}He, d) measurements. Tritons were recorded in Kodak type NTB emulsions 50 μm in thickness. A spectrum recorded at 50° is shown in Fig. 5. In this reaction it was found that cross sections for transitions leading to states at excitation energies greater than 2 MeV were very small due to the influence of the Coulomb barrier on the outgoing particle.

RESULTS

It is seen from Figs. 4 and 5 that the states most strongly excited in the stripping reaction are a series of doublets occuring at excitation energies closed to those of the known single-proton state in ^{209}Bi. Similarly, Figs. 2 and 3 show that the states most strongly excited in pickup form a series of multiplets centered at excitation energies close to those of the known neutron single-hole states in ^{207}Pb. Above an excitation energy of about 3 MeV many states are excited in the (^{3}He, d) reaction, some of them relatively strong-

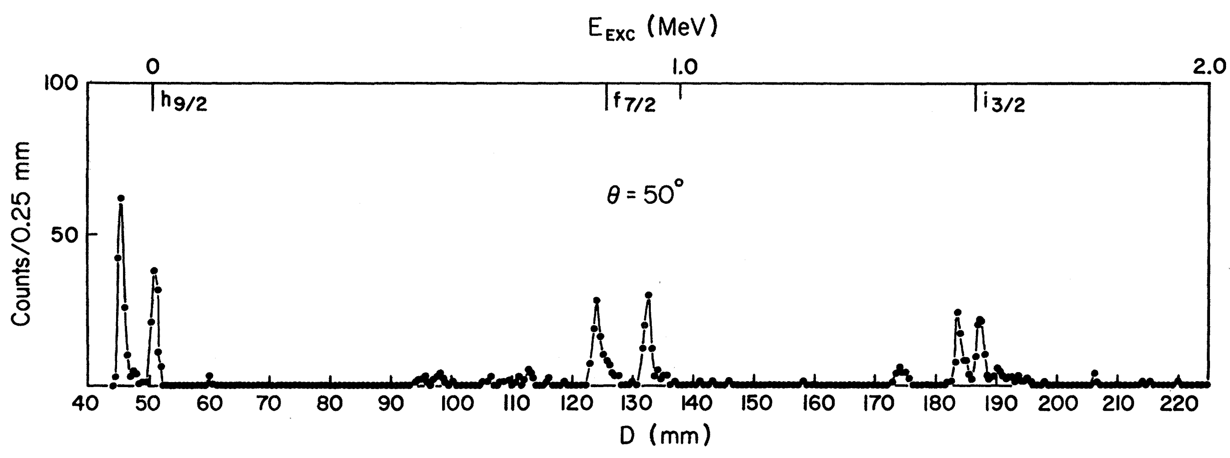

FIG. 5. Spectrum of tritons from the ^{207}Pb(α, t)^{208}Bi reaction at 30 MeV.

TABLE I. Energies and spectroscopic strengths for states observed in ^{208}Bi.

Energy (keV)	(d,t) l	j [a]	S_{expt}	$S_{\mathrm{p-h}}$	$(^3\mathrm{He},d)$ l	j [a]	G_{expt}	$G_{\mathrm{p-h}}$	J^{π}	Configuration p	n^{-1}
0	1	$\frac{1}{2}$	1.07	1.10	5	$\frac{9}{2}$	5.5	5.5	5^+	$h_{9/2}$	$p_{1/2}$
65	1	$\frac{1}{2}$	0.87	0.90	5	$\frac{9}{2}$	4.1	4.5	4^+	$h_{9/2}$	$p_{1/2}$
512	3	$\frac{5}{2}$	1.30	1.30					6^+	$h_{9/2}$	$f_{5/2}$
	3	$\frac{5}{2}$	0.70	0.90	5	$\frac{9}{2}$	0.76	0	4^+	$h_{9/2}$	$f_{5/2}$
603	+	+	+		or						
	1	$\frac{3}{2}$	0.10	0.00	3	$\frac{7}{2}$	0.09	0			
					5	$\frac{9}{2}$	0.83	0	$5^+ + 3^+$	$h_{9/2}$	$f_{5/2}$
631	3	$\frac{5}{2}$	1.85	1.80	or						
					3	$\frac{9}{2}$	0.11	0	3^+	$h_{9/2}$	$f_{5/2}$
652	3	$\frac{5}{2}$	1.40	1.50					7^+	$h_{9/2}$	$f_{5/2}$
890	1	$\frac{3}{2}$	1.28	1.10					5^+	$h_{9/2}$	$p_{3/2}$
930	3	$\frac{5}{2}$	0.48	0.50					2^+	$h_{9/2}$	$f_{5/2}$
939					3	$\frac{7}{2}$	3.4	3.5	3^+	$f_{7/2}$	$p_{1/2}$
963	1	$\frac{3}{2}$	0.84	0.90					4^+	$h_{9/2}$	$p_{3/2}$
1038	1	$\frac{3}{2}$	0.11	0.00	3	$\frac{7}{2}$	3.9	4.5	4^+	$f_{7/2}$	$p_{1/2}$
1075	1	$\frac{3}{2}$	0.68	0.70					3^+	$h_{9/2}$	$p_{3/2}$
1099	1	$\frac{3}{2}$	1.47	1.30					6^+	$h_{9/2}$	$p_{3/2}$
	3	b	0.005								
1467	or										
	1		0.002								
	3	b	0.005								
1534	or										
	1		0.002								
1565					3	b	0.18	0			
1574	6	$\frac{13}{2}$	2.51	2.10					10^-	$h_{9/2}$	$i_{13/2}$
1606			weak								
1630	6	b	~0.1	0	6	$\frac{13}{2}$	5.9	6.5	6^-	$i_{13/2}$	$pi_{1/2}$
1662	6	$\frac{13}{2}$	2.0	1.70					8^-	$h_{9/2}$	$i_{13/2}$
1673					6	$\frac{13}{2}$	5.6	7.5	7^-	$i_{13/2}$	$p_{1/2}$
1699	6 [c]	$\frac{13}{2}$		0.90					4^-		
1719	6 [c]	$\frac{13}{2}$	4.13	1.3+1.5	6		2.1	0	$6^- + 7^-$	$h_{9/2}$	$i_{13/2}$
1734			weak								
1791	6	$\frac{13}{2}$	2.07	1.90					9^-	$h_{9/2}$	$i_{13/2}$
1806					(3) [d]		0.31				
1842	6	$\frac{13}{2}$	1.09	1.10					5^-	$h_{9/2}$	$i_{13/2}$
	3		0.01								
1878	or										
	1		0.004								
1885					(3)		0.30				

TABLE I *(Continued)*

Energy (keV)	(d,t) l	j^a	S_{expt}	S_{p-h}	(^{3}He,d) l	j^a	G_{expt}	G_{p-h}	J^π	Configuration p	n^{-1}
1927	6	$\frac{13}{2}$	0.76	0.70					3^-	$h_{9/2}$	$i_{13/2}$
2132					(1)		0.08				
2345	3	$\frac{7}{2}$	1.38	1.50					7^+	$h_{9/2}$	$f_{7/2}$
2389	3	$\frac{7}{2}$	1.87	0.9+1.1					$4^+ + 5^+$	$h_{9/2}$	$f_{7/2}$
2413	3	$\frac{7}{2}$	1.38	1.30					6^+	$h_{9/2}$	$f_{7/2}$
2431	6	$\frac{13}{2}$	2.74	2.30					11^-	$h_{9/2}$	$i_{13/2}$
2462	3	$\frac{7}{2}$	0.68	0.70	(3)		0.17		3^+	$h_{9/2}$	$f_{7/2}$
2506	3	$\frac{7}{2}$	0.45	0.50	(3)		0.11		2^+	$h_{9/2}$	$f_{7/2}$
2665	3	$\frac{7}{2}$	1.60	1.70					8^+	$h_{9/2}$	$f_{7/2}$
2688			weak								
2716			weak						2^-	$h_{9/2}$	$i_{13/2}$
2888	3	$\frac{7}{2}$	0.31	0.30					1^+	$h_{9/2}$	$f_{7/2}$
2890					3	$\frac{5}{2}$	3.4	3.5	3^+	$f_{5/2}$	$p_{1/2}$
2915			weak								
2945					3	$\frac{5}{2}$	2.8	2.5	2^+	$f_{5/2}$	$p_{1/2}$
3070			weak		1	$\frac{3}{2}$	0.19			$p_{3/2}$	$p_{1/2}$
3173					1	$\frac{3}{2}$	0.76			$p_{3/2}$	$p_{1/2}$
3260					1	$\frac{3}{2}$	0.48			$p_{3/2}$	$p_{1/2}$
3270	5	$\frac{9}{2}$	1.05							$h_{9/2}$	$h_{9/2}$
3288					1	$\frac{3}{2}$	1.68	1.50	(1^+)	$p_{3/2}$	$p_{1/2}$
3323	5	$\frac{9}{2}$	0.97							$h_{9/2}$	$h_{9/2}$
3365	5	$\frac{9}{2}$	0.97							$h_{9/2}$	$h_{9/2}$
3393			weak								
3410					1	$\frac{3}{2}$	0.44			$p_{3/2}$	$p_{1/2}$
3412	5	$\frac{9}{2}$	2.67							$h_{9/2}$	$h_{9/2}$
3459	5	$\frac{9}{2}$	1.60							$h_{9/2}$	$h_{9/2}$
3460					1	$\frac{3}{2}$	0.32			$p_{3/2}$	$p_{1/2}$
3525	5	$\frac{9}{2}$	0.76							$h_{9/2}$	$h_{9/2}$
3535							weak				
3565	5	$\frac{9}{2}$	2.27	1.90						$h_{9/2}$	$h_{9/2}$
3612					1	$\frac{3}{2}$	0.60			$p_{3/2}$	$p_{1/2}$
3652			weak								
3683			weak								
3716	5	$\frac{9}{2}$	0.72							$h_{9/2}$	$h_{9/2}$

[a] Assignments based on location of single-particle states in ^{209}Bi or single-hole states in ^{207}Pb.

[b] Proposed assignments for weak transitions are shown in Table IV and discussed in the text.

[c] States only partly resolved.

[d] Quantities in parentheses assigned on the basis of calculations from Ref. 7.

[e] Seen in (^{3}He,α) results only.

ly, but no simple structure is obvious. In the pickup results, many weakly excited states were observed up to about 5 MeV, but no strong transitions were observed above about 3.5 MeV.

For the (^{3}He, d), (α, t), and (d, t) reactions excitation energies were measured relative to the ground-state group using a peak-fitting program. For the (^{3}He, d) and (d, t) measurements, excitation energies measured at different angles for well-resolved groups showed deviations of no more than 5 keV from the mean. For weak or poorly resolved groups deviations of up to 10 keV were seen. In the (^{3}He, α) reaction the ground-state transition was very weak, and energies were measured relative to the 512-keV group.

The results of the measurements are summarized in Table I. The first column lists all levels observed in either the (d, t) or (^{3}He, d) reactions, or both. In addition, Table I lists proposed assignments of l and j for many states, and measured spectroscopic factors (pickup) or spectroscopic strengths (stripping). Column 10 lists proposed spin and parity assignments based mainly on the measured transition strengths. Column 11 lists proposed assignments for the particle-hole multiplets giving rise to the observed states. Finally, columns 5 and 9 list spectroscopic factors or strengths for the pure particle-hole state proposed in columns 10 and 11.

The angular distributions observed for the strong transitions in the ^{209}Bi(d, t)^{208}Bi reaction are shown in Figs. 6–12. The error bars shown on the data points represent statistical uncertainties only. The over-all uncertainty in the absolute cross section is estimated to be about 20%.

In extracting spectroscopic strengths from these results, it is found that the transfer l value is generally well determined from the measured angular distribution. This is listed in the second column of Table I. The j value is not determined from the measurements, but has been assumed to

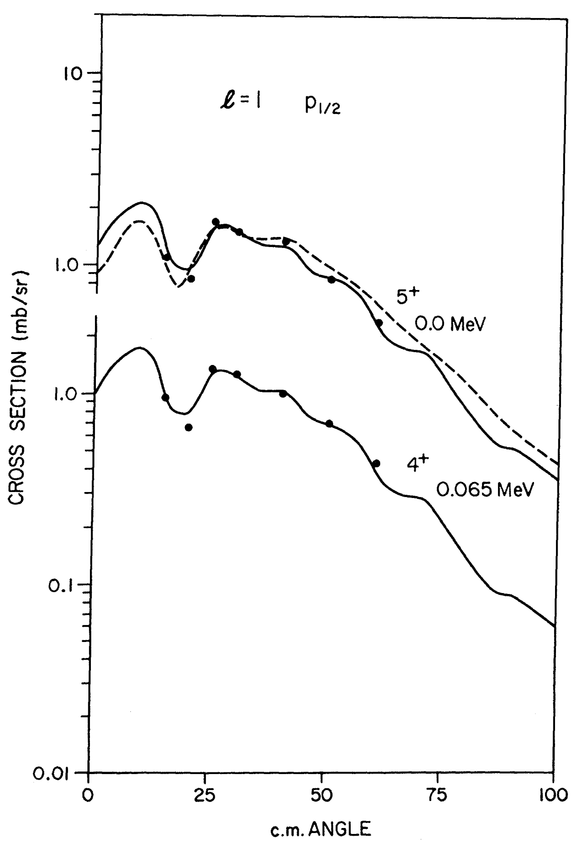

FIG. 6. Angular distributions for members of the $h_{9/2}\times p_{1/2}^{-1}$ multiplet observed in the ^{209}Bi(d,t)^{208}Bi reaction. The dashed curve is the result of a DWBA calculation. Solid curves are taken from measurements of the ^{208}Pb(d,t)^{207}Pb reaction.

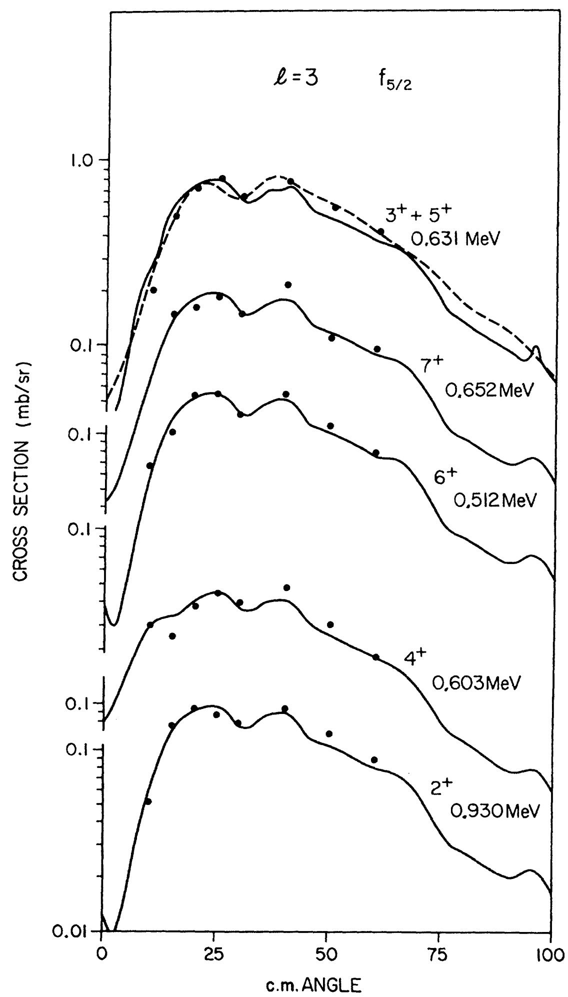

FIG. 7. Angular distributions for members of the $h_{9/2}\times f_{5/2}^{-1}$ multiplet observed in ^{209}Bi(d,t)^{208}Bi.

be that of the corresponding single neutron-hole state in ^{207}Pb.

A further complication arises from the fact that since the target has nonzero spin, in general two or more l values could contribute to the measured cross section. Comparison of measured and calculated cross sections was carried out using a computer program which adjusted the relative contributions from distorted-wave Born-approximation (DWBA) cross sections for two specified l values in order to provide the best least-squares fit to the measured cross section.

DWBA cross sections were calculated using the same parameters as had been used in fitting measurements of the ^{208}Pb(d, t)^{207}Pb reaction[2] at 20 MeV. The fit to the data shown by the dashed line in the figures was about as good as was obtained in the ^{208}Pb(d, t)^{207}Pb study, but the deviations were large enough to raise serious doubts about the significance of small admixtures of different l values which might be obtained in the analysis. It was observed that the actual angular distributions measured for ^{208}Pb(d, t)^{207}Pb were identical with those measured in the present work when

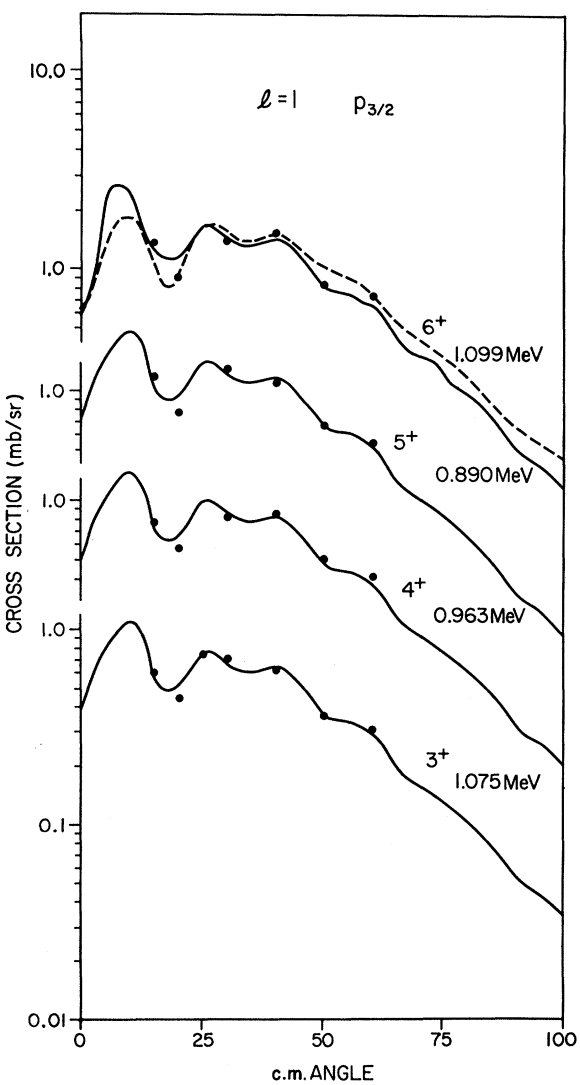

FIG. 8. Angular distributions for members of the $h_{9/2}\times p_{3/2}{}^{-1}$ multiplet observed in ^{209}Bi(d,t)^{208}Bi.

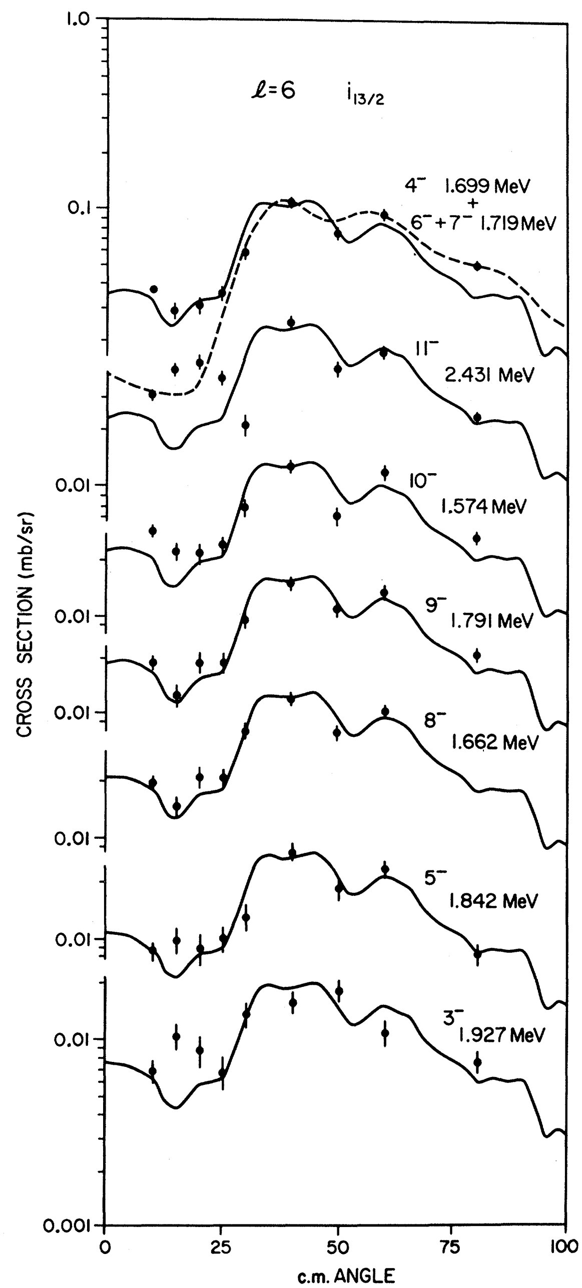

FIG. 9. Angular distributions for members of the $h_{9/2}\times i_{13/2}{}^{-1}$ multiplet observed in ^{209}Bi(d,t)^{208}Bi. Bars represent statistical uncertainties.

appropriate transitions were compared. For this reason, the computer program was also used to fit the present data, with the experimental angular distributions from ^{208}Pb used as the standard. The Q values for transitions to the neutron-hole states in ^{207}Pb are close to the centroid energies for corresponding multiplets. For multiplet members displaced more than 200 keV from the centroid Q dependence of the magnitude of the experimental cross section was estimated from DWBA calculation. The results of this comparison are shown as the solid curves on the figures. In making this comparison between the two sets of experimental data, it was found that most of the angular distributions could be fitted assuming that only a single l value contributed to the cross section. In fact, for only one transition (603 keV) is there a significant mixture of different l values required in order to fit the measured angular distribution.

The spectroscopic factors obtained by comparing the present results with those from the ^{208}Pb-$(d, t)^{207}$Pb reaction are shown in the fourth column of Table I. In carrying out this analysis, a single adjustment has been made to renormalise relative cross sections in the two measurements. A pre-

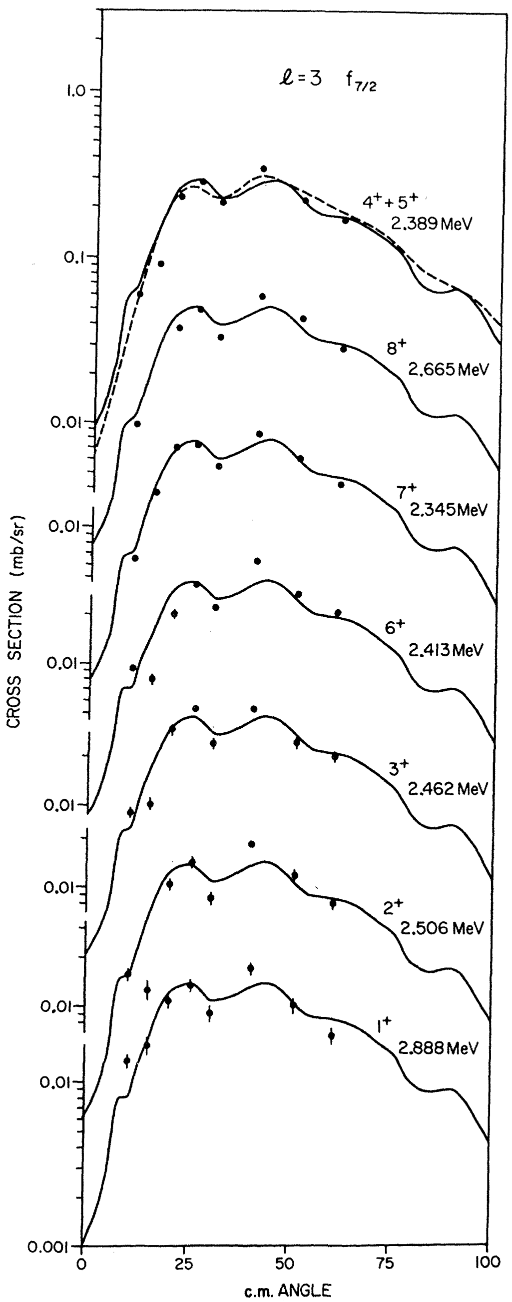

FIG. 10. Angular distributions for members of the $h_{9/2} \times f_{7/2}^{-1}$ multiplet observed in $^{209}\text{Bi}(d,t)^{208}\text{Bi}$.

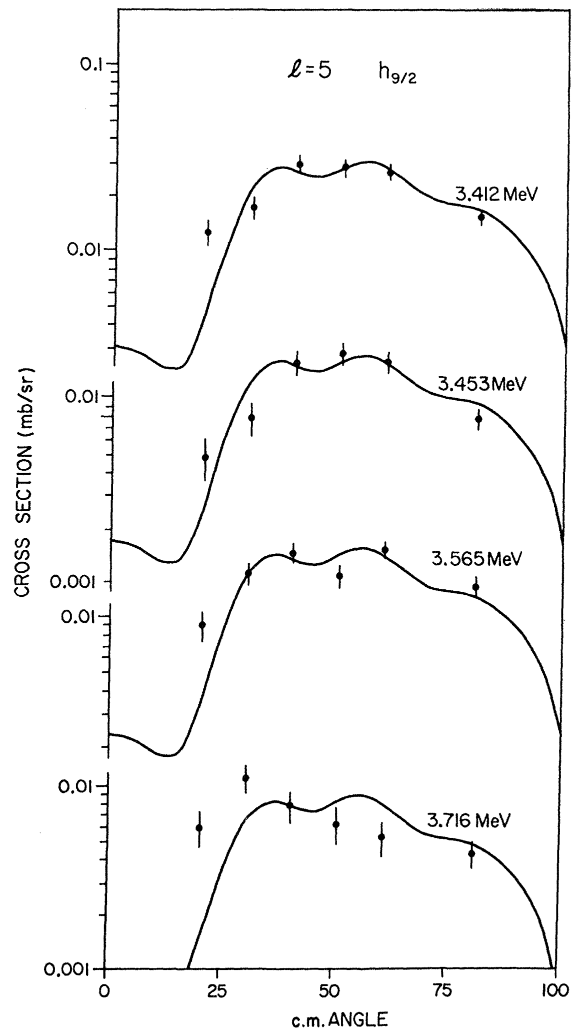

FIG. 11. Angular distributions for some of the stronger transitions identified as members of the $h_{9/2} \times h_{9/2}^{-1}$ multiplet. Spin assignments have not been made for these states.

liminary analysis showed that in the $h_{9/2}\times p_{1/2}^{-1}$, $h_{9/2}\times f_{5/2}$, and $h_{9/2}\times p_{3/2}^{-1}$ multiplets, the total strength was approximately 25% less than the sum-rule limit. It was assumed that this was caused by a discrepancy in the absolute cross sections reported for the two different measurements. It should be noted that such a discrepancy is within the combined uncertainties quoted for the two measurements. For calculating spectroscopic factors, we have arbitrarily reduced the measured cross sections for the ^{208}Pb reaction used as a standard, by a factor of 0.8.

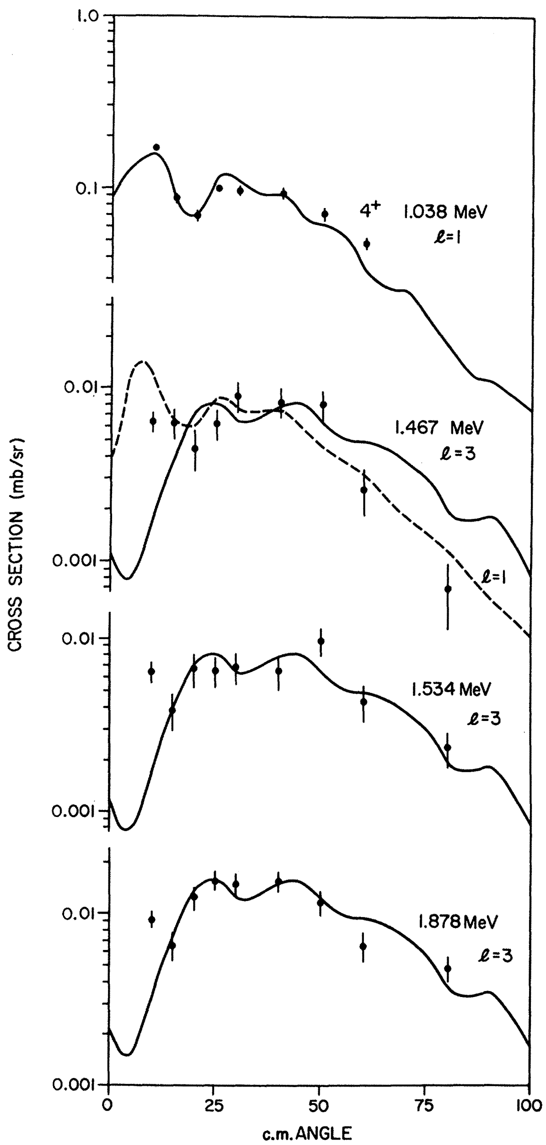

FIG. 12. Weak transitions observed in the ^{209}Bi(d,t)-^{208}Bi reaction. The curves are measured angular distributions for the ^{208}Pb$(d,t)^{207}$Pb reaction for $l=1$, $j=\frac{3}{2}$ or $l=3$, $j=\frac{7}{2}$.

Angular distributions were not measured for the (^{3}He, α) reaction, but the measured cross sections were useful in confirming the identification of some of the states observed in the (d,t) reaction. It has been found[13] that the (^{3}He, α) reaction has a large intrinsic cross section for high l transitions, which are quite weak in (d,t). In the present results, the (^{3}He, α) measurements provided clear identification of the $h_{9/2}\times i_{13/2}^{-1}$ 11^- state at 2.436 MeV which had been tentatively identified in the (d,t) results. The $h_{9/2}\times i_{13/2}^{-1}$ 2^- state had been tentatively assigned to a weak transition at 2910 keV in the (d,t) results. The (^{3}He, α) cross section showed that this could not be correct, and that the state is probably located at 2.716 keV. In addition, some of the weakly excited states seen in (d,t) between 3 and 4 MeV were excited relatively strongly in the (^{3}He, α) reaction. These have been identified in Table I as the $l=5$ transitions arising from $h_{9/2}$ pickup. Given this identification, strengths were determined from the (d,t) results even though angular distributions were often incomplete, due to poor resolution or high background at some angles.

The relatively strong transitions shown in Table I can be assigned as members of multiplets arising from coupling of a neutron hole to the $h_{9/2}$ proton, or a proton to a $p_{1/2}$ neutron hole. In addition, a number of weak transition were observed to states below 3 MeV for which no definite l assignments could be made. Though weak, these transitions are of considerable interest in indicating the extent of mixing of different particle-hole configurations.

The angular distributions measured in the ^{207}Pb-(^{3}He, d)^{208}Bi reactions are shown in Figs. 13–16. The error bars on the data points represent statistical uncertainties only. The uncertainty in the absolute cross sections is estimated to be about 20%. Missing data points at some angles represent regions where the group of interest was obscured by an impurity group. It is clear that at this energy the angular distributions do not determine l values very well. The most characteristic features appear at forward angles, where our data are the least accurate because of impurity groups.

Following the procedure used in analyzing the (d,t) results, spectroscopic strengths were obtained by comparison with measured cross sections for the ^{208}Pb(^{3}He, d)^{209}Bi reaction leading to the levels presumed to represent single-particle proton states in ^{209}Bi. Measurements of this latter reaction[14] were available at 30 MeV over the angular range from 20 to 45° and were fitted with DWBA calculations using the optical parameters obtained in fitting angular distributions observed in this reaction[15] at 51 MeV. The calculations used a zero-range local interaction and a radial cutoff of 8.8 fm. The fits to the shapes of the angular distributions were excellent. The absolute magnitudes of the DWBA cross sections were normalized to yield a spectroscopic factor of unity for each of the states in ^{209}Bi, and these were then used as "experimental" distorted-wave (DW) cross sections for comparison with the ^{207}Pb results. The relative normalization of the cross sections for ^{207}Pb and ^{208}Pb was provided by the observation of transitions arising from the 5.5% ^{208}Pb impurity in the ^{207}Pb target material. The final result of this procedure is to yield spectroscopic strengths for ^{207}Pb(^{3}He, d)^{208}Bi relative to those for ^{208}Pb(^{3}He, d)^{209}Bi with an over-all uncertainty of no more than 10%, provided that the transferred l and j values are known.

The assignments of l values presented some difficulties, as noted earlier. For the strong

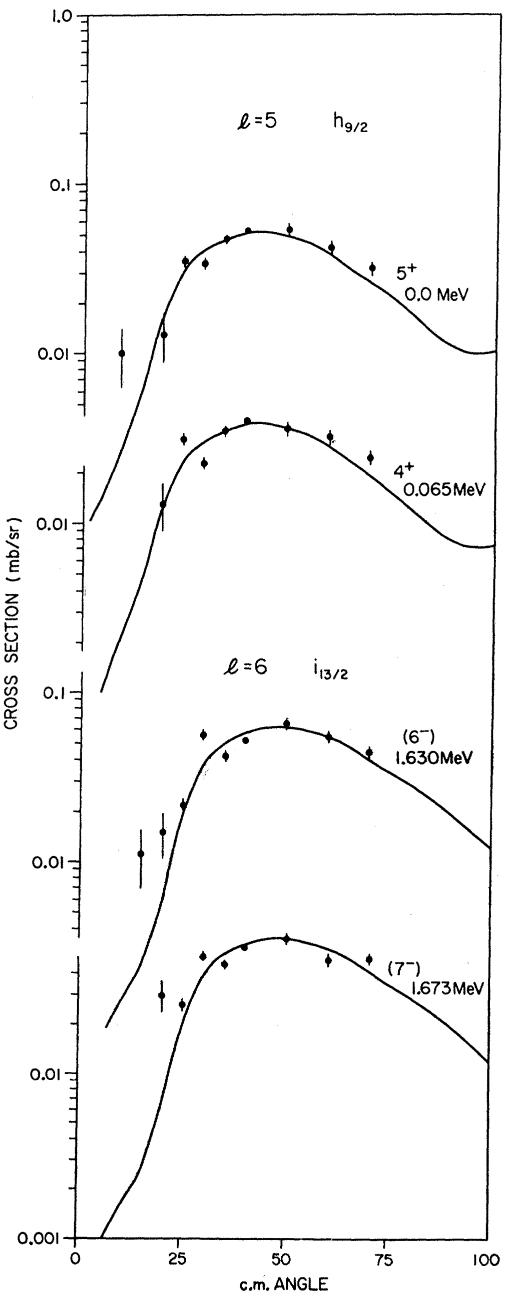

FIG. 13. Angular distributions for $h_{9/2} \times p_{1/2}{}^{-1}$ and $i_{13/2} \times p_{1/2}{}^{-1}$ doublets in the ^{207}Pb(^{3}He, d)^{208}Bi reaction. The solid curves are the result of DWBA calculations for the (l, j) values indicated.

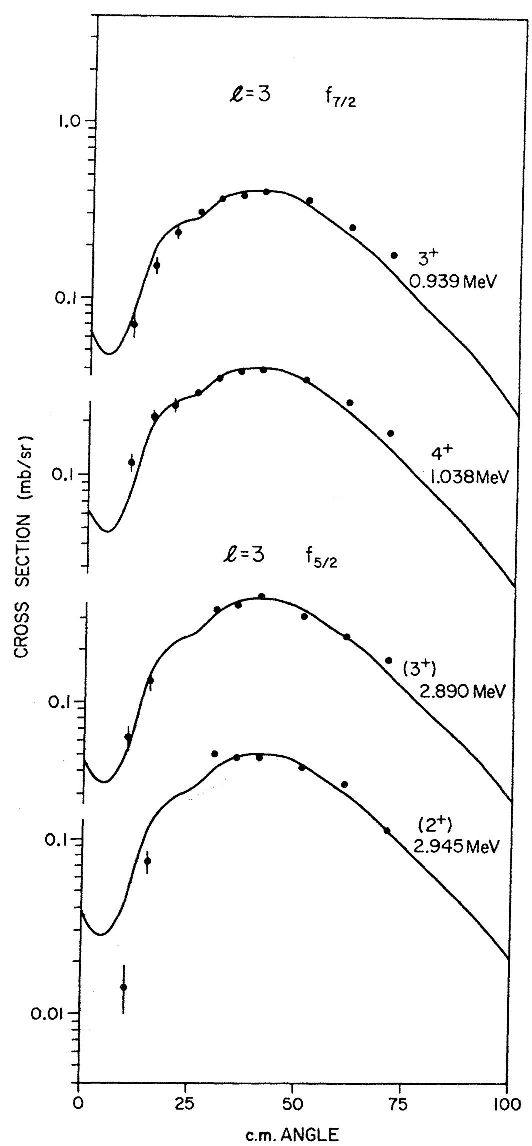

FIG. 14. Angular distributions for $f_{7/2} \times p_{1/2}{}^{-1}$ and $f_{5/2} \times p_{1/2}{}^{-1}$ doublets in the ^{207}Pb(^{3}He, d)^{208}Bi reaction. The solid curves are the result of DWBA calculations for the (l, j) values indicated.

doublets, values were assumed to be the same as for the corresponding single-particle state in ^{209}Bi. For other states, some help was provided by a comparison of (^{3}He, d) and (α, t) cross sections. The (α, t) reaction provides an enhancement of high l transitions in the same fashion as in the (^{3}He, α) reaction. Although statistical uncertainties are large, the present results suggest that the states at 603 and 631 keV were populated by $l=3$ rather than $l=5$ transitions. The state at 1720 keV is seen in (α, t) as well as (^{3}He, d) indicating that this is probably an $l=6$ transition arising by mixing of $h_{9/2}\times i_{13/2}^{-1}$ and $i_{13/2}\times p_{1/2}^{-1}$ configurations. In addition to the present (α, t) measurements, cross sections for this reaction have been measured at 42 MeV by Tickle.[16] His results confirm the $l=3$ assignment for transitions to the 603- and 631-keV states and indicate that the states above 3-MeV excitation arise from $l=1$ transitions with no more than 10% admixture of $l=3$ strength from the $f_{5/2}\times p_{1/2}^{-1}$ doublet near 2.9 MeV. Proposed assignments are given in column 6 of Table I. Measured spectroscopic strengths for specified and j values are also shown in column 8 of Table I.

DISCUSSION

In assigning states as particular members of a given multiplet, we were guided by the observation that admixtures are generally quite small. For pure particle-hole states we expect the cross section, in both stripping and pickup, to be proportional to the statistical factor $2J+1$, where J is the spin of the final state. In Table I, we have listed values of S for pickup and G $\{\equiv[(2J+1)/$

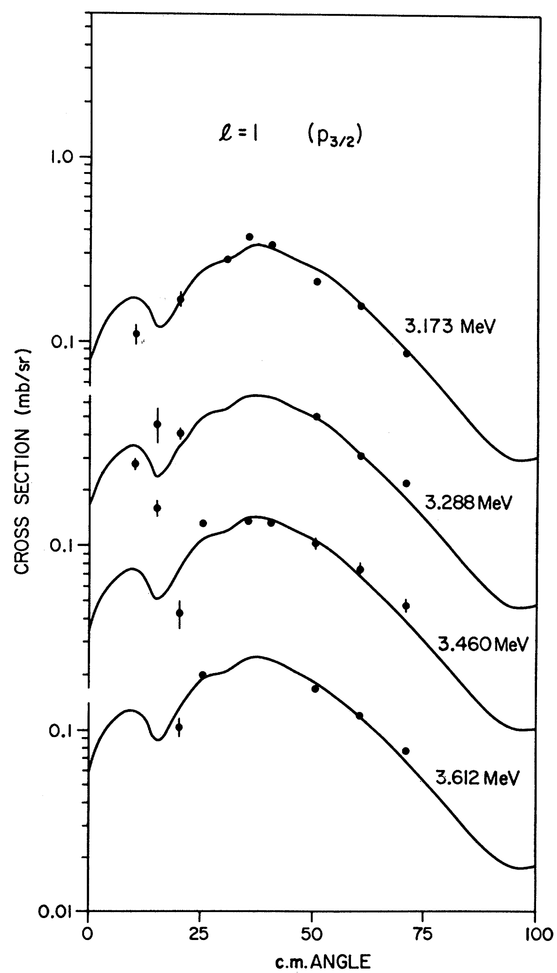

FIG. 15. Angular distributions for some strong $l=1$ transitions observed in ^{207}Pb(^{3}He, d)^{208}Bi.

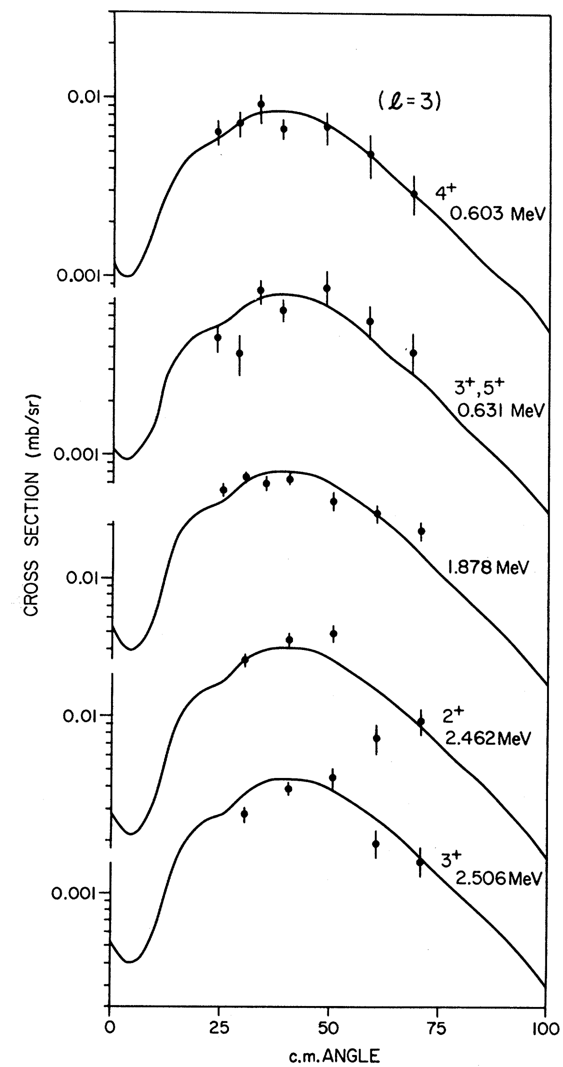

FIG. 16. Weak transition observed in ^{207}Pb(^{3}He, d)^{208}Bi. The curves are DWBA calculations for $l=3$, $j=\frac{7}{2}$ for the states at 0.603 and 0.630 MeV, $j=\frac{5}{2}$ for the others.

$(2I+1)]c^2S\}$ for stripping, which represent the cross section with the kinematic factors removed. These obey the sum rules: $\Sigma S=(2j+1)$ for filled orbits in pickup and $\Sigma G=(2j+1)$ for empty orbits in stripping, with j the angular momentum of the orbit. Table I also lists the values of S and G for pure particle-hole states of the J assigned, along with the observed values. It is seen that each of the strong transitions to states below 3-MeV excitation can be accounted for as a member of one of the expected multiplets. It is clear from Table I, however, that many J values are not uniquely determined from the cross sections and that an uncertainty of perhaps ±1 should be assigned to the values of J assigned on this basis, particularly for the higher J values where one depends on smaller percentage differences for an assignment. It is gratifying to observe that for the levels up to 652-keV excitation, plus the levels at 930, 1075, and 1574, independent determinations have been made which are consistent with our assignments.

While all the states strongly excited in either transfer reaction have been assigned to a member of a multiplet, in some cases unusually large cross sections suggest unresolved doublets. In particular, the 631-keV state is thought to be a 3^+-5^+ doublet (and has actually been resolved[17] into a 5^+ state at 628.4 keV and a 3^+ state at 632.4 keV); the 1719-keV level seems to be a 6^--7^- doublet with a partially resolved third level 1699 keV, and the 2389-keV state is assigned as a 4^+-5^+ doublet. Some small discrepancies should be noted. The strength observed for the 5^+ and 6^+ states of $h_{9/2}\times p_{3/2}^{-1}$ is about 15% high, and no explanation for this has been found. For most states of the $h_{9/2}\times i_{13/2}^{-1}$ multiplet, the observed strength is greater than predicted, but this can be ascribed to the fact that these transitions have small cross sections and are most subject to uncertainty from background or contaminant groups. Such effects are clearly seen at forward angles in the angular distribution for the 11^- state at 2431 keV. Here the cross section is increased by contributions from the strong $l=3$ transition at 2413 keV. The measured strength in the $i_{13/2}\times p_{1/2}^{-1}$ doublet is definitely less than predicted. This is quite significant when it is noted that the small intrinsic cross sections for these $l=6$ transitions makes them most likely to be perturbed by background or unresolved weak groups. The $f_{5/2}\times p_{1/2}^{-1}$ doublet shows more strength than predicted, slightly more than the sum-rule limit in fact. This probably indicates that unresolved states are contributing to these cross sections.

The results for the strongly excited states are summarized in Table II. Here we have listed the low-lying multiplets involving a proton coupled to a $p_{1/2}$ neutron hole which should be excited in stripping, or a neutron hole coupled to a $h_{9/2}$ proton, which should be excited in pickup. The experimental energy is the energy of the observed state assigned as the expected member. The calculated energy is the value predicted for the corresponding state in a detailed calculation by Kuo.[7] The agreement between theoretical and experimental energies is generally very good, although similar agreement is obtained with calculations involving any reasonable residual two-body interaction. The full justification for the proposed assignments lies in the fact that an observed state with a reasonable spectroscopic factor can be correlated with each predicted state, and that all

TABLE II. Particle-hole multiplets in ^{208}Bi.

Configuration	J^π	E	E (Ref. 7)
$h_{9/2}\times p_{1/2}^{-1}$	4^+	65	39
	5^+	0	0
$h_{9/2}\times f_{5/2}^{-1}$	2^+	930	1064
	3^+	631	686
	4^+	603	533
	5^+	631	595
	6^+	512	423
	7^+	652	738
$h_{9/2}\times p_{3/2}^{-1}$	3^+	1075	1073
	4^+	963	935
	5^+	890	808
	6^+	1099	1075
$f_{7/2}\times p_{1/2}^{-1}$	3^+	939	1016
	4^+	1038	1038
$i_{13/2}\times p_{1/2}^{-1}$	6^-	1630	1604
	7^-	1673	1634
$h_{9/2}\times i_{13/2}^{-1}$	2^-	2716	3176
	3^-	1927	1913
	4^-	1699	1972
	5^-	1842	1775
	6^-	1719	1783
	7^-	1719	1749
	8^-	1662	1693
	9^-	1791	1794
	10^-	1574	1626
	11^-	2431	2227
$f_{5/2}\times p_{1/2}^{-1}$	2^+	2945	2920
	3^+	2890	2903
$h_{9/2}\times f_{7/2}^{-1}$	1^+	2888	3005
	2^+	2506	2527
	3^+	2462	2453
	4^+	2389	2401
	5^+	2389	2346
	6^+	2413	2415
	7^+	2345	2292
	8^+	2665	2666

the strongly excited states up to about 3-MeV excitation can be accounted for this way.

Further confirmation for the correctness of this interpretation is shown in Table III. Here the measured strength sums for each multiplet have been taken from Table I. In addition, the sum-rule limits are shown for the pure multiplets. It is seen that the interpretation of the observed states as members of pure multiplets is an excellent approximation.

Another approach to the analysis of such multiplet structure is provided by a multipole decomposition. Such an analysis has been published for these results[18] and shows similar behavior of the multipole coefficients for each multiplet. A striking feature of this analysis was the fact that the average experimental coefficients were in excellent agreement with a δ-function force with spin exchange for the residual particle-hole interaction.[19] More sophisticated calculations, including the most recent one by Kuo[7] using a "realistic" interaction, provided equally good, but no better agreement than that using the simple δ-function force.

In addition to the multiplet components identified in Tables II and III, a number of relatively strong transitions have been observed to states between 3- and 4-MeV excitation. In the (d,t) measurements a number of $l=5$ transitions have been identified and are listed in Table I. These are presumably members of the $h_{9/2}\times h_{9/2}^{-1}$ multiplet. The measured strengths are probably all somewhat high since these states are weak, and often poorly resolved. The measured strengths are not consistent with any simple pure multiplet structure. It is rather striking, however, that the total $l=5$ strength is close to that observed in the ^{208}Pb$(d,t)^{207}$Pb reaction ($S_{9/2}=11$, compared with a sum-rule limit of 10) even though the $h_{9/2}^{-1}$ state in ^{207}Pb is known to be significantly fragmented.

In the $(^3$He$,d)$ results, the strong $l=1$ transitions proceed to seven states between 3 and 3.6 MeV. Four states should arise from the pure $p_{3/2}\times p_{1/2}^{-1}$ and $p_{1/2}\times p_{1/2}^{-1}$ multiplets. The 0^+ state would be expected to be fragmented with some strength going into the isobaric analog state at high excitation. The observed structure clearly indicates that considerably greater fragmentation of the expected states is now taking place. The total strength in the observed transitions ($\Sigma G_{3/2}=4.47$) is somewhat greater than the sum-rule limit of 4 for $p_{3/2}$ proton transfer. The difference may represent contributions from $p_{1/2}$ transfer, or possible unresolved states.

The calculations of Kuo predict that these three multiplets expected above 3 MeV should be rather strongly perturbed. Figure 17 shows the predicted strength distributions for levels excited by $l=5$ pickup. The observed results are also shown, and it is clear that there is definite disagreement between the experimental and theoretical results.

Since the state of highest spin of each multiplet comes at high excitation, it is probable that the state at 3565 keV is the 9^+ member of the multiplet. Other high-spin states occur at 3453 and 3412 keV, and the latter must be at least a doublet to account for the total strength. A more detailed assignment of states is probably not profitable.

Figure 18 shows Kuo's predictions for the distribution of $l=1$ strength in the $(^3$He$,d)$ reaction, along with the experimental results. The measurements extended slightly beyond 4-MeV exci-

TABLE III. Summed strengths in ^{208}Bi multiplets.

Configuration	$S_{\rm exp}$	$S_{\rm p-h}$	$G_{\rm exp}$	$G_{\rm p-h}$
$h_{9/2}\times p_{1/2}^{-1}$	1.94	2	9.6	10
$h_{9/2}\times f_{5/2}^{-1}$	5.73	6		
$h_{9/2}\times p_{3/2}^{-1}$	4.27	4		
$h_{9/2}\times i_{13/2}^{-1}$	15.30	14		
$h_{9/2}\times f_{7/2}^{-1}$	7.67	8		
$f_{7/2}\times p_{1/2}^{-1}$			8.3	8
$i_{13/2}\times p_{1/2}^{-1}$			11.4	14
$f_{5/2}\times p_{1/2}^{-1}$			6.2	6
$p_{3/2}\times p_{1/2}^{-1}$ $p_{1/2}\times p_{1/2}^{-1}$			4.47	6

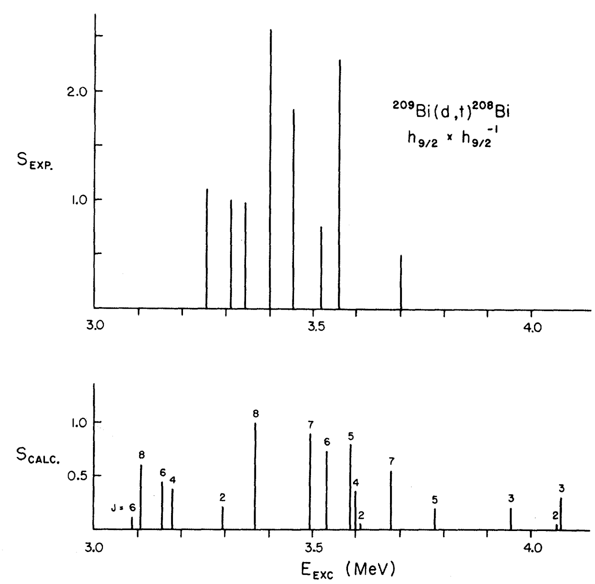

FIG. 17. Measured and predicted strength distributions for states excited by $l=5$ pickup. States with a predicted spectroscopic factor less than 0.05 are not shown.

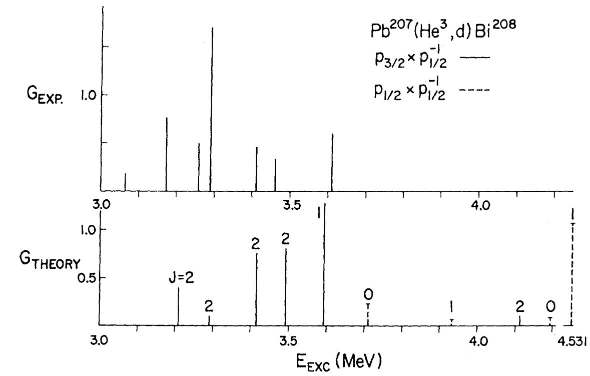

FIG. 18. Measured and predicted strength distributions for states excited in $l=1$ stripping. The experimental results do not discriminate between $j=\frac{1}{2}$ and $j=\frac{3}{2}$.

tation, but no strong transitions were observed and no quantitative analysis was attempted beyond 3.6 MeV. No transition is seen corresponding to the $p_{1/2}\times p_{1/2}^{-1}$ 1^+ state. For the other states, the degree of fragmentation is comparable in both the measured and calculated distributions, but detailed agreement is not very good. No spins are known for the observed states. It is probable that the level at 3288 keV is the $p_{3/2}\times p_{1/2}^{-1}$ 1^+ state, since this is predicted to be rather pure, while the 2^+ state is fragmented.

Off-diagonal matrix elements giving rise to configuration mixing may prove to be the best test of model calculations. The observed weak transitions which may be ascribed to relatively small admixtures between low-lying configurations are shown in Table IV. In most cases, transfer l values could not be determined from the measured angular distributions. Those shown were determined by comparison of cross sections in different reactions, as discussed in connection with Table I, or are chosen to be consistent with model predictions. In some cases, spectroscopic factors are shown for two different possible l values.

The experimental results in Table IV are compared there with transition strengths calculated from Kuo's wave functions. In addition to the mixing shown in Table IV, the calculations predict mixtures of a few percent in intensity amongst levels of the multiplets which are strongly excited in the (d,t) reaction. These mixtures would in most cases be undetectable in present measurements. For example, the $f_{5/2}$ and $f_{7/2}$ components of the 2^+ state predicted at 1064 keV would both contribute an $l=3$ transition to the total cross section. In fact the predicted mixtures are all below the limit of detectability in the present measurement except for the $h_{9/2}\times p_{3/2}^{-1}$ mixture in the 4^+ state of $h_{9/2}\times f_{5/2}^{-1}$ which is predicted at 533 keV and observed at 603 keV. The reciprocal mixing of $h_{9/2}\times f_{5/2}^{-1}$ in the 4^+ state of $h_{9/2}\times p_{3/2}^{-1}$

TABLE IV. Weak transitions to states in ^{208}Bi.

E_{exc}	(d,t) l	S_{exp}	S_{calc} (Ref. 7)	$(^3He,d)$ l	G_{exp}	G_{calc} (Ref. 7)	Dominant configuration [a] p n^{-1}	J^{π} [a]	E_{calc} (Ref. 7)
603	1	0.10	0.13	5	0.76	0.09	$h_{9/2}\times f_{5/2}$	4^+	533
	3	0.70	0.77	or 3	0.09	0.00			
631		strong [b]		5	0.83	0.11	$h_{9/2}\times f_{5/2}$	5^+	595
1038	1	0.11	0.002		strong [b]		$f_{7/2}\times p_{5/2}$	4^+	1038
1467	3	0.005	0.000				$(f_{7/2}\times f_{5/2})$		
	or 1	0.002	0.000						
1534	3	0.005	0.000				$(f_{7/2}\times f_{5/2})$		
	or 1	0.002	0.000						
1565				3	0.18	0.14	$(f_{7/2}\times f_{5/2})$	(4^+)	1431
1606		weak							
1630	6	0.1	0.0		strong [b]		$i_{13/2}\times p_{1/2}$	6^-	
1719		strong [b]		6	2.1	0.0	$h_{9/2}\times i_{13/2}$	6^-+7^-	
1734		weak	0.04				$(f_{7/2}\times f_{5/2})$	(3^+)	1893
1806				3	0.31	0.27	$(f_{7/2}\times p_{3/2})$	(4^+)	1806
1878	3	0.01	0.02				$(f_{7/2}\times p_{3/2})$	(5^+)	1986
	or 1	0.004	0.000						
1885				3	0.30	0.15	$(f_{7/2}\times p_{3/2})$	(2^+)	2130
2132				1	0.08	0.08	$(f_{7/2}\times f_{5/2})$	(1^+)	2089
2462		strong [b]		3	0.17	0.04	$h_{9/2}\times f_{7/2}$	3^+	
2506		strong [b]		3	0.11	0.00	$h_{9/2}\times f_{7/2}$	2^+	
2688		weak							

[a]Assignments in parentheses based on Kuo calculation only.

[b]Particle-hole multiplet component. See Table II.

should be barely detectable, although the present results indicate a pure $l=3$ transition to the observed state at 963 keV.

The state at 1038 keV is strongly excited in the ^{207}Pb(^{3}He, d)^{208}Bi reaction and is probably the 4^+ state of $f_{7/2}\times p_{1/2}{}^{-1}$. The observed mixture with $h_{9/2}\times p_{3/2}{}^{-1}$ is much larger than predicted. Part of this may be explained by the fact that the two states involved in fact fall closer in energy than in Kuo's calculations, but a substantial discrepancy remains. The 3^+ state of $f_{7/2}\times p_{1/2}{}^{-1}$ is predicted to be observed in pickup, but the transition would be obscured by the strong transition to the 2^+ state of $h_{9/2}\times f_{5/2}{}^{-1}$ which is only 9 keV away. The states observed at 1734 and 1878 keV are probably the predicted 3^+ and 5^+ states of $f_{7/2}\times p_{3/2}{}^{-1}$, a configuration which should not be strongly excited in either stripping or pickup to levels in ^{208}Bi. The levels at 1467, 1534, and 1606 keV are probably states of the $f_{7/2}\times f_{5/2}{}^{-1}$ configuration, which are predicted in this energy region. Mixing is not predicted by the calculation, but is in fact observed to be very small. The state at 1630 keV is probably one component of the $i_{13/2}\times p_{1/2}{}^{-1}$ doublet seen in ^{207}Pb(^{3}He, d)^{208}Bi. The observed transition was very weak, but because of the small intrinsic cross section for $l=6$ transitions, it could represent 10 to 20% of the strength of a pure state and is in disagreement with calculations.

In the (^{3}He, d) reaction, the states at 603 and 631 keV are predicted to be excited by an $l=5$ transition. A comparison of (^{3}He, d) and (α, t) cross sections for these states indicated $l=3$ as the more likely assignment. The measured strength assuming $l=5$ transitions is much greater than the model prediction and would be inconsistent with the strength observed in transitions to the ground-state doublet. It is more likely that these states are excited by small mixtures of the $f_{7/2}\times p_{1/2}{}^{-1}$ states.

Predicted transitions to the 4^+ and 3^+ states of $h_{9/2}\times p_{3/2}{}^{-1}$ would be obscured by the tails of the strong transitions to the 3^+ and 4^+ members of $f_{7/2}\times p_{1/2}{}^{-1}$ which occur at slightly lower energy in each case. The state at 1719 keV is seen in both the (^{3}He, d) and (α, t) reactions, and probably is one of the $h_{9/2}\times i_{13/2}{}^{-1}$ states at 1719 keV excited by mixing with $i_{13/2}\times p_{1/2}{}^{-1}$. The observed cross section is small, but it represents an appreciable amount of strength for an $l=6$ transition. The extent of the mixing indicated by the observation of this state is consistent with the measured strength for the $i_{13/2}\times p_{1/2}{}^{-1}$ doublet. It is much greater than predicted. No unique correspondence can be suggested between the three states seen at 1806, 1878, and 2132 keV and the five even-parity states predicted in this region, but it is striking that the number of states and their strength are roughly comparable. The states at 2462 and 2560 keV, 3^+ and 2^+ of $h_{9/2}\times f_{7/2}{}^{-1}$, are excited somewhat more strongly than predicted, probably by mixing with the states of $f_{5/2}\times p_{1/2}{}^{-1}$.

A comparison between observed and predicted mixing is shown in Fig. 19 for the (^{3}He, d) results.

To summarize, we may say that the observed mixing is in rough qualitative agreement with that predicted by Kuo's calculation, but detailed quantitative agreement is not obtained.

CONCLUSION

The low-lying states of ^{208}Bi have been recognized for some time as examples of a relatively simple particle-hole structure. The present results provide a quantitative assessment of the

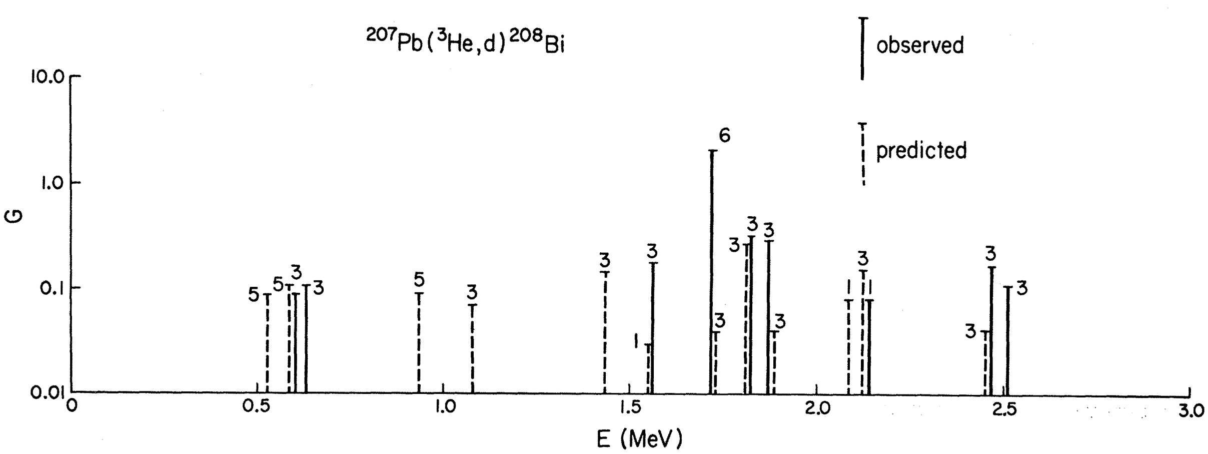

FIG. 19. Measured and predicted strengths for states weakly excited by configuration mixing in stripping. The numbers on each state indicate the transfer l value.

validity of such a model and extend earlier results to higher excitation energies.

By comparison with results of measurements of the ^{208}Pb(^{3}He, d)^{209}Bi and ^{208}Pb(d,t)^{207}Pb reactions, it is found that the total spectroscopic strength in each multiplet is close to that found in the corresponding single-particle or single-hole state. Such a comparison is not subject to most of the usual uncertainties of a DWBA analysis. The mixing between different configurations is shown to be small, generally less than a few percent in intensity for states below 3 MeV.

A comparison with a model calculation shows excellent agreement for the strong transitions, provided spin assignments based on observed strengths are correct. Only for the states of the $i_{13/2}\times p_{1/2}{}^{-1}$ doublet does there appear to be a significant discrepancy. For weak transitions, the identification of the observed states is often conjectural. The general qualitative magnitude of the mixing agrees with predictions, but a number of significant discrepancies have been noted.

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*Presently at Argonne National Laboratory, Argonne, Illinois 64039, and University of Chicago, Chicago, Illinois 60637.

†Present address: Physics Department, Stanford University, Stanford, California 94305.

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