Laser-Induced Multiphoton Processes in e^--p Scattering

Helmut Brehme*
Institute of Theoretical Physics, University Freiburg, Freiburg, Germany
(Received 26 March 1970; revised manuscript received 4 December 1970)

In order to obtain information about the role of multiphoton processes in recently reported laser-induced nuclear fusion, the cross section for simultaneous net absorption of n photons by a system of two point charges has been calculated numerically for e^--p scattering in a ruby laser beam at an incident relative energy of 100 eV. The results show that at laser intensities involved in fusion experiments multiphoton processes apparently play an important role, partly contributing to the same order within certain n intervals.

I. INTRODUCTION

What is the role of multiphoton processes in recently reported laser-induced nuclear fusion?[1] As transfer of laser energy to a plasma is mainly due to net photon absorption by electron-ion systems undergoing free-free transitions, useful information will be obtained from the calculation of such processes. Nuclear fusion has been achieved at photon fluxes of about $\rho(\omega)\approx 10^{32}-10^{33}$ (cm^{-2} sec^{-1}), while fluxes up to $\rho(\omega)\approx 10^{36}$ have already been reported.[2]

Clearly, no perturbation calculation in the laser field is possible in this (the fusion) region. We therefore employ the following method, the principle of which has been used by several authors[3-6] in connection with related problems, and which for the present case is due to Bunkin and Fedorov[7] and Arnett,[8] while a detailed discussion, covering the whole field of interest, has been given by Eberly.[9] Starting from the exact solution of Schrödinger's equation for two mutually noninteracting point charges in a space-independent monochromatic electromagnetic field (dipole approximation) and treating the Coulomb interaction as a first-order perturbation, the expression for the cross section for simultaneous net absorption of n photons by a system of two point charges is derived. Next, numerical values are given for the case of e^--p scattering in a ruby laser beam, assuming a special experimental situation. At photon fluxes above $\rho(\omega)\approx 10^{34}$, the curves for the cross sections reveal a strong oscillatory dependence on the laser intensity, which leads to crossing among curves for different n. As this situation becomes more distinct the higher the incident particle energy and the smaller the frequency of light, it is not due to one of the approximations performed, but must be considered to be inherent in the model chosen for the description of the scattering process (namely, using asymptotic states which describe particles in a classical light field).

To get more precise information, the above-mentioned derivation of the cross-section formula is redone, starting now from the corresponding solutions of the relativistic wave equation for the equivalent one-body problem in a space-dependent electromagnetic field. Using the new formula for $n=1$ in the region $10^{32}\leqslant\rho(\omega)\leqslant 6\times 10^{34}$, the qualitative behavior of the curve is found to remain unchanged. On the other hand, for higher photon fluxes and large n, considerable modification is to be expected. Unfortunately, numerical computations will be very difficult in this region. Nevertheless, as can be shown by a short inspection of the microwave case, oscillation and curve crossing are already found in regions where relativistic effects are completely negligible.

II. DERIVATION OF CROSS SECTIONS; NONRELATIVISTIC CASE

Since the center-of-mass motion separates out in the dipole approximation, one easily finds the Schrödinger equation for the relative motion of the mutually noninteracting point charges (Z_1e, m_1), (Z_2e, m_2) in a space-independent electromagnetic field $\vec{A}(t)$ to be

$$\frac{[\vec{p}-\bar{Z}e\vec{A}(t)/c]^2}{2\mu}\psi(\vec{r},t)=i\hbar\frac{d}{dt}\psi(\vec{r},t)\,, \tag{1}$$

where

$$\mu=m_1m_2/(m_1+m_2)\,, \tag{2}$$

$$\bar{Z}=(m_2Z_1-m_1Z_2)/(m_1+m_2)\,, \tag{3}$$

$$\vec{p}=-i\hbar\vec{\nabla}_{\vec{r}}\,, \tag{4}$$

and $\vec{r}$ is the relative coordinate. The electromagnetic field is assumed to be linearly polarized:

$$\vec{A}(t)=2\vec{A}_0\cos\omega t\,, \tag{5}$$

$$\vec{A}_0=[2\pi\hbar c\rho(\omega)/\omega]^{1/2}\vec{e}_\omega\,, \tag{6}$$

where $\vec{e}_\omega$ is the polarization vector, ω is the angular frequency, and $\rho(\omega)$ is the photon flux. The well-known solution of (1) reads

$$\psi_i(\vec{r}, t) = \exp[i(\vec{k}_i\vec{r} - \omega_i t)] \exp\left[-\frac{i}{\hbar}\int_{t_0}^{t} d\tau I_i(\tau)\right], \tag{7}$$

where

$$I_i(\tau) = [\vec{p}_i - \bar{Z}e\vec{A}(\tau)/c]^2/2\mu, \tag{8}$$

$$\omega_i = \hbar^2 k_i^2/2\mu, \tag{9}$$

and

$$\vec{p}_i = (m_2\vec{p}_{1,i} - m_1\vec{p}_{2,i})/(m_1 + m_2) \tag{10}$$

is the incident relative momentum before the electromagnetic field (2) is switched on at $t = t_0$.

Perturbing (7) to first order in the Coulomb interaction

$$V(r) = Z_1 Z_2 e^2/r \tag{11}$$

and projecting on a state

$$\psi_f(\vec{r}, t) = \exp[i(\vec{k}_f\vec{r} - \omega_f t)] \exp\left[-\frac{i}{\hbar}\int_{t_0}^{t} d\tau I_f(\tau)\right], \tag{12}$$

one finds the scattering amplitude to be

$$S_{if} = \sum_n S_{if}^{(n)}, \tag{13}$$

where

$$S_{if}^{(n)} = -i(2\pi/\hbar)\delta(\omega_f - \omega_i + n\omega) J_n(x_{if}) K_{if} \tag{14}$$

and

$$K_{if} = (Z_1 Z_2 e^2)[4\pi/(\vec{k}_f - \vec{k}_i)^2], \tag{15}$$

$$x_{if} = 2(\bar{Z}e/\mu c)(\vec{k}_f - \vec{k}_i)(\vec{A}_0/\omega), \tag{16}$$

while J_n is the Bessel function of integer order n. It should be noted that the term $\propto A^2$ (canceling out during calculation) is of no physical significance in this approximation and consequently does not appear in (14). Relativistic corrections due to this term are discussed in Sec. V.

Going over to the cross section in the usual way, one has

$$\sigma = \sum_n \sigma^{\pm(n)}, \tag{17}$$

where

$$\sigma^{\pm(n)} = 4\left(\frac{Z_1 Z_2}{a_0}\right)^2 \frac{k_f^{\pm(n)}}{k_i} \int d\Omega \frac{J_n^2(x_{if})}{(\vec{k}_f^{\pm(n)} - \vec{k}_i)^2}, \tag{18}$$

$$a_0 = \hbar^2/(\mu e^2), \tag{19}$$

$$k_f^{\pm(n)} = |\vec{k}_f^{\pm(n)}| = [k_i^2 \mp n(2\mu c/\hbar)k_\omega]^{1/2}, \tag{20}$$

and $d\Omega$ is the differential solid angle in the c.m. system.

The expression (18) is the total cross section for simultaneous net absorption (−) or net emission (+) of n photons. While one can expand the Bessel functions for sufficiently low laser intensities (ω, k_i fixed), allowing further analytical calculation[7] of (18), this is no longer possible in the fusion region where a numerical calculation becomes necessary, results of which are given in the next section.

III. NUMERICAL RESULTS

Formula (18) has been calculated for e^--p scattering at a relative energy of 100 eV in a ruby laser beam ($\hbar\omega = 1.8$ eV), assuming a special experimental situation which is shown in Fig. 1. The results for $n = 1$ to 10, and $n = 10^2$, 5×10^2, 10^3 are shown in Fig. 2 (abscissa: photon flux in cm^{-2} sec^{-1}, logarithmic scale; ordinate: total cross section $\sigma^{-(n)}$ in cm^2, logarithmic scale).

As is clear, in the breakdown region $10^{28} \lesssim \rho(\omega) \lesssim 10^{31}$ net processes of order $n > 1$ become probable, contributing to the same order within certain n intervals. This is in qualitative agreement with present theory based on calculations by Bebb and Gold,[10] according to which electrons, triggering optical breakdown of gases, are generated by

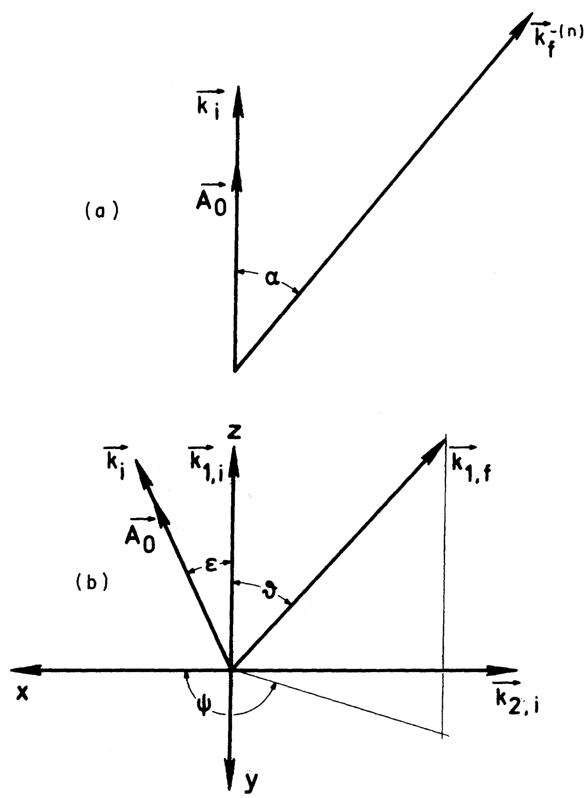

FIG. 1. The special experimental situation chosen for the calculation: (a) situation in the c.m. system: $d\Omega_{\text{c.m.}} = -2\pi d(\cos\alpha)$, $(\vec{A}_0 \| \vec{k}_i)$; (b) corresponding situation in the lab system: $d\Omega_{\text{lab}} = -d(\cos\theta)d\psi$ (observing particle 1) $[\vec{k}_{1,i} \perp \vec{k}_{2,i}$; vectors $\vec{A}_0$, $\vec{k}_i$, $\vec{k}_{1,i}$, $\vec{k}_{2,i}$ coplanar; $\tan\epsilon = (m_1 k_{2,i})/(m_2 k_{1,i})]$.

multiphoton ionization. When the laser intensity is increased, this situation becomes much more distinct; in the fusion region a 100-, 500-, 1000-photon process is probable.

By inspection of (18) and (20) it is seen that the strong oscillations which lead to curve crossing are due to the resonant behavior of the Born denominator at $\alpha = 0$ in connection with the well-known properties of the Bessel functions. It is shown in the next section that this feature is not, in principle, due to the approximations involved in the derivation of (18).

IV. DERIVATION OF CROSS SECTIONS; RELATIVISTIC CASE

Dirac's equation for the spinor particle $(Z_1 e, m_1)$ in an electromagnetic field $\vec{A}(\vec{r}, t)$ reads

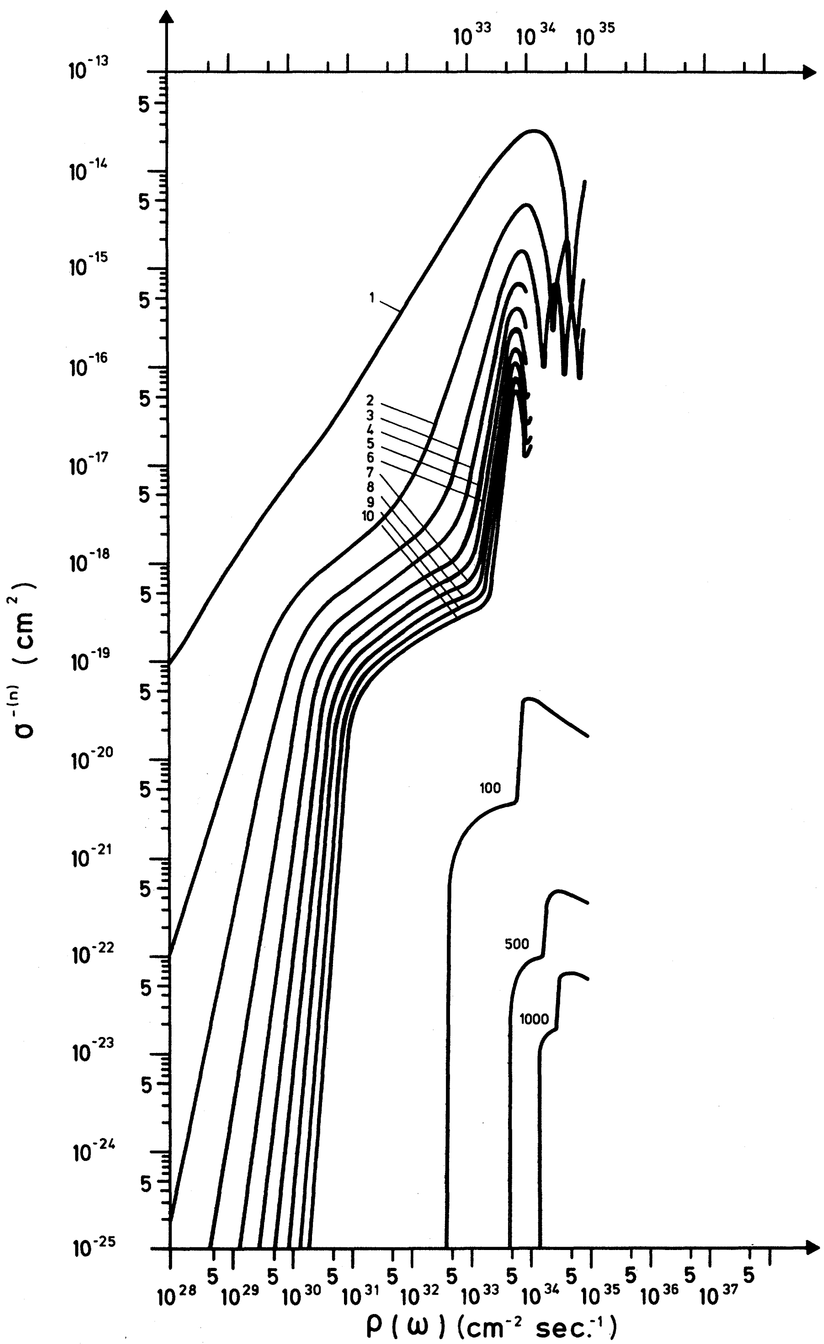

FIG. 2. Total cross section $\sigma^{-(n)}$ in cm^2 for simultaneous net absorption of n ruby-laser photons ($\hbar\omega = 1.8$ eV) by an e^{-}-p system in a 100-eV scattering state (geometrics given in Fig. 1) versus photon flux $\rho(\omega)$ in $\text{cm}^{-2}\,\text{sec}^{-1}$.

$$\left\{\beta m_1 c^2 + c\vec{\alpha}\cdot\left[\vec{p} - \frac{Z_1 e}{c}\vec{A}(\vec{r},t)\right]\right\}\psi(\vec{r},t) = i\hbar\frac{d}{dt}\psi(\vec{r},t). \tag{21}$$

Choosing a light wave traveling along the x axis,

$$\vec{A}(\tau) = 2\vec{A}_0\cos\omega\tau, \tag{22}$$

$$\tau = x/c - t, \tag{23}$$

one obtains the well-known Volkov solutions[4,11]

$$\psi_{i,\lambda}(\vec{r},t) = [1 + g_i(\tau)]\exp i(\vec{k}_i\vec{r} - \omega_i t) \times\exp\left[-\frac{i}{\hbar}\int_{\tau_0}^{\tau} d\tau' I_i(\tau')\right]u_\lambda, \tag{24}$$

where

$$\omega_i = E_i/\hbar = (m_1^2c^4 + c^2p_i^2)^{1/2}/\hbar, \tag{25}$$

$$\lambda_i = E_i - cp_{x,i}, \tag{26}$$

$$I_i(\tau) = (1/2\lambda_i)[2Z_1ec\vec{p}_i\vec{A}(\tau) - Z_1^2e^2A^2(\tau)], \tag{27}$$

and $g_i(\tau)$ is a matrix function due to the coupling between spin and electromagnetic field. Neglecting spin effects completely, one arrives at the corresponding solutions of the Klein-Gordon equation:

$$\Phi_i(\vec{r},t) = \exp[i(\vec{k}_i\vec{r} - \omega_i t)]\exp\left[-\frac{i}{\hbar}\int_{\tau_0}^{\tau} d\tau' I_i(\tau')\right]. \tag{28}$$

Performing the steps equivalent to those in the nonrelativistic case, one finds the scattering amplitude to be

$$S_{if} = \sum_n S_{if}^{(n)}, \tag{29}$$

where

$$S_{if}^{(n)} = -i(2\pi/\hbar)\delta(\omega_{if}^{(n)})\left(\sum_j \alpha_{j,n-2j}\right)K_{if} \tag{30}$$

and the corresponding cross section

$$\sigma = \sum_n \sigma^{\pm(n)}, \tag{31}$$

where now

$$\sigma^{\pm(n)} = 4\left(\frac{Z_1Z_2}{a_0}\right)^2\frac{1}{k_i}\int d\Omega\left(\sum_j \alpha_{j,n-2j}\right)^2\frac{k_f^{\pm(n)}}{(\vec{k}_f^{\pm(n)} - \vec{k}_i \pm n\vec{k}_\omega)^2}. \tag{32}$$

Here

$$\alpha_{j,n-2j} = J_j(x_L)J_{n-2j}(x_{if}), \tag{33}$$

$$k_f^{\pm(n)} = [(\omega_f^{\pm}/c)^2 - (m_1c/\hbar)^2]^{1/2}, \tag{34}$$

$$\omega_f^{\pm} = \omega_i \mp n\omega \pm \omega_L, \tag{35}$$

$$\omega_L = \frac{Z_1^2e^2A_0^2}{\hbar}\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_f}\right), \tag{36}$$

$$x_L = \omega_L/\omega, \tag{37}$$

$$x_{if} = \left|2Z_1ec\left(\frac{\vec{k}_f^{\pm(n)}}{\lambda_f} - \frac{\vec{k}_i}{\lambda_i}\right)\cdot\left(\frac{\vec{A}_0}{\omega}\right)\right|. \tag{38}$$

V. DISCUSSION OF CORRECTIONS; CONCLUSION

Equation (32) is to be compared with (18). When $x_L \ll 1$, (32) reduces to (18) apart from the modification of the Born denominator, which is due to allowing for the photon's momentum. For the situation in Sec. III, one has (see Fig. 2 for $n = 1$) $(x_{if})_{\max} \approx 3.83$ at $\rho(\omega) \approx 5.9\times 10^{34}$ cm^{-2} sec^{-1}, where $x_L \approx 6\times 10^{-3}$, from which it follows that the curve remains practically unchanged in this region. This has been verified by numerical recomputation, using Eq. (32), retaining only the first five products of Bessel functions ($j = 0, 1, 2, 3, 4$) which is sufficient up to $\rho(\omega) \approx 6\times 10^{34}$. Of course, for higher photon fluxes and large n, Eq. (32) will change the curves considerably. In general, however, one can always choose ω, A_0 such that $(x_{if})_{\max}$ is in the oscillation region of the Bessel functions while $x_L \approx 0$. [For instance, in the microwave case $\omega \approx 10^{10}$ (sec^{-1}), $\rho(\omega) \approx 10^{20}$ (cm^{-2} sec^{-1}), where A_0 is about a factor 10^{-5} smaller than the corresponding value for $\omega \approx 10^{15}$ and $\rho(\omega) \approx 10^{35}$, while the ratio A_0/ω is the same in both cases.]

In conclusion, as oscillation and curve crossing become more distinct the higher the incident particle energy (that is, the better the Born approximation) and the smaller the frequency of light (that is, the better the dipole approximation), it is evident that these results are inherent in the model chosen for description of the scattering process in a very strong electromagnetic field. In general, multiphoton processes can be assumed to play an important role in heating a plasma by strong laser fields, which recently led to nuclear fusion.

VI. ACKNOWLEDGMENTS

The author is grateful to Dr. H. Krüger and Dr. W. Demtröder for valuable discussions of this topic, as well as to Dr. F. Wille for his assistance with numerical calculations.

*Present address: Messerschmitt-Bölkow-Blohm GmbH, Space Division, Muenchen, Germany.

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