Influence of Breakup Channels on the Analysis of Deuteron Stripping Reactions

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(Received 24 August 1970)

It is shown by explicit calculations that the adiabatic treatment of certain deuteron breakup channels predicts differential cross sections for (d,p) and (p,d) reactions, involving 20-MeV deuterons and a wide range of targets, similar to results obtained with the distorted-wave method but with large suppression of the contributions from low partial waves. An improved fit to angular distributions is obtained in many cases.

I. INTRODUCTION

An approximate treatment of the contribution from deuteron breakup channels to the transition amplitude for deuteron stripping has been proposed recently by Johnson and Soper.[1] It was shown that the adiabatic treatment of channels involving ^3S states of the neutron-proton system and the ground state of the target nucleus leads, in a three-body model, to a rather straightforward modification of the usual distorted-wave method (DWM).[2] It was also shown that when applied to deuteron elastic scattering by Ni at 21 MeV, the adiabatic theory produced a systematic improvement over the predictions of a model which ignored these breakup channels completely. In this paper the adiabatic theory is investigated further by applying it along the lines indicated in Ref. 1 to the calculation of cross sections and polarizations for a number of (d,p) and (p,d) reactions involving deuterons in the 20-MeV region.

II. FORMULATION

In the adiabatic approximation used here[1] the (d,p) transition matrix takes a form which, as far as its numerical evaluation is concerned, is precisely the same as the DWM matrix element. The key difference is that the role played by the deuteron distorted wave in the distorted-wave Born approximation (DWBA) is played by a distorted wave $(\bar{\chi})$ which is generated by a potential $(\bar{V})$ differing in a significant way from potentials that fits elastic deuteron scattering. In the three-body model discussed in Ref. 1, this potential is related to the nucleon optical potentials V_n and V_p corresponding to one half of the incident deuteron kinetic energy by the formulas

$$\bar{V}(\vec{R}) = D_0^{-1}\int d\vec{r}\,[V_n(\vec{R}+\tfrac{1}{2}\vec{r}) + V_p(\vec{R}-\tfrac{1}{2}\vec{r})]V_{np}\phi_d(r), \tag{1}$$

$$D_0 = \int d\vec{r}\, V_{np}\phi_d(r), \tag{2}$$

where $\vec{R}$ and $\vec{r}$ are, respectively, the position vector of the deuteron center of mass relative to the target and the position vector of the neutron relative to the proton, and ϕ_d is the deuteron internal wave function.[3]

Although the structure of the stripping amplitude in the adiabatic theory is similar to the DWM amplitude the physical interpretation of the function $\bar{\chi}(\vec{R})$ is quite different from the deuteron distorted wave used in the DWM approach. This point was discussed earlier,[1] and it was pointed out there that while $\bar{\chi}(\vec{R})$ describes the motion of the c.m. of the neutron and proton they may not be in the form of a deuteron. For brevity, the function $\bar{\chi}(\vec{R})$ will often be referred to as the "deuteron" distorted wave. The manner in which these differences enter the calculations reported in the next sections is through the detailed difference between the potential $\bar{V}(R)$ (to be referred to as the adiabatic potential) and the deuteron optical potential (often referred to as the conventional potential in the following). This comparison is made in Fig. 1 for a typical case. The nucleon optical potentials used to generate $\bar{V}$ in this case were the proton potential of Perey[4] and the local equivalent of the nonlocal neutron potential of Perey and Buck.[5] The potential was calculated for use in the analysis of the ^{54}Fe$(d,p)^{55}$Fe reaction at 23 MeV reported later, and the deuteron optical potential shown in Fig. 1 is that used in the original analysis of the reaction.[6] The product $V_{np}\phi_d(r)$ in Eq. (1) was represented by the Hulthén estimate

$$V_{np}\phi_d(r) = [-\epsilon_d + (\hbar^2/m)\nabla^2]\phi_d(r), \tag{3}$$

with

$$\phi_d(r) = N(e^{-\gamma r} - e^{-\beta r})/r, \tag{4a}$$

$$\epsilon_d = (\hbar^2/M)\gamma^2 = 2.23 \text{ MeV}, \quad \beta/\gamma = 6. \tag{4b}$$

It can be seen that while there are small differences in depths the main difference between the potentials shown in Fig. 1 lies in their geometries.

It is shown later that it is the latter differences which are more significant for stripping calculations.

There are definite physical reasons for some of the differences shown in Fig. 1. Thus, according to Ref. 1, the manner in which the absorptive part of the deuteron optical potential extends to large distances is mainly associated with breakup of the deuterons into ^3S states of low relative momentum. It is precisely these channels which contribute explicit components, in addition to the elastic component, to the distorted wave $\bar{\chi}$ used in the adiabatic approach. It is natural therefore that the imaginary part of the potential $\bar{V}(R)$ which generates $\bar{\chi}(R)$ should be confined to smaller distances than the imaginary part of the potential which accounts for absorption out of the elastic channel. The greater diffuseness displayed by the real part of the conventional potential can also be understood from the correspondence[1] between this potential and the result obtained by averaging nucleon optical potentials over the deuteron internal wave function rather than the very short-range averaging used to generate $\bar{V}$ according to Eq. (1).

It is clear that the numerical calculation of stripping amplitudes is no more complicated in the adiabatic theory than in the conventional theory, once the potential $\bar{V}(R)$ is known. This potential must be evaluated numerically by a double integration. In the case when the neutron and proton optical potentials have the same radius and diffuseness, the potential $\bar{V}(R)$, in zero-range approximation,[7] also has this same radius and diffuseness, and a depth equal to the sum of the depths of the nuclear potentials. This situation arises when using the Perey proton potential[4] and the local equivalent of the Perey-Buck nonlocal neutron potential.[5] In this simplified case, the numerical evaluation of $\bar{V}(R)$ in finite-range approximation can be avoided by the use of the following formulas, which have been found to be quite accurate when compared with an exact calculation.

$$\bar{V}_0 = V_0^p + V_0^n, \quad \bar{r}_0^{s,i} = r_0^{s,i}, \quad \bar{a}_0^{s,i} = a_0^{s,i} + 0.04,$$
$$\bar{W}_0 = (W_0^p + W_0^n) a_0^s / \bar{a}_0^s, \tag{5}$$

where V_0^p and V_0^n are, respectively, the depths of the real parts of the neutron and proton potentials and W_0^p and W_0^n the depths of their surface-absorption potentials, $r_0^{s,i}$ and $a_0^{s,i}$ are the radius and diffuseness of the potentials. The superscripts "s" and "i" refer to surface and volume parameters, respectively. In the case of nuclear potentials where the radius and diffuseness are isospin dependent, the above formulas are still quite accurate, if the average of the nuclear radii are used instead of $r_0^{s,i}$ and the average of the diffusenesses in place of $a_0^{s,i}$. All of the parameters in Eq. (5) refer to a standard Woods-Saxon form. It is important to note that while the prescription (5) does give a useful account of the effects of the finite range of V_{np} in Eq. (1), in fact these effects are found to be unimportant compared with other uncertainties in the theory.

Following Ref. 1, the form of the adiabatic prescription used here neglects the contribution from ^1S states of the neutron-proton system. It has been shown[8,9] that this approximation is entirely consistent with our present knowledge of the isospin dependence of the nucleon optical potential.

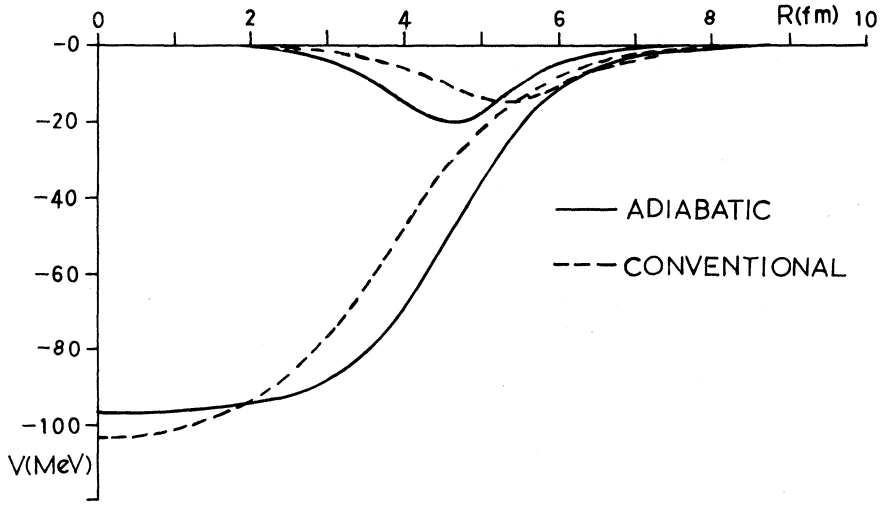

FIG. 1. A comparison of a typical conventional deuteron optical potential (with $r_0 = 1.02$ fm, $a = 0.86$ fm, $r_0' = 1.42$ fm, $a' = 0.65$ fm, $V = 105$ MeV, $W = 15$ MeV) with the calculated finite-range adiabatic potential (with approximate parameters $r_0 = 1.25$ fm, $a = 0.69$ fm, $r_0' = 1.25$ fm, $a' = 0.50$ fm, $V = 97$ MeV, $W = 20$ MeV) for the reaction ^{54}Fe$(d,p)^{55}$Fe at 23 MeV.

III. RESULTS OF THE ADIABATIC THEORY

The program used for numerical calculations, was a modified version of the DWM Algol code written by Santos,[10] which can handle a large number of refinements of the DWM stripping theory. Finite-range effects in the integration of the stripping matrix element can be handled in the adiabatic theory[1] in the same way as they are in DWM. All the finite-range calculations reported here include these effects in the standard approximate fashion.[11] It should be noted that the range of the potential V_{np} also enters the adiabatic calculations through the potential $\bar{V}$ [Eq. (1)].

The implementation of the adiabatic theory involves the use of nucleon optical potentials evaluated at half the incident deuteron energy, and once these potentials are fixed the adiabatic theory contains no adjustable parameters apart from the absolute normalization of the cross section which is determined by the spectroscopic factor just as in DWM. In this way we avoid completely difficulties associated with ambiguities in deuteron optical potentials. There remain, however, ambiguities in the nucleon optical potentials as currently determined, which are reflected in ambiguities in the predictions of the adiabatic theory. Two sets of optical potential parameters have been used in this work, the proton optical potential of Perey[4] in combination with the equivalent local potential of the Perey-Buck nonlocal neutron potential[5]; and the Bechetti-Greenlees nucleon optical potentials.[12] The former are referred to as the Perey potentials, and the latter as the BG potentials.

The results presented here concentrate on reactions in which the energy in the deuteron channel is around 20 MeV. It was at this energy that the adiabatic approximation was found to give a good description of the elastic scattering of deuterons in Ref. 1. The adiabatic theory is expected to be more reliable as the energy in the deuteron channel is raised, since the adiabatic approximation becomes better, and the nuclear optical potentials become more reliable as the energy at which they are evaluated is raised (this is half the deuteron energy).

One suitable set of data for evaluating the adiabatic theory is that for the stripping of deuterons incident on ^{54}Fe at 23 MeV.[6] The measured $l=1$ cross sections are particularly interesting because their rate of falloff at large angles is very steep, and cannot be reproduced by the conventional DWM calculation. Yntema and Ohnuma[6] obtained a good fit using the conventional theory, by the rather arbitrary device of increasing the ranges of nonlocality of the optical potentials to approximately twice their accepted value, which has the effect of severely damping the interior contributions. Calculations performed using the adiabatic theory, for zero- and finite-range V_{np}, and using the Perey and BG optical potentials, in all cases produced a cross section with the required falloff, without the use of a cutoff or of nonlocality corrections. Typical results are shown in Figs. 2 and 3 for the $(\frac{1}{2}^-)$ transition, where the adiabatic theory can be compared with the conventional theory, both with and without the large nonlocality parameters ($\beta_p=2$ fm, $\beta_d=1$ fm). Similar results were found for the $(\frac{3}{2}^-)$ transition.

A similar phenomenon occurs in the analysis of the reaction ^{40}Ca$(p,d)^{39}$Ca at 30 MeV. In this case the asymmetries associated with a polarized proton beam were also measured accurately.[13] Chant and Nelson[13] had great difficulty in fitting their measured angular distributions for this reaction, the predicted cross sections showing little falloff

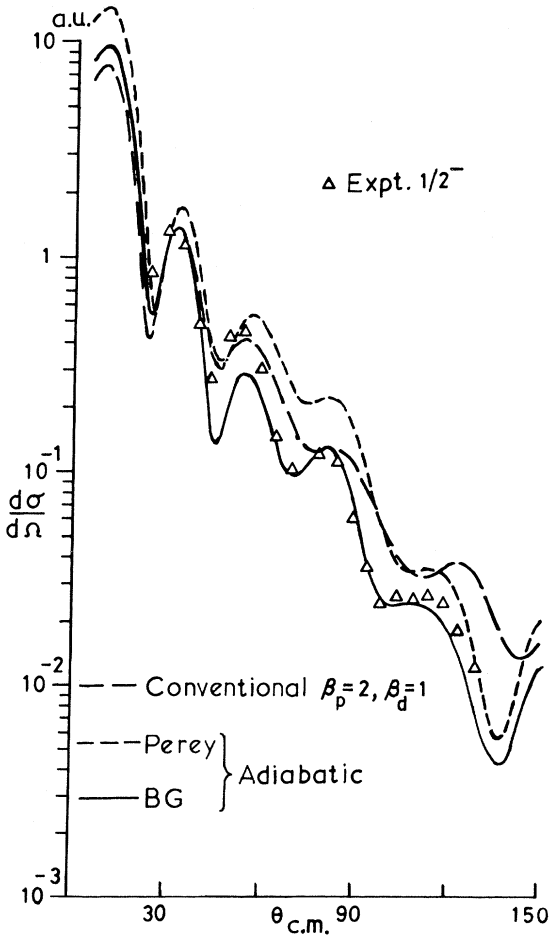

FIG. 2. A comparison of the predictions of the adiabatic theory (using Perey and BG potentials) with the conventional theory predictions (with large nonlocality parameters). The experimental points are taken from Ref. 6 for the ^{54}Fe$(d,p)^{54}$Fe reaction at 23 MeV.

from the main peak, while the experimental distributions were quite strongly forward peaked. Reasonable agreement was obtained by using a massive volume absorption strength of 60 MeV for the deuteron. This overwhelmingly strong absorption potential effectively causes the deuteron wave function to vanish in the nuclear interior. The results of the adiabatic theory with two sets of nucleon optical potentials and without nonlocality corrections are compared with the analyzing power and differential cross section for this reaction in Figs. 4 and 5. Several interesting features emerge, firstly the results of the adiabatic theory reproduce the general features of the differential cross section quite naturally, without any means of damping the contributions from the interior being necessary. The results of using the adiabatic theory with the Perey potentials are simialr to the results of using the conventional theory with the large volume absorption (Fig. 4). The predictions of the adiabatic theory using the Perey potentials, and especially of the conventional theory, for the asymmetries are poor. Comparing the use of the Perey and BG potentials in the adiabatic theory in Fig. 5, reveals that the BG potentials have introduced violent diffraction oscillations into the cross section, which are not seen experimentally but has produced a dramatic improvement in the fit to the asymmetries, all the oscillations being reproduced.

Calculations using the conventional theory for neutron pickup reactions at proton energies of around 30 MeV on light nuclei[13-16] have encountered similar difficulties to those mentioned earlier, large radial cutoffs being necessary to reproduce the observed falloff of the cross section. Calculations for $(l=1)(p,d)$ reactions on N^{14} with a proton beam of 30 MeV, using the adiabatic theory, showed that the latter reproduced the falloff required by the data, but showed great sensitivity to the nucleon optical potentials used, and introduced strong diffraction oscillations into the cross section in some cases.

Some examples of the results obtained using a reasonable set of local nucleon optical potentials are shown in Fig. 6. The proton optical at 30 MeV was that obtained by Satchler[17] from a fit to the N^{14} proton-scattering data. In order to evaluate the adiabatic potential it was assumed that the same potential could be used at 10 MeV for both neutron and proton, but with a change in depth derived from the change in energy, using an assumed energy dependence of $dV/dE = -\frac{1}{3}$. In view of the uncertain nature of the nucleon optical potentials for this reaction, only the general features of the prediction are of interest. For comparison, the conventional calculation prediction for the $\frac{1}{2}^-$ distribution is also shown. The deuteron optical potential used in this calculation was the average "Z2" potential of Satchler,[18] obtained from a study of elastic scattering on C^{12}. Results similar to those shown in Fig. 6 have been obtained by Satchler.[17]

IV. COMPARISON OF DWM AND ADIABATIC CALCULATIONS

In many of the reactions which have been studied, it has been apparent that the use of the adiabatic potential instead of the deuteron optical-model potential, has had a similar effect to that obtained by suppressing the interior contributions to the stripping matrix element. In the application of the conventional theory, this suppression of interior contributions, has in the past been obtained in a variety of ways, either by the use of extreme values of the absorption potential or the nonlocality parameters, or by the use of a radial cutoff. In at-

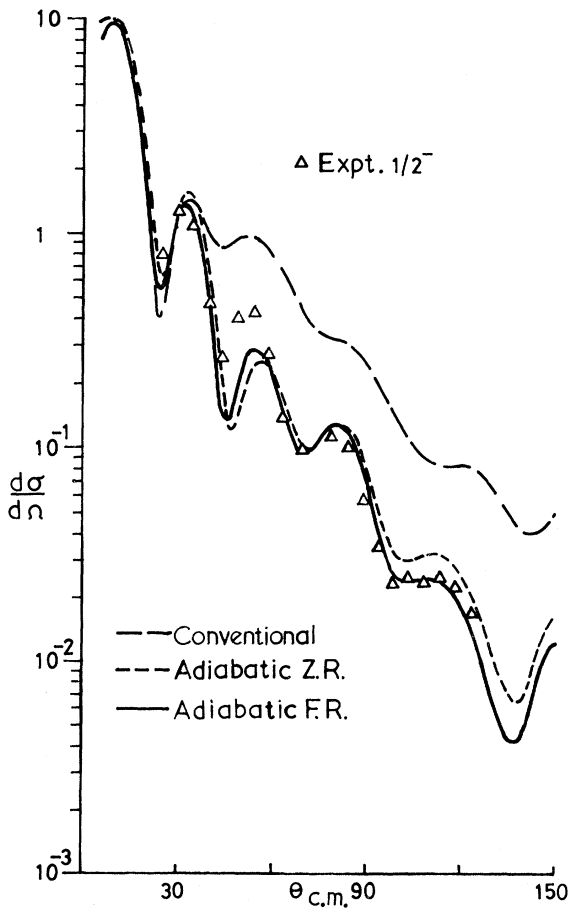

FIG. 3. A comparison of zero- and finite-range adiabatic predictions (using the BG potentials) for the ^{54}Fe-$(d,p)^{55}Fe$ reaction at 23 MeV. Also shown is the curve predicted by the conventional theory without nonlocality corrections.

tempting to understand the mechanism for this apparent suppression of interior contributions in the adiabatic theory, it is useful to examine the radial integrals which determine the cross section in both the conventional and the adiabatic calculations. These are defined in terms of the radial parts, $U_{l_p j_p}$ and $U_{l_d j_d}$ of, respectively, the proton and deuteron (or "deuteron" in the adiabatic case) channels, and the radial part of the transferred neutron's wave function. They are proportional to the integrals

$$I(l_p j_p, l_n j_n, l_d j_d) = \int dR\, U_{l_p j_p}(R) R_{l_n j_n}(R) \times U_{l_d j_d}(R) \Lambda(p, d, R), \quad (6)$$

where the factor $\Lambda(p, d, R)$ incorporates finite-range and nonlocality corrections.

Concentrating upon the reaction $Fe^{54}(d,p)Fe^{55}$ at 23 MeV, two sets of radial integrals for the conventional calculation are shown in Fig. 7. The diagram shows the radial integrals, with and without the large nonlocality parameters used in Ref. 6. In both cases, the radial integrals with the nonlocality correction included, are greatly damped for low partial waves, compared with those without the nonlocality corrections. This is easily understood, since the main contributions to the low partial-wave radial integrals come from the interior and surface regions where they are greatly damped by the correction factor. The corresponding radial integrals for the adiabatic calculation are shown in Fig. 8, and they show a similar structure to those of the conventional calculation, with the large nonlocality corrections. It is in this respect, that the adiabatic calculation gives apparent suppression of the interior contributions.

Examination of the actual wave functions generated by the adiabatic and deuteron optical potentials shows that for all partial waves the adiabatic wave functions are just as large in the region of the nucleus as the optical-model wave functions. Typical examples are shown in Fig. 9 where $|U_{l_d j_d}|$ is plotted against R. It should be realized, however, that the detailed behavior of these functions well inside the nucleus (for $R \leq 4$ fm in the case shown)

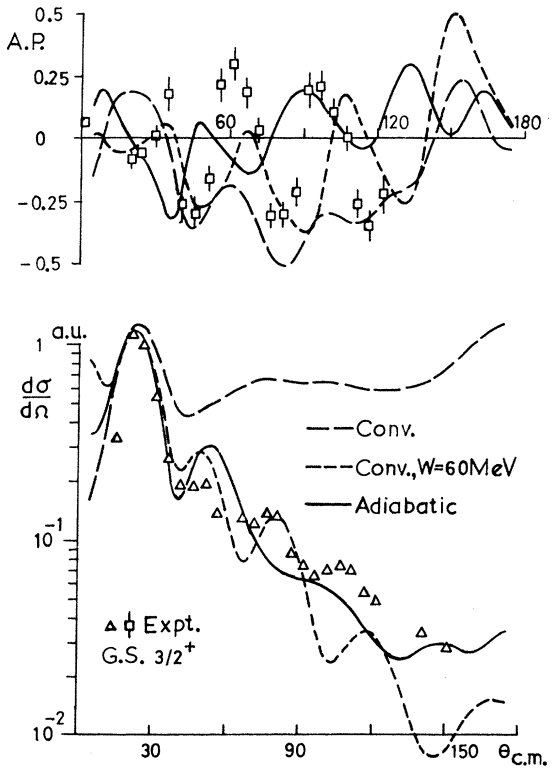

FIG. 4. A comparison of the adiabatic theory predictions (using the Perey potentials) for the differential cross section and analyzing power, with the standard conventional theory predictions, and also with the predictions for the conventional theory using 60-MeV volume absorption in the deuteron channel. The deuteron optical potentials and data were taken from Ref. 13 for the $^{40}Ca(p,d)^{39}Ca$ reaction at 30.5 MeV.

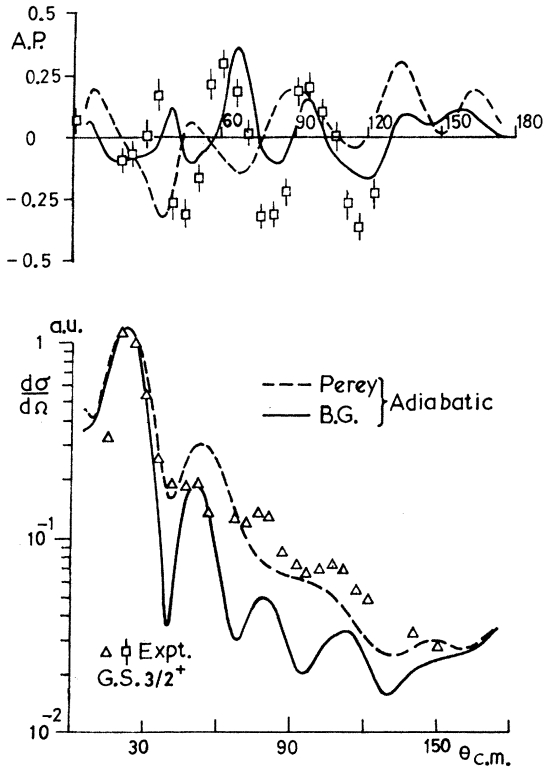

FIG. 5. A comparison of the adiabatic theory predictions using BG and Perey potentials for the analyzing power and differential cross section in the $^{40}Ca(p,d)^{39}Ca$ reaction at 30.5 MeV.

is not very important for the stripping matrix elements because the wavelength associated with the proton distorted waves in this region is about twice the wavelength apparent from Fig. 9, so that the net contribution to a radial integral from this region of configurations space is very small. The major contributions to the matrix element come from the immediate vicinity of the nuclear surface. The main difference between the adiabatic and optical-model wave functions in this region is that the optical-model potential, because of the smaller value of the radius parameter of its real part, is associated with local wavelengths that have a similar magnitude to the wavelengths associated with the proton distorted wave in this region. That is, the equality,

$$\mathrm{Re}[M_d \bar{V}(r)] = \mathrm{Re}[M_p V_p(R)], \tag{7}$$

where M_d and M_p are reduced masses corresponding to the incident and outgoing channels, and V_p is the optical potential used to generate the outgoing proton distorted wave, is very hard to satisfy at any finite value of R in a consistent calculation. On the other hand, the equality,

$$\mathrm{Re}[M_d V_d(R)] = \mathrm{Re}[M_p V_p(R)], \tag{8}$$

with V_d a deuteron optical potential with real part close to the real part of

$$\langle \phi_d | V_n + V_p | \phi_d \rangle, \tag{9}$$

is usually closely satisfied somewhere in the nuclear surface.

In the ^{54}Fe case considered, the main reason that condition (8) is easier to satisfy than condition (7) is that the radius of $\mathrm{Re}V_d$ is smaller than that of $\mathrm{Re}\bar{V}$. It would be expected therefore that the suppression of the low partial-wave integrals would disappear if the radius of the adiabatic potential was reduced to that of the deuteron optical potential. The radial integrals produced by such a calculation are shown in Fig. 8 by the dashed line. It is clear that the low partial-wave radial integrals are no longer small compared with the surface partial-wave radial integrals. In addition, the rate of falloff of the cross section is found to be reduced to a similar value to that of the DWM calculation.

This effect of the presence or absence of waves of similar local wavelength in the two channels in the region of the nuclear surface can be further enhanced in some cases by the fact that the real and imaginary parts of deuteron optical potentials obtained from fits to elastic deuteron scattering have radii which can differ by up to a factor of 2, whereas, for example, when the Perey potentials are used the real and imaginary parts of the nucleon op-

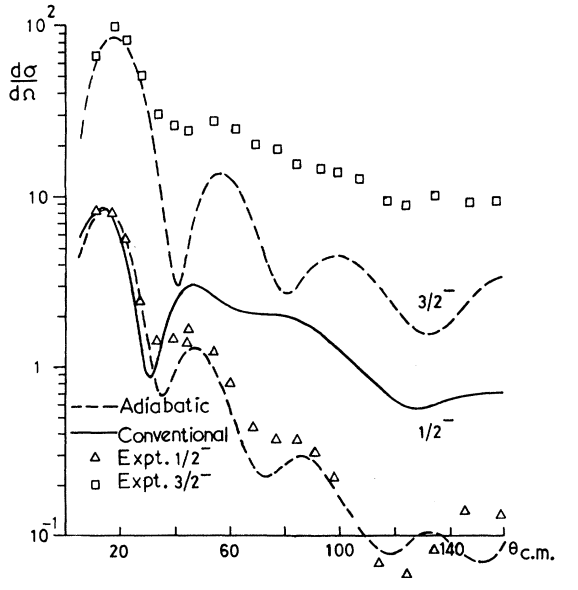

FIG. 6. Adiabatic and conventional theory predictions for the ^{14}N$(p,d)^{15}$N reaction at 30.3 MeV for the $(\frac{1}{2}^-)$ ground-state transition and the $(\frac{3}{2}^-)$ 7.38-MeV transition.

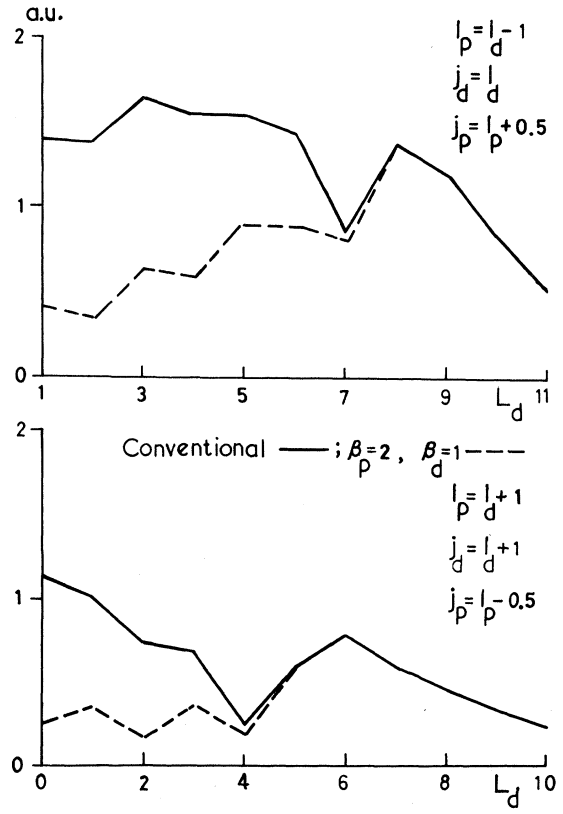

FIG. 7. Radial integrals yielded by the conventional stripping calculation for the ^{54}Fe$(d,p)^{55}$Fe reaction at 23 MeV, with and without the large nonlocality corrections.

tical potentials, and hence the real and imaginary parts of the adiabatic potential, have closely similar radius parameters. To understand the consequences of this it is convenient, following Austern,[19] to think of the radial wave functions as linear combinations of the form

$$U_l(R) = e^{-iK_lR} - \eta_l e^{iK_lR}, \tag{10}$$

where $K_l(R)$ is identified with the local wave number at R and we assume that $\eta_l(R)$ and $K_l(R)$ change slowly in a wavelength. This sort of approximation has been used previously in the discussion of reactions involving deuterons in this energy region.[20] For deuteron low partial waves (see, e.g. Fig. 9) $|\eta_l(R)| \approx 1$ for R well inside the nucleus where the absorptive potential is negligible, and $|\eta_l(R)| \approx 0$ for large R. The detailed way $\eta_l(R)$ behaves in the surface region depends, among other things, on the radial dependence of the absorptive potential. Sensitivity of the stripping integrals to

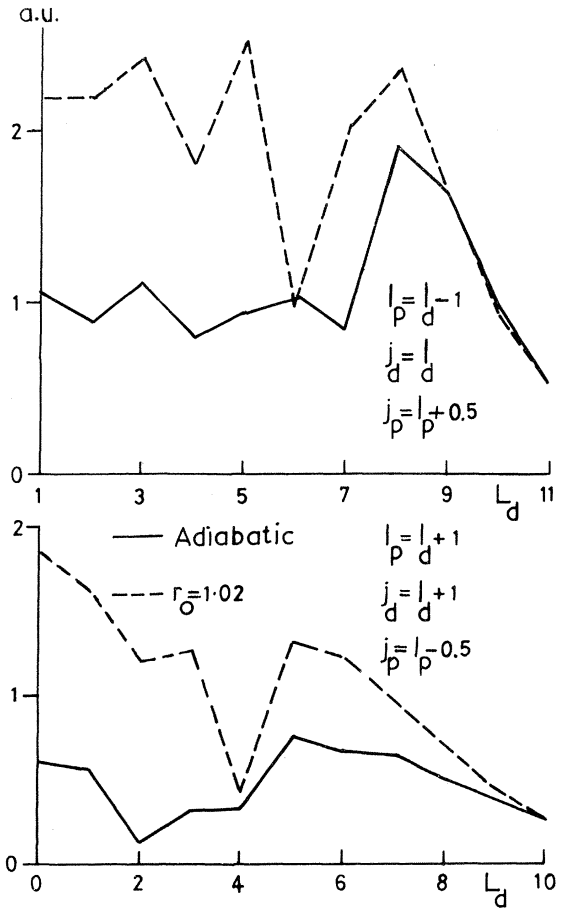

FIG. 8. Radial integrals yielded by the adiabatic stripping calculations for the $Fe^{54}(d,p)Fe^{55}$ reaction at 23 MeV, and by the adiabatic calculation with radius of the real part of the potential (r_0) reduced to 1.02 fm.

the difference between the adiabatic and deuteron optical potentials can clearly arise in this model through the outgoing-wave term in Eq. (10). However, this term will only be present in appreciable amounts in the region where the wave numbers in the proton and incident channels are closely similar and where this term can interfere constructively with the incoming wave component of the proton wave function, provided the absorptive potential is not too large in this region. The latter situation often holds more closely in the DWM matrix element with conventional deuteron optical potentials than in the adiabatic case.

The case of neutron pickup from N^{14} using a 30-MeV proton beam mentioned above is an example. The radii of the real and imaginary parts of the deuteron optical potential used in the DWM calculations in this case were widely separated being approximately 2.1 and 4.1 fm, respectively. Satchler[17] has reported that radial cutoff of up to 2 F had little effect on the angular distributions. The cutoff required in order to produce the observed falloff with angle was between 3 and 4 F. In the light of the success of the adiabatic prescription in producing the required falloff with angle without the use of any cutoff or other suppression factor it would appear that the need for such large cutoffs in the DWM is associated with the need to eliminate large contributions to the DWM matrix elements from the region lying between the real and imaginary parts of the deuteron optical potential. It is also reasonable to predict that the use of the adiabatic theory will resolve many of the difficulties with interior contributions encountered by Snelgrove and Kashy[16] in their recent investigation of (p,d) reactions on ^{16}O and ^{14}N at a range of energies from 20 to 45 MeV. It has been suggested recently,[21] that the increased damping of the interior contributions required to obtain agreement with these experiments, when the conventional theory is used, could be derived from a density dependent interaction V_{np}. The authors found, however, that including every damping agent available, the interior contributions were still too large when compared with required damping found empirically. It may be reiterated that the use of the adiabatic theory eliminates the need for extreme damping of the interior and surface contributions.

V. SUMMARY AND CONCLUSIONS

The first point to be noted in connection with the adiabatic prescription for stripping-reaction calculations, is that it produces an understanding of why the class of deuteron optical potentials having a depth of roughly twice that of the nucleon potentials, gives the best results, when used in stripping

reactions. Although it is not immediately clear that the use of the adiabatic potential will give significantly different results from the use of the conventional optical potential, when this is chosen from the class mentioned above, numerical investigation has shown that the results of the two theories can differ markedly in some cases. The major differences between the two sets of theoretical predictions which emerged were that the cross sections generated by the adiabatic prescription tended to have a greater falloff and stronger diffraction pattern than those generated by the conventional theory. These features can be related to one general feature which distinguishes the adiabatic calculation from the conventional calculation. This feature is the increased l-space localization of the radial integrals yielded by the adiabatic calculations, when compared with the conventional calculation. The increased l-space localization of the radial integrals in the former calculation is in turn associated with the relation between the radii of the real and imaginary parts of the potential used to generate the wave function in the deuteron channel, and their relationship with the distorting potential in the proton channel. In many cases where the difference in these radii is large, the conventional stripping calculation does not yield a sufficiently strong l-space localization of the radial integrals to produce the observed falloff of the cross section. The adiabatic theory always has the radius parameters of the "deuteron" distorting potential approximately equal to the radius parameters of the proton distorting potential, and does not require suppression of the interior and surface contributions to produce the required l-space localization of the radial integrals. It has already been pointed out at the end of Sec. II that these geometrical differences in the potentials are associated with rather definite physical effects not accounted for in the DWM.

A further general feature of the use of the adiabatic theory is that although the ambiguity in the deuteron optical potential is avoided, great sensitivity is sometimes seen to the nucleon optical potentials used (a typical example is shown in Fig. 5). This sensitivity is related to the severity of the l-space localization produced by the adiabatic theory, and the uncertainties in the nucleon optical potentials are reflected in differences in the low-partial-wave radial integrals, which are the end re-

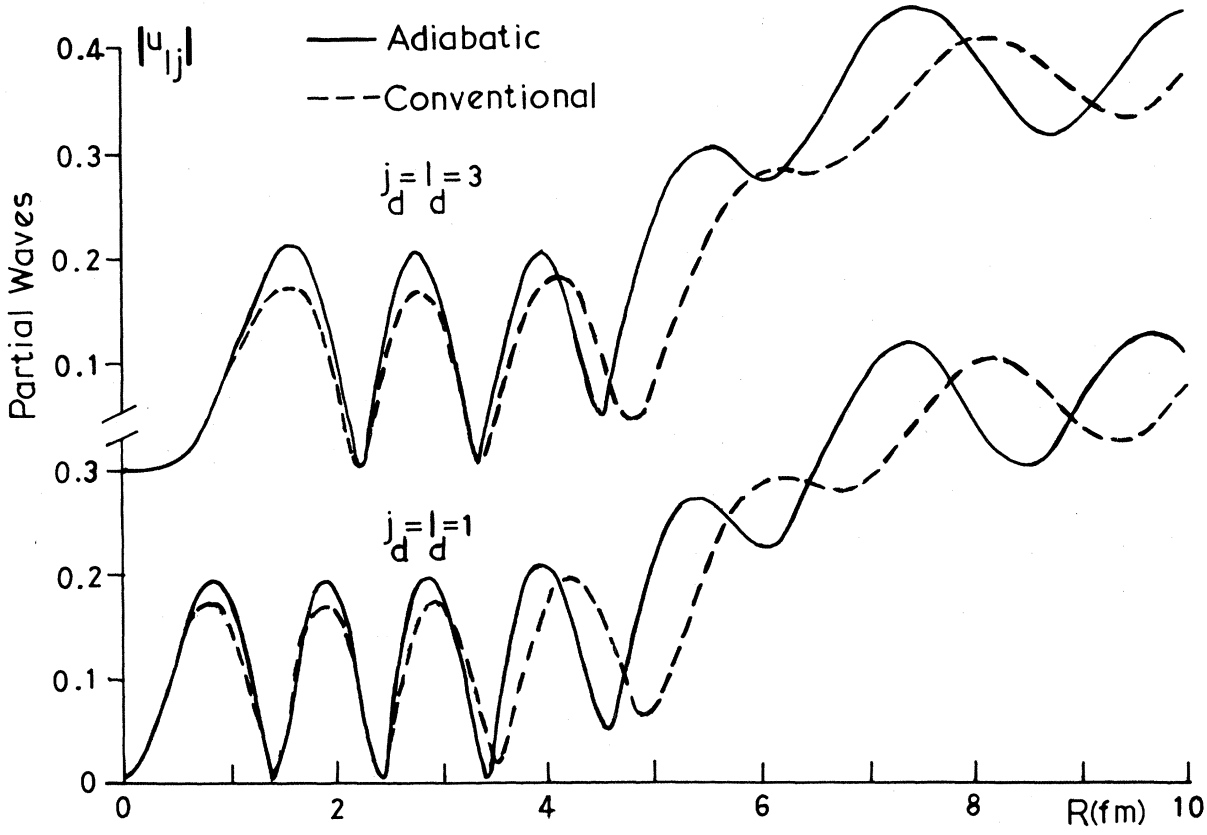

FIG. 9. Comparison of the radial wave functions yielded by the adiabatic and conventional deuteron potentials for two angular momenta. The potentials used were those shown in Fig. 1 for the ^{54}Fe$(d,p)^{55}$Fe reaction at 23 MeV.

sults of considerable cancellations. The adiabatic theory requires the use of effective two-body potentials for the interaction of nucleons in the deuteron with the target, and these are taken as optical potentials evaluated at half the incident deuteron energy. This procedure is approximate, and is chosen mainly on the grounds of simplicity. In view of the sensitivity of the adiabatic calculations to the nucleon optical potentials used, it is clear that many-body corrections could have a significant effect on the adiabatic predictions.

The adiabatic theory predicts an l-space localization of the radial integrals appearing in a DWM calculation which is too great in some cases, and this can lead to a falloff in the cross sections which is too great, and to too strong a diffraction pattern. It is encouraging to note, however, that the inclusion of higher relative angular momentum in the neutron-proton system in the conventional calculation,[3] tends to have the effect of filling in minima in the cross section.

The general result that the adiabatic theory produces an l-space localization of the radial integrals, which can often only be obtained in the context of the conventional theory by rather artifical means, constitutes the evidence in support of the use of the adiabatic theory in analyzing deuteron stripping reactions. The predictions of the adiabatic theory are expected to improve as the energy in the deuteron channel is raised, and the adiabatic approximations become better founded; this should be borne in mind in any application of the theory. While the use of this theory will not alter the assignment of the l value of a particular transition, it can affect the extracted spectroscopic factors, which are generally increased somewhat.

Probably the major deficiency of the adiabatic theory in its present form stems from the inaccuracy of the approximations used for the effective potential. The corrections to the effective potential introduced by many-body effects, which were estimated by Johnson and Soper,[1] applied to the potentials used in the description of elastic scattering, and do not apply directly to the adiabatic potential. It is significant to note, however, that these authors found that these corrections affected mainly the low partial waves, and in the reaction which has been analyzed in the most detail[8] the $Ca^{40}(p,d)$-Ca^{39} reaction, the sensitivity of the different nucleon optical potentials was reflected in a sensitivity of the low-partial-wave radial integrals. The sensitivity of the adiabatic theory to the nucleon optical potentials used is intimately related to the many-body corrections to the effective potential and presents a problem which will have to be solved before detailed fits to the experimental cross sections can be expected.

Finally, it is interesting to consider the relation between this work and the approach of Rawitscher[22] and Ichimura *et al.*[23] In the first place, the latter authors obtain much smaller corrections to the DWBA than the corrections to the DWM reported here. Secondly, although the "non-orthogonality" present in the work of these authors[22,23] make a direct comparison with this work difficult, it does appear that the two approaches describe different physical effects. Thus, on the basis of the calculations reported here, it can be concluded (see, e.g. Fig. 8) that the dominant contributions to stripping to low angular-momentum states, comes from proton and deuteron orbital angular momenta satisfying $l_p, l_d \gg l_n$. The stripping component of the complete wave function for the system is dominated, therefore, by terms in which the relative angular momentum of the neutron and proton is nonzero. As discussed in Ref. 1, these are just the components neglected in the adiabatic treatment, because they are likely to have a small overlap with the neutron-proton potential that appears in the form of the stripping matrix element used in the adiabatic approach.

ACKNOWLEDGMENTS

We wish to thank Dr. F. D. Santos who provided the basic DWM code. We are also grateful for computing assistance from personnel at the University of Surrey and the SRC Atlas Computer, Chilton. One of us (JDH) is grateful to the Association of Commonwealth Universities for a Scholarship held while he was at Surrey.

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PHYSICAL REVIEW C VOLUME 3, NUMBER 2 FEBRUARY 1971

Nuclear Level Densities and Reaction Mechanisms from Continuum Neutron Spectra*

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(Received 14 September 1970)

The neutron spectra produced by the $^{51}V(p,n)^{51}Cr$, $^{59}Co(p,n)^{59}Ni$, $^{48}Ti(\alpha,n)^{51}Cr$, and $^{56}Fe(\alpha,n)^{59}Ni$ reactions were measured at five angles between 15 and 135° for proton energies between 7.8 and 14.7 MeV, and for α energies between 11.5 and 22.7 MeV. Spectra at low energies were used to obtain the spin-weighted level density of the residual nuclei ^{51}Cr and ^{59}Ni. Comparison of these spectral shapes with those obtained at higher energies made possible a separation of the higher-energy spectra into compound and noncompound contributions.

The deduced compound-nuclear cross sections to given groups of levels were related to the integrals of the level densities of the residual nuclei; the variation of these cross sections with energy was used to extend the level-density measurements beyond the neutron binding energy. A constant-temperature level-density form is found to be appropriate for ^{51}Cr and ^{59}Ni up to residual excitation energies of 14 MeV. Values of the moment of inertia of the residual nuclei were extracted from the magnitude of the asymmetry of the compound-nuclear angular distributions. The characteristics of the noncompound portion were compared with those expected from direct- and pre-equilibrium-reaction mechanisms. It is concluded that no convincing evidence for a pre-equilibrium component is observed in the (α,n) spectra; the (p,n) data show behavior consistent with contributions from both pre-equilibrium- and direct-reaction mechanisms. A value of approximately 160 keV was obtained for the widths of the participating doorway states from a model-dependent calculation based on the magnitude of the pre-equilibrium (p,n) cross section.

I. INTRODUCTION

Analyses of nuclear emission spectra have yielded much information about nuclear level densities, including both the functional form and parameters of the level-density distribution for specific nuclei.[1-7] The statistical theory[8] predicts that the differential cross section for emission of particles of energy E integrated over angle can be related to the level density $\rho(U)$ of the residual nucleus as follows:

$$\sigma(\epsilon) \propto \epsilon\sigma_c(\epsilon)\rho(U), \quad (1)$$

where ϵ is the channel kinetic energy and $\sigma_c(\epsilon)$ is the capture cross section for the inverse reaction at an energy ϵ.

The consequence of the statistical theory is that nuclear level-density parameters obtained for a specific residual nucleus should be independent of (1) bombarding energy, and (2) entrance channel. Experimental results,[1,2,5,7] however, have sometimes contradicted this prediction. Often, individual spectra have been fit with a specific level-density form, but the parameters so obtained have depended on bombarding energy as well as residual excitation.

In addition to the excess of high-energy particles produced by direct reactions, i.e., above the num-