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Study of the Low-Lying Excited States of ^{29}Al. III. Mean Lifetimes and Interpretation*

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Lifetimes or, alternatively, limits for lifetimes of low-lying levels in ^{29}Al have been obtained employing the Doppler-shift-attenuation method. The levels were excited by the ^{27}Al$(t,p)^{29}$Al reaction and measurements were made of the γ-ray spectrum in coincidence with reaction protons. Measured mean lifetimes, or limits, [$E_x(\tau_m)$, in MeV(psec)] for the first six levels of ^{29}Al are: 1.402($3.3^{+2.2}_{-1.0}$), 1.762(<0.05), 2.228(<0.08), 2.875(<0.15), 3.071(<0.05), and 3.191(0.21 ± 0.10). A partial level scheme of ^{29}Al for excitation energy $E_x \leq 4.5$ MeV based on previously reported information is presented. Properties of these levels are interpreted according to the Nilsson model on the basis of (1) a direct-reaction investigation and (2) γ-ray angular-correlation investigations as previously reported. The properties of ^{29}Al are shown to be consistent with a prolate deformation characterized by $\eta \approx +3$.

I. INTRODUCTION

The present series of measurements to determine lifetimes of the excited states of ^{29}Al was undertaken to supplement previous work in this laboratory on the same nucleus.[1-3] This work consisted of (1) a direct-reaction investigation and subsequent interpretation of the differential cross-section measurements,[1] (2) γ-ray branching-ratio determinations and measurements of γ-ray angular correlations in a collinear geometry,[2] and (3) investigation of the delayed γ-rays following the β decay of ^{29}Al.[3] Since completion of the first phases of our investigation, three publications have appeared pertaining to the structure of the levels of this nucleus.[4,5] These are the work of Kean and his collaborators[4] and that of Hirko.[5] Both used the ^{26}Mg$(\alpha,p\gamma)^{29}$Al reaction and made measurements of γ-ray branchings and γ-ray angular correlations for the low-lying levels of ^{29}Al. Their results and those of Ref. 2 are essentially in agreement as regards spin assignments. However, the reported branchings of the 3.19- and 3.65-MeV levels disagree. Hirko[5] reports a (9 ± 6)% branch from the 3.19 → 1.40 transition. This branch is not seen in the other investigations. The branching of the 3.65-MeV level is reported by Hirko[5] as (83 ± 12)% to the ground state and (17 ± 12)% to the 1.76-MeV state. Jones, Becker, and McDonald[2] report these branches as (56 ± 3)% and (44 ± 3)%, respectively. The investigations of Kean *et al.*[4] did not include this level. No ready explanation for this discrepancy is available. Weighted averages of the branching-ratio determinations of Refs. 2, 4, and 5 are given in Fig. 1. To arrive at the quoted values, due allowance was made for a possible ±15% error in the results of Kean *et al.*[4] as quoted in their paper.

Kean *et al.*[4] speculate as to the applicability of an intermediate-coupling model and also a strong-coupling model to ^{29}Al and conclude that the information presently available on the nucleus does not warrant detailed comparison with nuclear models. The work of Hirko,[5] on the other hand, includes a Nilsson-model[6] description with assignment of low-lying levels to rotational bands based on this model. Hirko's assignments are, however, based entirely on level position, spin and parity values, and γ-ray branching modes. In addition to the information used by Hirko, both spectroscopic factors and electromagnetic transition rates are extremely important tests of the Nilsson model, and it was our intention to complete the present series of investigations of ^{29}Al by measuring the lifetimes of the low-lying states and then make a detailed comparison with the Nilsson model, incorporating all the available information on the nucleus. The lifetime experiments were cut short, however, since the characteristics of the Ge(Li) γ-ray detector were radically altered by radiation damage arising from the strong $^{27}Al(t,n)^{29}Si$ reaction. As a consequence the number of counts obtained in each full-energy absorption peak had a large percentage statistical error. (Typically, as shown in Sec. II, peak centroid shifts were determined to $\simeq$8% accuracy.) Measured lifetimes were obtained only for the 1.402- and 3.191-MeV levels. This experiment is described in Sec. II. Comparison of the properties of ^{29}Al with the Nilsson model is then presented in Sec. III, using the spectroscopic information extracted from the direct-reaction investigation,[1] the branching-ratio and angular-correlation information[2,4,5] and the presently obtained lifetime information.

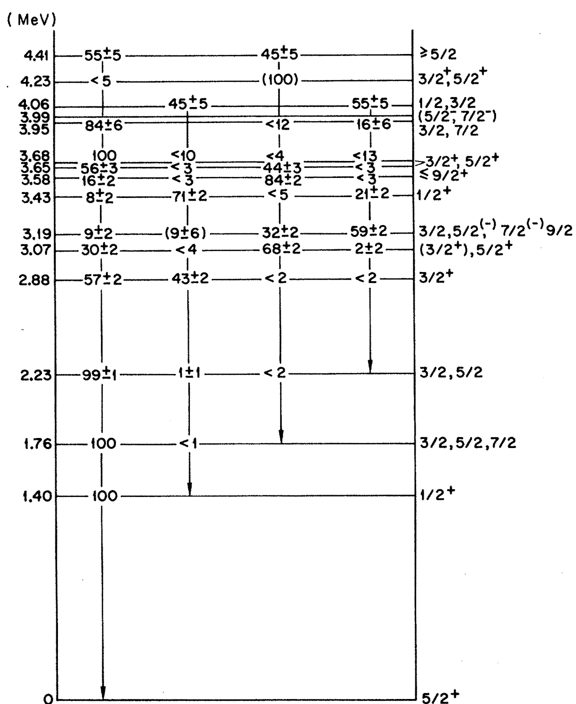

FIG. 1. Partial energy level diagram for ^{29}Al. The γ-ray branching ratios shown are weighted averages from Refs. 2, 4, and 5, except for the 3.95-MeV level where the branching ratios of Ref. 2 are quoted. The (9 ± 6)% branch shown for the 3.19 → 1.40 transition is from Ref. 5 only. Spin and parity assignments are from Refs. 1, 2, and 5.

II. EXPERIMENT

The Doppler-shift-attenuation method[7] was used to measure, or place limits on, lifetimes of the levels in ^{29}Al with excitation energy $E_x \leq 3.19$ MeV. The levels were populated by the $^{27}Al(t,p)^{29}Al$ reaction, and time coincidence measurements were made of reaction protons and γ rays. Tritons of incident energy 2.54 MeV accelerated by the Lockheed 3.5-MeV Van de Graaff generator were used to bombard a target consisting of a 3.3-mg/cm^2-thick foil of aluminum. The reaction protons were detected in an annular silicon semiconductor counter placed at 170 ± 5° to the incident beam. A foil of aluminum in front of the counter stopped elastically scattered tritons from entering the detector. γ radiation was detected in a 25-cc Ge(Li) spectrometer system. Signal processing of both particle and γ-ray pulses was carried out with conventional modular electronics and dual 4096-channel analog-to-digital converters. Data were processed, as previously described[8,9] with the aid of an SEL 810A computer and were stored on magnetic tape.

The attenuated Doppler shifts were obtained by recording particle–γ-ray two-parameter spectra at γ-ray angles of 90 and 0° with respect to the incident particle direction, and subsequently analyzing the data by obtaining from the magnetic-tape records γ-ray spectra in coincidence with the various particle groups. The thick target precluded unambiguous identification of particle groups but previous knowledge of the γ-ray branchings[2,4,5] ensured correct identification of the γ-ray emitting level. The positions of γ-ray full-absorption peaks were obtained by fitting a smooth background below the peaks and then calculating the centroid of the remainder. The maximum possible shift of the γ rays was determined from a knowledge of the beam energy, reaction Q value,[10] and angle of detection of the protons. The measured attenuation of the Doppler shifts is shown in Table I in terms of the quantity $F(\tau)$, defined as

$$F(\tau) = \frac{\text{Observed shift of } \gamma \text{ ray}}{\text{Maximum shift of } \gamma \text{ ray}}. \quad (1)$$

There have been many detailed accounts recently in the literature of the theoretical calculation of $F(\tau)$.[11,12] We will here merely give the salient features with the parameters used in the present calculation.

The stopping of the excited ^{29}Al ions due to collisions with the electrons in the ^{27}Al target is treated as follows[11]:

$$\left(\frac{dE}{dx}\right)_{\text{electronic}} = K_e\left(\frac{v}{v_0}\right) - K_3\left(\frac{v}{v_0}\right)^3. \qquad (2)$$

Here, K_e and K_3 are constants, v is the ion velocity and $v_0 = c/137$. K_e is evaluated according to the formulism of Lindhard and Scharff[13] but adjusted to correspond to the experimental measurements of Ormrod, MacDonald, and Duckworth.[14] These latter authors report measurements on the stopping of ions with $Z \leq 11$ in aluminum and $Z \leq 19$ in carbon. The stopping cross section from these data exhibits very similar oscillatory structure about the Lindhard-Scharff[13] estimate for both C and Al stoppers, and the value of K_e was decreased from the Lindhard estimate for the stopping of ^{29}Al in aluminum by 30% in accord with the same percentage decrease observed for the stopping of ^{27}Al in carbon. The value of K_e thus obtained was $K_e = 2.01 \pm 0.20$ keV cm^2/μg. K_3 is a constant that accounts for any deviation from linearity of the de/dx versus v curve. This is only important for high incident velocities, and K_3 was found to be insignificant in this calculation.

The nuclear stopping is taken directly from Lindhard, Scharff, and Schiøtt[15] as quoted by Blaugrund.[16] (We have used the expression for $d\epsilon/d\rho$ quoted for $\epsilon > 1$ in the caption to Fig. 1 in Ref. 16 and assumed $d\epsilon/d\rho$ constant for $\epsilon \leq 1$.) Large-angle scattering of the ^{29}Al ions resulting from collisions with ^{27}Al nuclei is taken into consideration according to the formulism of Blaugrund[16] [Eqs. (15) and (15a)], with the over-all magnitude of the nuclear stopping being multiplied by 0.7 following the further work of Blaugrund *et al.*[17] (Ref. 17, Table I; data for ^{24}Na ions stopping in ^{27}Al). Accordingly, we find $\tau_m(1.40) = 3.3^{+2.2}_{-1.0} \times 10^{-12}$ sec, and $\tau_m(3.19) = 2.1 \pm 1.0 \times 10^{-13}$ sec, together with the other limits as set down in the last column of Table I. The large errors of these two mean lifetimes are mainly due to the limited statistics of the data.

Figures 2 and 3 show predicted line shapes obtained for the γ rays originating from the levels at 1.40 and 3.19 MeV. No lifetime information was extracted from the line-shape analysis, and the fits merely show that they are consistent with the lifetimes deduced from the centroid analysis. To generate these line shapes, one further parameter is introduced into the analysis and that is the full width at half maximum of a Gaussian curve that is assumed to represent the shape of the unshifted line.

TABLE I. Lifetimes of excited states of ^{29}Al.

Level (MeV)	$F(\tau)$ measured	τ_m deduced (sec)
1.40	0.17 ± 0.06	$(3.3^{+2.2}_{-1.0}) \times 10^{-12}$
1.76	1.05 ± 0.06	$<5 \times 10^{-14}$
2.23	0.98 ± 0.06	$<8 \times 10^{-14}$
2.88	0.93 ± 0.08	$<15 \times 10^{-14}$
3.07	1.08 ± 0.08	$<5 \times 10^{-14}$
3.19	0.81 ± 0.08	$(2.1 \pm 1.0) \times 10^{-13}$

III. INTERPRETATION OF ^{29}Al IN TERMS OF THE NILSSON MODEL

The strong-coupling model of Nilsson[6] has been used with good success to explain nuclear structure for odd-mass nuclei in the beginning of the nuclear $2s$-$1d$ shell. We present an application of this model to ^{29}Al in this section. To begin with, the spin and parity assignments in Al^{29} are discussed; levels without firm spin assignments are given an assignment suggested by the data for the purposes of discussion and as a guide to further experiments.

Of the low-lying excited states of ^{29}Al, definite spin-parity assignments have been made for the ground state ($J^\pi = \frac{5}{2}^+$),[18,19] 1.40-MeV state ($J^\pi = \frac{1}{2}^+$), 2.88-MeV state ($J^\pi = \frac{3}{2}^+$), and the 3.43-MeV state ($J^\pi = \frac{1}{2}^+$).[1,2] The 3.07-, 3.65-, and 3.68-MeV states are restricted to have $J^\pi = \frac{3}{2}^+$ or $\frac{5}{2}^+$.[1] The spin of the 3.07-MeV state is most probably $J^\pi = \frac{5}{2}^+$ from the observation of an $L = 0$ transfer to the state in the ^{27}Al$(t,p)^{29}$Al reaction investigated at 3.34-MeV incident triton energy.[5] A $J = \frac{5}{2}$ assignment is also

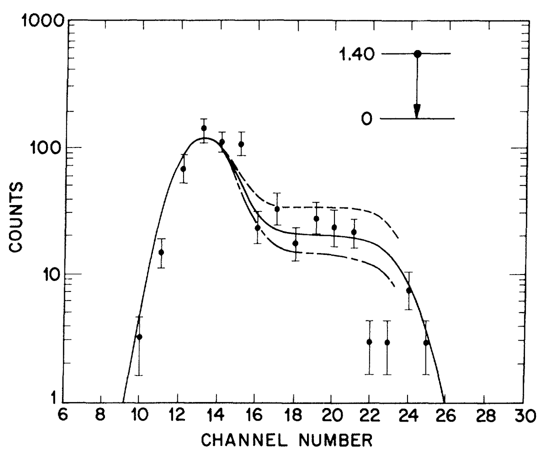

FIG. 2. Pulse-height distribution for the full-energy absorption peak of the 1.40→0-MeV transition measured at $\theta_\gamma = 0°$. Background has been subtracted from the data. The mean lifetime of this level deduced from a centroid analysis is $\tau_m = 3.3^{+2.2}_{-1.0}$ psec. The continuous line shows the predicted line shape for $\tau_m = 3$ psec, while the dashed line and dot-dash line show the predictions for $\tau_m = 2$ and 4 psec, respectively.

suggested from arguments[1] based on the strength of the observed cross section in the ^{30}Si(t, α)^{29}Al direct-reaction investigation. However, definite spin-parity assignments have not been made for the 1.76-, 2.23-, 3.19-, 3.58-MeV, and higher states.

The γ-ray angular-correlation data on the 1.76 → 0 and 2.23 → 0 transitions[2,4,5] do not distinguish between various spin possibilities ranging from $J=\frac{3}{2}$ to $J=\frac{7}{2}$ for both these states. The best evidence for the spins of these states comes from analyses of relative total cross-section data based on a statistical compound-nucleus theory; this results in $J=\frac{7}{2}$ for the 1.76-MeV state, $J=\frac{3}{2}$ for the 2.23-MeV state, and $J=\frac{9}{2}$ for the 3.58-MeV state,[1] and these spins will be assumed for this discussion. Referring to Fig. 1 and Table I, these spin assignments are consistent with the γ-ray branching and lifetime data. The $J=\frac{7}{2}$ assignment for the 2.23-MeV state is ruled out by the observed γ-ray decay of the $J^\pi=\frac{1}{2}^+$ state at 3.43 MeV, which decays to both the $J^\pi=\frac{1}{2}^+$ state at 1.40 MeV and the 2.23-MeV state.[2,5] The 3.58-MeV state has γ-ray branches 86% to the 1.76-MeV state and 14% to the $J^\pi=\frac{5}{2}^+$ ground state, which is not inconsistent with its proposed spin. Also if $J(2.23)=\frac{3}{2}$ and $J(1.76)=\frac{7}{2}$ then the observed nonzero quadrupole/dipole multipole mixing ratios for the ground-state transitions suggest that the states have even parity. The 3.58-MeV state has even parity from the $L=2$ transfer to the state observed in the ^{27}Al(t,p)^{29}Al reaction by Jaffe.[19]

It was suggested in Ref. 1 that the 3.19-MeV state might have odd parity, since the proton pick-up angular distribution in the ^{30}Si(t, α)^{29}Al direct-reaction investigation, although weak, is best described by $l=3$. Thus the 3.19-MeV level would have $J^\pi=\frac{5}{2}^-$ or $\frac{7}{2}^-$. If the spin of the 2.23-MeV state is $J=\frac{3}{2}$, however, then the γ-ray angular-correlation data of Hirko[5] requires $J=\frac{5}{2}$ for the 3.19-MeV level whereas the work of Refs. 2 and 4 allows $J=\frac{3}{2}$ or $\frac{5}{2}$. Hirko reports a $(9\pm 6)\%$ γ-ray branch from the 3.19-MeV state to the $J^\pi=\frac{1}{2}^+$ level at 1.40 MeV. The evidence on the presence of this γ-ray branch is conflicting. In Ref. 2 an upper limit of 6% is placed on this transition, while Kean *et al.*[4] also do not report the presence of the transition. If the γ-ray branch is present, it tends to negate the possible negative-parity assignment, as this would imply the 3.19 → 1.40 transition has multipole mixing $M2/E1$. A way to check the parity assignment would be remeasurement of the L value of the transferred neutrons in the ^{27}Al(t,p)^{29}Al reaction to this state. This distribution, as obtained by Hirko,[5] does not show an unambiguous pattern. For the present arguments, in view of the conflicting evidence on the properties of this state, we take the spin of the level to be $J=\frac{3}{2}$ or $\frac{5}{2}$ and make no assumptions as regards the parity.

The spins and parities given to the low-lying states of ^{29}Al that are to be considered in the rest of this paper are summarized in Fig. 4.

A. Ground-State Band

For odd-mass nuclei with either Z or $N=13$, the ground state is identified with Nilsson orbit 5, $(J^\pi, K)=(\frac{5}{2}^+, \frac{5}{2})$, and the nuclear deformation is

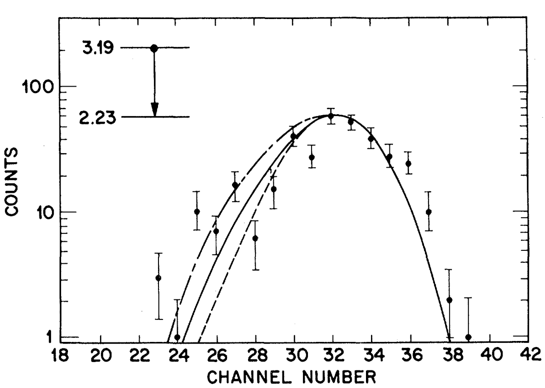

FIG. 3. Partial pulse-height distribution obtained at $\theta_\gamma=0°$ for the full-energy absorption peak of the 3.19 → 2.23 transition. Background has been subtracted from the data. The mean lifetime of this level deduced from the centroid analysis is $\tau_m=0.21\pm 0.10$ psec. The continuous line shows the predicted line shape for $\tau_m=0.3$ psec, while the dashed and dot-dash line show the predictions for 0.2 and 0.4 psec, respectively.

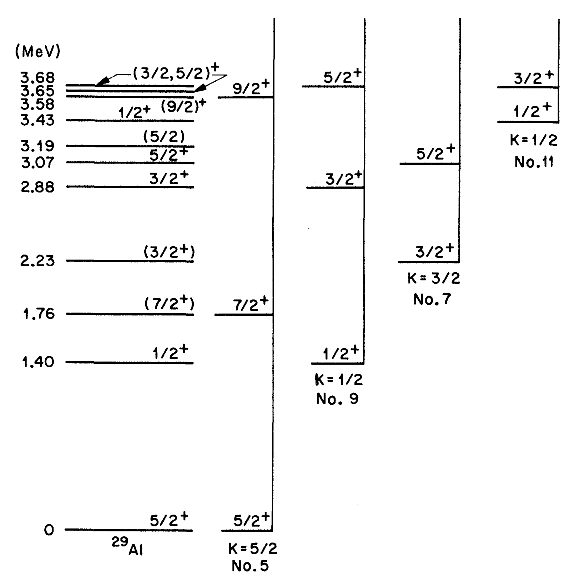

FIG. 4. Partial energy level diagram for ^{29}Al, with the assignment of states to bands based on the appropriately numbered Nilsson orbits. Ascription of spins to the states is discussed in the text.

prolate.[20] In ^{29}Al the $J^{\pi}=\frac{7}{2}^{+}$ and $\frac{9}{2}^{+}$ states at 1.76 and 3.58 MeV would be the next two members of this band. To see if this is a reasonable description, we begin by comparing this proposed band with similar states in ^{25}Al and ^{27}Al. Figure 5 presents the partial level scheme for ^{29}Al, as well as for ^{25}Al and ^{27}Al. ^{25}Al has many of the properties of a collective nucleus,[20] while ^{27}Al has not been successfully interpreted in terms of this model.[21] When the excitation energy of the levels shown in Fig. 5 are fitted to the expansion

$$E(J)=AJ(J+1)+BJ^2(J+1)^2 \tag{3}$$

the values of A and B included in Fig. 5 are obtained. A small value of B, a term that represents a vibration-rotation interaction and is a measure of the deviation of the levels from those of a pure rotator, is needed to describe the levels in both ^{25}Al and ^{29}Al whereas for ^{27}Al the value of B is much larger. The values of $A=\hbar^2/2I$, where I is the moment of inertia of the nucleus, imply the permanent ^{29}Al deformation is slightly less than that of ^{25}Al. The value of A for ^{27}Al is seen to be in bad agreement with the other two isotopes and is part of the evidence used to conclude that the properties of ^{27}Al are not entirely consistent with a strong-coupling collective model.

It would then seem advantageous to make more detailed attempts to apply the unified model of Nilsson to ^{29}Al as has been done successfully[20] for ^{25}Al. We further note that a $J^{\pi}=\frac{5}{2}^{+}$ ground state implies a prolate deformation for ^{29}Al with $0\leq\eta\leq 4$ for a value of the spin-orbit coupling term $\chi=0.05$. For $\chi=0.10$, $0\leq\eta\leq+3$. Evidence suggests[22] that for odd-A nuclei with either Z or $N=13$, $\chi=0.08$ is the best value, which would imply $0\leq\eta\leq+3.5$ for the present case. The possibility of the $\frac{5}{2}^{+}$ state being the lowest-energy state of $K=\frac{1}{2}^{+}$ band based on a higher orbit, 9 or 11, is ruled out by the large decoupling parameter a needed for such a situation.[20]

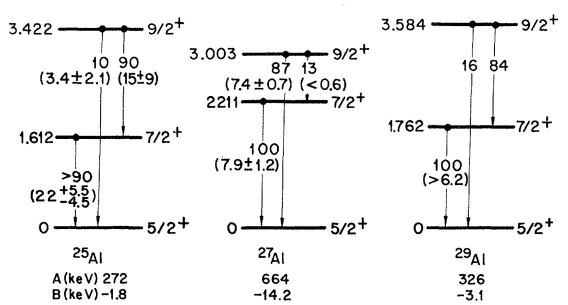

FIG. 5. Properties of the identified $K=\frac{5}{2}$ bands in ^{25}Al and ^{27}Al compared with the suggested members of the same band in ^{29}Al. The bracketed numbers shown underneath the branching ratios are the $E2$ transition rates in Weisskopf units. The values of A and B shown are deduced from fitting the positions of the energy levels to the expansion $E(J)=AJ(J+1)+BJ^2(J+1)^2$. Energy levels are in MeV.

A prolate deformation for this band also can be inferred from the observed sign of the $E2/M1$ mixing ratio for the $J=\frac{7}{2}$ to $J=\frac{5}{2}$ transition using the realtionship $\mathrm{sgn}[\delta(E2/M1)]=-\mathrm{sgn}[(g_K-g_R)/Q_0]$.[23] The quantities g_K, g_R, and Q_0 denote the intrinsic gyromagnetic ratio, the collective gyromagnetic ratio, and the quadrupole moment. For a proton in orbit 5, $g_K=1.916$ (independent of the deformation), while $g_R=0.45$ (estimated as Z/A). Thus g_K-g_R is always positive, and the sign of the deformation, viz., the sign of Q_0 is opposite to the sign of the observed mixing ratio. Since for $J(1.76)=\frac{7}{2}$, $\delta(E2/M1)=-0.19\pm0.09$ from Ref. 2, the ground-state band has prolate deformation.

One of the characteristics of a rotational band is that of enhanced $E2$ transitions between members of the band. This is indeed the situation observed for ^{25}Al as shown in Fig. 5. The properties of ^{29}Al are also not inconsistent with this picture although the evidence is as yet rather tenuous, only a lifetime limit being available for the $J^{\pi}=\frac{7}{2}^{+}$ state and no information being known about the lifetime of the $J^{\pi}=\frac{9}{2}^{+}$ state. The similarity of the known properties of this band in ^{29}Al compared with the similar band[10] in ^{25}Al [$|\delta(E2/M1)_{\frac{7}{2}\to\frac{5}{2}}|=0.19$ for ^{25}Al, 0.19 for ^{29}Al, the branchings of the $J=\frac{9}{2}$ levels, and also the similarity of the static properties, as shown in Fig. 5] is good reason for applying a collective model to ^{29}Al.

It is interesting to note one reason why ^{25}Al and ^{29}Al are similar but have different properties from ^{27}Al. As seen from Fig. 6, the last proton in all these isotopes occupies the $K^{\pi}=\frac{5}{2}^{+}$ orbit No. 5. The last pair of neutrons in ^{25}Al is in orbit No. 7 ($K=\frac{3}{2}$), while in ^{29}Al they are in orbit No. 9 ($K=\frac{1}{2}$). The energy gap between the closed neutron orbits No. 7 and No. 9 and the next available orbits is similar for positive deformations $0<\eta<+6$ and for masses 25–29. This gap is predicted as a few MeV. Thus, as the effects of neutron excitations are not observed in the low-energy spectrum of the mass-25 nuclei, they are not expected to manifest themselves in mass-29 nuclei either. The situation in ^{27}Al is different and neutron excitations between orbit No. 5 ($K=\frac{5}{2}$) and No. 9 ($K=\frac{1}{2}$) are more likely because of the proximity of the band-head energies. Therefore the wave functions of ^{27}Al can probably not be considered as proton single-particle states but would have two components demonstrating the probability of obtaining zero-coupled neutron pairs in orbits No. 5 and No. 9, respectively. The transition rates would differ from those predicted by the Nilsson-model wave functions and also the position of the energy levels would be perturbed by the interaction of the nearby states. It would be instructive to further investigate these possibilities with a view to a consistent explanation of the properties of ^{27}Al in terms of the Nilsson model.

We turn now to a discussion of the $Si^{30}(t, \alpha)Al^{29}$ pickup reaction. The nucleus ^{30}Si is in a mass region where the nuclear deformation is generally considered as changing from prolate to oblate.[24] Therefore, the order in which the Nilsson orbitals are filled could be different from that of ^{29}Al, which we have shown to possess prolate properties. In the following sections we assume an oblate deformation for the ^{30}Si ground state, but note that the arguments do not alter even if ^{30}Si were a prolate nucleus, since the only difference would be the reversal of the order of filling orbits No. 8 and No. 11. The Nilsson orbitals in ^{30}Si are filled up to orbit No. 9 for neutrons and orbits No. 5, No. 6, and No. 7 are filled with protons (see Fig. 6). To interpret the pickup data, as summarized in Table II, we propose that in the ground state of ^{30}Si there is a probability $\pi V_9{}^2$ of the last two protons being excited to orbit No. 9. (Also for later discussion we propose a probability $\pi V_8{}^2$ and $\pi V_{11}{}^2$ of the last two protons being excited to orbits No. 8 and No. 11, respectively.) The notation here is that π or ν denotes a proton or neutron, respectively, and V_n denotes occupation of a Nilsson orbital No. n. This occupation of higher proton orbitals (No. 9, No. 8, and No. 11) is indeed necessary for

TABLE II. Observed spectroscopic factors in the $^{30}Si(t,\alpha)^{29}Al$ reaction.

Level (MeV)	l value	Probable J^π	Spectroscopic factor
0	2	$\frac{5}{2}^+$	3.26
1.40	0	$\frac{1}{2}^+$	0.31
2.23	2	$\frac{3}{2}^+$	≤0.05
2.88	2	$\frac{3}{2}^+$	0.32
3.07	2	$\frac{5}{2}^+$	1.32
3.19	2	$\frac{5}{2}^+$	0.07
	3	$\frac{5}{2}^-$	0.15
3.43	0	$\frac{1}{2}^+$	0.11
3.65	2	$\frac{3}{2}^+, \frac{5}{2}^+$	0.17
3.68	2	$\frac{3}{2}^+, \frac{5}{2}^+$	0.10

any interpretation of the pickup experiment. Thus, finally, it can be noted that the $^{30}Si(t, \alpha)^{29}Al$ pickup data[1] is consistent with the predicted properties of the ground-state band. The model predicts that all the pickup strength should go into the ground state and none into the higher $J^\pi = \frac{7}{2}^+$ and $\frac{9}{2}^+$ states. This is realized in the experiment. The absolute magnitude of the pickup strength is discussed in the next section.

B. $K = \frac{3}{2}$ Hole-State Band

In the $^{30}Si(t, \alpha)^{29}Al$ direct-reaction investigation,[1] the strongest transition observed in the spectrum is that to the ground state with a spectroscopic

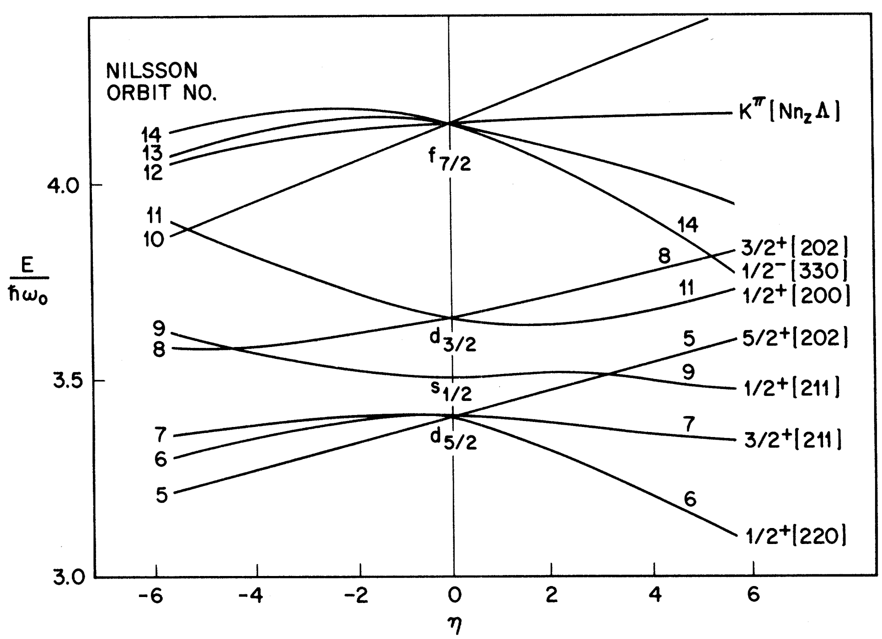

FIG. 6. Energies of a single particle in a deformed oscillator potential as a function of the deformation parameter η. N, n_z, and Λ are the asymptotic quantum numbers describing each orbit as numbered. For a detailed treatment and derivation of this figure see Ref. 6.

strength $S = 3.26$. One other state with a large pickup strength is observed – the 3.07-MeV state with $S = 1.32$. This large strength is consistent only with this state being a hole state. Possible bands on which this state can be based are the $K^\pi = \frac{3}{2}^+$ (orbit No. 7) and $K^\pi = \frac{1}{2}^+$ (orbit No. 6) bands. Orbit No. 7 is energetically preferred and it is believed in the mass-25 nuclei that the hole state from this orbit lies at 4.2 MeV.[25] The only possible contender for the $J^\pi = \frac{3}{2}^+$ band-head member is the state at 2.23 MeV (moment of inertia, $A = 167$ keV). The 2.88-MeV state is ruled out by the large moment of inertia obtained if this state were the band head ($A = 32$ keV) and also by the pickup strength to this state – an order of magnitude larger than predicted for the $J^\pi = \frac{3}{2}^+$ state of orbit No. 7. In fact, for $0 < \eta < +6$, more than 90% of the spectroscopic strength should go to the $J^\pi = \frac{5}{2}^+$ member of this band. Therefore, the negligible pickup strength observed to the 2.23-MeV level is consistent with it being the head of a hole-state band with $K = \frac{3}{2}$.

The sum of spectroscopic strengths to members of the ground-state band and the $K = \frac{3}{2}$ hole-state band should be the same and have the value $\sum S = 2$ if the bands are completely filled. (As shown below this is not entirely true.) For the ground-state band $\sum S = 3.26$ and for the $K = \frac{3}{2}$ band $\sum S = 1.37$. A possible explanation for this discrepancy could lie in mixing of the $K = \frac{3}{2}$ and $K = \frac{5}{2}$ bands by the Coriolis force.[26] Thus the total summed spectroscopic strength is a more realistic check on the nature of the states. This sum, 4.58, is near the value of 4.0 expected theoretically. The slightly higher value could come from mixing with other bands, notably the $K = \frac{1}{2}$ band (orbit No. 6). The discrepancy, however, is like that observed in the pickup reactions leading to ^{25}Mg, where a similar anomalously high strength to the ground state is observed with a correspondingly much weaker transition to the $(J^\pi, K) = (\frac{5}{2}^+, \frac{3}{2})$ state.[25]

It is instructive to calculate the total pickup strength for the $J = \frac{5}{2}$ state from orbits No. 5, No. 6, and No. 7 – the ones that originate from the $1d_{5/2}$ spherical shell-model state. Pickup calculations based on the Nilsson model predict for $\eta = 3$ that ~90% of the available strength should manifest itself in the $J = \frac{5}{2}$ states. The occupation of the $1d_{5/2}$ shell is 82%, from Ref. 1. If it is assumed that orbits No. 5 and No. 6 are fully occupied, and depletions in orbit No. 7 account for the 82% occupation, then the Nilsson model predicts $\sum S = 4.30$. (Alternatively, if we assume that orbits No. 5, No. 6, and No. 7 share equally the 82% population, then $\sum S = 4.35$.) The experimentally observed strength, $S = 4.58$, is largely to the $J = \frac{5}{2}$ ground and 3.07-MeV states; thus the $J = \frac{5}{2}$ state based on a proton in orbit No. 6 is not expected to be observed with appreciable intensity.

Further suggestion of mixing of the $K = \frac{5}{2}$ ground-state band and the $K = \frac{3}{2}$ hole band comes from the ^{27}Al$(t,p)^{29}$Al direct-reaction investigation of Hirko.[5] If the bands were unmixed in ^{29}Al then a strong double-stripping transition would be observed to the ground state and no strength to the $(J^\pi, K) = (\frac{5}{2}^+, \frac{3}{2})$ hole state. In fact, a strong transition is observed in the experiment to the 3.07-MeV state as well as the ground state. The relative intensity with respect to the ground state is 1 : 3 which would naively suggest (neglecting kinematic factors) that the bands were mixed ~75:25 by intensity.

C. $K^\pi = \frac{1}{2}^+$ Bands

Having identified a band based on the $J^\pi = \frac{5}{2}^+$ ground state and shown that the properties of the 2.23- and 3.07-MeV states are consistent with identification as the $J^\pi = \frac{3}{2}^+$ and $\frac{5}{2}^+$ states of a $K^\pi = \frac{3}{2}^+$ hole-state band based on Nilsson orbit No. 7, we can now try to identify further bands that we expect to see in the low-lying level spectrum. These would be based on protons being excited into the two $K^\pi = \frac{1}{2}^+$ orbits No. 9 and No. 11 and also a $K^\pi = \frac{1}{2}^+$ band based on a hole in orbit No. 6. We can probably identify the $J^\pi = \frac{1}{2}^+$ state at 1.40 MeV with a particle in orbit No. 9. For all prolate deformations and also for $\eta > -4$, orbit No. 9 is lower in energy than orbit No. 11, and also orbit No. 6 is expected to be at a higher excitation than the suggested head of a hole-state band based on orbit No. 7 at 2.23 MeV (see Fig. 6). In order to identify the $J^\pi = \frac{3}{2}^+$ state of orbit No. 9, we need to investigate the allowed values of the decoupling parameter a for various prolate deformations. This varies from $a = 1.0$ for $\eta = 0$ to $a = 0$ for $\eta = +4$. To estimate the excitation energy of the $\frac{3}{2}^+$ state, we take these as extreme values and assume the moment-of-inertia parameter A for this band lies between 326 keV (the value for the ground-state band) and 163 keV – an *ad hoc* value of one half the ground-state value. (Such a trend is observed in mass-25 nuclei, where for the $K = \frac{1}{2}$ band based on the first excited state A is 176 keV compared with $A = 272$ keV for the ground state.)[20] We can now predict that the $J^\pi = \frac{3}{2}^+$ state will lie between 1.85 and 3.25 MeV. From Fig. 3, possible candidates for the state are then the 2.88-MeV state, which has been positively identified as having $J^\pi = \frac{3}{2}^+$, and the 3.19-MeV state. The 2.23-MeV state can also be considered. To differentiate between these possibilities we consider, in turn, two separate pieces of evidence: (a) differential cross-sections measured in the ^{30}Si$(t, \alpha)^{29}$Al direct-reaction investigation; and (b) the γ-ray branchings of these levels.

The data can be seen to be consistent with the 2.88-MeV state being the $J^{\pi}=\frac{3}{2}^{+}$ member of the $K=\frac{1}{2}$ band built on the 1.40-MeV state, since neither the 2.23- nor the 3.19-MeV levels have the required pickup strength. The relative values of the 1.40- and 2.88-MeV spectroscopic factors imply a value of η (by interpolation) of ~3.5. For this deformation, the strength of the $J^{\pi}=\frac{5}{2}^{+}$ member of the band is predicted to be ~0.7 times that of the other two members, and so either the 3.65- or 3.68-MeV states could be the $J^{\pi}=\frac{5}{2}^{+}$ member of this band.

The γ-ray branching of the 2.88-MeV state, as shown in Fig. 1, is entirely consistent with it being the $J^{\pi}=\frac{3}{2}^{+}$ member of this band; while on the other hand the γ decay of the other candidates, the 2.23- and 3.19 MeV levels, does not include a branch to the 1.40-MeV level.

In the lifetime measurements described earlier, a value of $\tau_m=3.3^{+2.2}_{-1.0}$ psec was obtained for the 1.40-MeV state. This implies an $E2$ rate of $8.4^{+3.7}_{-3.4}$ Weisskopf units (W.u.) for the transition to the ground state. This value of 8.4 W.u. is to be compared with a value[27] of 0.8 W.u. obtained in ^{25}Mg. One difference, however, between these two nuclei which may explain the large $E2$ rate is the position of the $K=\frac{3}{2}$ orbit No. 7 band head in these nuclei which lies at a lower excitation in ^{29}Al (2.23 MeV) than in ^{25}Mg (~4.0 MeV). In fact, mixing of the $(J^{\pi},K)=(\frac{5}{2}^{+},\frac{5}{2})$ ground state and the $(J^{\pi},K)=(\frac{5}{2}^{+},\frac{3}{2})$ state at 3.07 MeV is required for interpretation of the ^{27}Al$(t,p)^{29}$Al direct-reaction study, as mentioned earlier. Also the $(J^{\pi},K)=(\frac{5}{2}^{+},\frac{1}{2})$ state based on Nilsson orbit No. 9, whose major component is suggested to lie at 3.65 or 3.68-MeV, could be mixed in the ground state to a greater extent than in mass-25 nuclei because of the presence at lower excitation energy in ^{29}Al of the $(J^{\pi},K)=(\frac{5}{2}^{+},\frac{3}{2})$ state. Thus a detailed band mixing calculation is needed to see whether the measured 1.40 to ground-state $E2$ rate is inconsistent with the proposed model for the nucleus.

D. Other $K^{\pi}=\frac{1}{2}^{+}$ Bands

One other $J^{\pi}=\frac{1}{2}^{+}$ state is positively identified in the spectrum at 3.43 MeV. This state could be the $(J^{\pi},K)=(\frac{1}{2}^{+},\frac{1}{2})$ state resulting from an odd particle in orbit No. 11 or a hole in orbit No. 6. We can, in fact, attempt to distinguish between the possibilities, using the results of Ref. 1. The ratio of the $l=0$ strengths to the 1.40- and 3.43-MeV states is given by

$$\pi V_9^2 S_9/\pi V_{11}^2 S_{11}=0.31/0.11\,, \tag{4}$$

or

$$\pi V_9^2 S_9/\pi V_6^2 S_6=0.31/0.11\,. \tag{5}$$

We can make reasonable estimates on the ratios of the population parameters. As orbit No. 11 lies at a higher excitation than orbit No. 9, a reasonable lower limit is $\pi V_9^2/\pi V_{11}^2=1/1$, whereas orbit No. 6 is ostensibly filled so $\pi V_9^2/\pi V_6^2$ can be reasonably given an upper limit of 1/2. This then yields

$$S_9/S_{11}\leq 2.8\,, \tag{6}$$

or

$$S_9/S_6\geq 5.6\,. \tag{7}$$

Obviously the condition set out in Eq. (6) conforms to Table III for $\eta\gtrsim 2$ while Eq. (7) applies for $\eta\lesssim 1.5$. The information on orbit No. 9 derived in the last section favors a value of η of +3.5. This would then favor the assignment of the 3.43-MeV state to orbit No. 11. If we take a reasonable estimate for $\pi V_9^2/\pi V_{11}^2$ of 2/1 then agreement with theory is obtained for a value of η of ~3 – in accord with the properties of orbit No. 9. This conclusion, of course, is only valid if orbits No. 9 and No. 11 have similar distortions, and assumes unmixed bands. We have previously surmised that the $J^{\pi}=\frac{5}{2}^{+}$ state of orbit No. 6 is mixed with $J^{\pi}=\frac{5}{2}^{+}$ states of other bands and this is, of course, one limiting factor on the present analysis.

As a possible contender for the $J^{\pi}=\frac{3}{2}^{+}$ member of orbit No. 11 we have either the 3.65- or 3.68-MeV states. At a value of η of +3 the strength of the $J^{\pi}=\frac{3}{2}^{+}$ member is predicted to be some 50% greater than the $J^{\pi}=\frac{1}{2}^{+}$ member. Thus either the

TABLE III. Predicted pickup spectroscopic factors for $K=\frac{1}{2}$ states of ^{29}Al.

Nilsson orbit No.	Proposed states E_x (MeV)	J^{π}	Spectroscopic factors [a] $\eta=0$	$\eta=+2$	$\eta=+4$
9	1.40	$\frac{1}{2}^{+}$	2.0	1.30	0.56
	2.88	$\frac{3}{2}^{+}$	0	0.36	0.92
	3.65 3.68	$\frac{5}{2}^{+}$	0	0.34	0.52
11	3.43	$\frac{1}{2}^{+}$	0	0.39	0.94
	3.65 3.68	$\frac{3}{2}^{+}$	2.0	1.61	0.96
		$\frac{5}{2}^{+}$	0	~0	0.10
6		$\frac{1}{2}^{+}$	0	0.30	0.48
		$\frac{3}{2}^{+}$	0	0.06	0.13
		$\frac{5}{2}^{+}$	2.0	1.64	1.39

[a]Note that the spectroscopic strengths for orbits No. 9 and No. 11 must be multiplied by the occupation values of the respective orbits to give the absolute values, whereas for orbit No. 6 the values should be absolute for an unmixed band (orbit already filled).

3.65- or 3.68-MeV state could be this state. Assuming the moment-of-inertia parameter for this band to be between $A = 326$ and $A = 163$ keV, we deduce that the decoupling parameter a will be between −0.76 and −0.52. For orbit No. 11, a is predicted to be −0.6 for $\eta = +3.5$. Thus, there is good agreement with the static properties of orbit No. 11. The $J^\pi = \frac{5}{2}^+$ state of this orbit is now predicted, for $a = -0.6$, to lie between 5.0- and 6.6-MeV excitation energy.

In the discussion and identification of members of the $K = \frac{1}{2}^+$ bands we have neglected the effects of rotational particle coupling (RPC)[25] of states within these bands. Some of our evidence for proposing a prolate distortion for the bands in ^{29}Al comes from the static properties of the bands. Such an approach in ^{25}Al produced entirely consistent results without inclusion of RPC, and as the separation of the $J^\pi = \frac{1}{2}^+$ members of the $K = \frac{1}{2}$ band based on orbits No. 9 and No. 11 in ^{29}Al is 2.03 MeV, compared with a similar separation of 2.05 MeV in ^{25}Al, then the neglect of RPC (between the $K = \frac{1}{2}$ bands) can be expected to be no less significant in ^{29}Al than it was in ^{25}Al. However, the picture in ^{29}Al is complicated by the presence at a lower excitation of the $K = \frac{3}{2}$ hole band, which will mix with other low-lying states, probably to a stronger degree in ^{29}Al than in mass-25 nuclei.

IV. SUMMARY AND CONCLUSIONS

It seems fruitless at this stage, with the current lack of knowledge of definite spin-parity values, to further assign higher states to bands. The members of the $K^\pi = \frac{1}{2}^+$ band based on a hole in Nilsson orbit No. 6 would be expected to appear in the spectrum at $E_x > 3.43$ MeV, and the close proximity of several states with the same spin and parity but with K differing by 0 or ±1, would result in mixing of the states. This would change both the static and dynamic properties of the bands and would destroy the simple concept of a state having a definite K quantum number.

In our proposed scheme, we have fitted all the states with $E_x < 3.7$ MeV into a band picture, except for the 3.19-MeV state. It is certainly possible that this state belongs to an additional positive-parity band and has been pushed down to its present excitation energy by interaction with another state. However, it is interesting to speculate on the other structures for this state. The lowest odd-parity state in ^{25}Al is a $\frac{3}{2}^-$ state at 3.08 MeV. This state arises from a $K = \frac{1}{2}^-$ band based on orbit No. 14.[20] The 3.19-MeV state in ^{29}Al is then possibly the equivalent state if the spin is $J^\pi = \frac{3}{2}^-$. However, the contradictory evidence on the spin-parity of this state does not allow any definite conclusion to be drawn. There are two $J^\pi = 3^-$ states in ^{30}Si at ~5-MeV excitation. Invoking a weak-coupling model, negative-parity states in ^{29}Al can be formed by coupling a $J^\pi = \frac{5}{2}^+$ hole to this spin. Such a state will, as observed, show no pickup strength in the ^{30}Si$(t, \alpha)^{29}$Al reaction. A positive-parity state in ^{29}Al of $J = \frac{3}{2}$, $\frac{5}{2}$, or $\frac{7}{2}$ can be formed as follows:

$$\{[|\nu_{11}\rangle\tfrac{1}{2}|\nu_9\rangle\tfrac{1}{2}]_1 |\pi 5\rangle\tfrac{5}{2}\}_{3/2,\,5/2,\,7/2}\,.$$

Such a state will not show a pickup width and to make the structure feasible as a prediction for a low-lying state in ^{29}Al, a low-lying $J^\pi = 1^+$ state in ^{30}Si is required. Such a state exists at 3.77 MeV in this nucleus. Whether the 3.19-MeV state can be considered as having one of the above proposed configurations will, above all, require some more positive evidence on its spin and parity.

The static properties of the low-lying states of ^{29}Al, that is the level positions, extracted moment-of-inertia parameter A, and decoupling parameter, a, for $K = \frac{1}{2}$ bands can all be reconciled with a prolate nucleus with $\eta \simeq +3$. Analysis of the pickup results to members of the suggested bands is again consistent with a similar value of η. The similarity of the structure of ^{29}Al to ^{25}Al and ^{25}Mg is striking, the one essential difference being the appearance at a lower excitation in ^{29}Al of the $K = \frac{3}{2}$ band based on a hole in Nilsson orbit No. 7.

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