PHYSICAL REVIEW C VOLUME 3, NUMBER 2 FEBRUARY 1971

Semiempirical Generalized One-Boson-Exchange Potentials*

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(Received 17 August 1970)

Four generalized one-boson-exchange-potential (GOBEP) models are examined. The first is a highly realistic model which includes only well-established mesons and which takes account of the resonance width of the isoscalar scalar ϵ meson. Two others are adaptions of this model, each containing a phenomenological term to help improve the fits to high-energy phase shifts. The final model does not assume charge independence of the nuclear force. Meson-nucleon form factors derived from a generalized field theory are used throughout, and a new fitting procedure is introduced which makes allowances for the uncertainties in the theoretical model. The models are compared and evaluated in terms of both the F test for a fit to the phase shifts and the second-derivative matrix method for a fit to observable data. Our basic intention is to provide realistic GOBEP models suitable for nuclear-structure and nuclear-matter calculations.

I. INTRODUCTION

Realistic descriptions of the nucleon-nucleon (N-N) force have been provided by one-boson-exchange potentials (OBEP), especially by generalized OBEP's (GOBEP)[1-3] which include meson-nucleon form factors. Since Green and Sawada's work[1] achieved reasonable fits to all N-N scattering data up to about 150 MeV with only two adjustable parameters, it appears that the GOBEP model is qualitatively correct. However, as an approximate model, it cannot be expected to provide exact agreement with all experimental data at all energies.

Here we discuss four models which are in good agreement with experiment and thus provide potentials which should be useful for nuclear-structure and nuclear-matter calculations. While the various phenomenological potentials or energy-dependent phase-shift formalisms may be very accurate in interpolating on-energy-shell data, there is no assurance that they retain this accuracy when extrapolated into the off-shell regions. To provide the accurate off-shell transition amplitudes required by nuclear many-body problems, what is needed is an exact model which realistically incorporates all important ingredients of the nuclear force. As a step toward the eventual formulation of such a model, we examine an approximate GOBEP model which is in quantitative agreement with scattering data up to the inelastic threshold while involving only well-established mesons. The other models we discuss are modifications of this basic model.

Section II discusses the manner in which we derive meson-nucleon form factors from a generalized field theory. A new fitting procedure which provides a reasonable compromise between the high accuracy of some of the phase shifts and the inherent uncertainties is described in Sec. III. In Sec. IV we utilize these form factors and the new fitting procedure in a model which also takes account of the instabilities of the recently discovered $I=0$, $S=0$ ϵ meson. This differs from most previous models in that it does not contain any fictitious mesons such as the weakly-coupled σ used in earlier realistic GOBEP.[1,2]

Since GOBEP is an approximate model, we must still use some empirical adjustment to provide a precise fit to the data. The results of the Green-Sawada[1] two-parameter model indicate that the greatest discrepancies are to be expected in the high-energy region. This is not at all surprising since one would expect multiple meson exchange and relativistic effects to become more important as the energy is increased. In Sec. V we introduce a phenomenological L^2 force and in Sec. VI we introduce a contact potential in an attempt to obtain improved agreement with experiment.

Another problem which has plagued many of our previous models relates to the statistical errors of the experimental data. While the p-p scattering data have been determined quite precisely, the n-p data are much less well known.[4,5] Consequently, if the traditional χ^2 goodness-of-fit tests are used to adjust model parameters, the p-p data are weighted much more heavily than the n-p data. Our models generally would fit the p-p data better than one could reasonably expect from an approximate model while some of the fits to the n-p data are quite poor. In an attempt to circumvent this problem, we have attempted fits to the p-p and n-p data separately, the results of which are described in Sec. VII together with a discussion of the possibility of charge dependence of the nuclear force. In Sec. VIII the models are compared and discussed with respect to their agreement to both phase shifts and observable data.

II. FORM FACTORS

The generalized meson-field theory of Green[6] which incorporates arbitrarily high derivatives in the field Lagrangian leads to a nonsingular, or regularized, generalized meson potential. It has recently been shown[7] that this same regularized potential may be obtained from the usual meson-field theory by introducing a form factor at each meson-nucleon vertex of the form

$$F(k^2)=\prod_{i=1}^{n}(1+k^2/\Lambda_i^2)^{-1/2}, \tag{1}$$

where the form-factor parameters Λ_i are often called regulator masses.

We have given our form factors the normalization $F(0)=1$, as used in the work of Ueda and Green.[2,7] The coupling constant convention used in the well-regulated potential of Green and Sawada[1,8] resulted in form factors having the normalization $F(\mu^2)=1$. For purposes of comparison, g^2, the square of a meson-nucleon coupling constant from one of these early papers, should be multiplied by $\prod_{i=1}^{n}(1-\mu^2/\Lambda_i^2)^{1/2}$ to bring it into agreement with our present convention. This also applies to the recent publication of Stagat, Riewe, and Green.[9]

The form factor with $n=1$, i.e. "monopole regularization," results in a unrealistic form factor $(1+k^2/\Lambda^2)^{-1/2}$. Nevertheless, it is sufficient to eliminate the singularities from numerical calculations without the use of a hard-core cutoff and has been used for this purpose in several of the earlier models.[3,8]

Form factors which are more realistic may be obtained by choosing the Λ_i of Eq. (1) to be equal in pairs. This results in the expression

$$F(k^2)=\prod_{i=1}^{N}(1+k^2/\Lambda_i^2)^{-1}. \tag{2}$$

Note that for $N=1$ we have a "monopole" form factor, or dipole regularization. Setting $N=2$ with $\Lambda_1=\Lambda_2$ results, as pointed out by Ueda and Green,[7] in the familiar Hofstadter-Wilson dipole form factor, or quadrupole regularization.

In most GOBEP models, attempts were made to use regulator masses as adjustable parameters in order to provide greater flexibility in fitting experimental N-N phase shifts. However, the calculations were found not to be as sensitive to changes in the regulators as had been hoped. Any small change in the regulator masses could be compensated for by readjusting the coupling constants. Furthermore, two or more unequal regulators could be set equal to some properly chosen intermediate value without altering the results. Allowing the regulators to take on different values for each meson also resulted in little improvement, except in some cases where the pion regulator was set slightly higher than the rest.

Thus, we conclude that the fit to the N-N phase shifts is not very sensitive to the precise values of the regulators as long as they are constrained to lie in the region of about 1300 MeV $<\Lambda_i<$ 2500 MeV. That is, the χ^2 surface, viewed as a function of the regulator masses, apparently possesses a very broad minimum in this region. Consequently, so as not to overburden ourselves with a profusion of adjustable parameters, we have chosen to set $N=2$ and take $\Lambda_1=\Lambda_2=2M_n$ for all mesons, where M_n is the nucleon mass.

III. MODIFIED FITTING PROCEDURE

The most serious discrepancies in the pure OBEP fits achieved to this point occur at high energies in $^1P_1$, $^1D_2$, $^3D_2$, and ρ_1. Since three of these four are $T=0$ states, one is led to suspect that the over-all quality of our fits may be strongly influenced by the fact that the standard errors for the $T=1$ phase shifts are approximately only $\frac{1}{4}$ of those for the $T=0$ case.[4,5] This, of course, is because the $T=1$ phase parameters are obtainable from the very precise p-p data whereas for the $T=0$ states one must rely on the much poorer n-p data. As a result, a standard χ^2 minimization procedure will force a very precise fit to the $T=1$ states while tolerating rather large discrepancies in the $T=0$ states. Now since the one-boson-exchange model is admittedly an approximate theory one cannot realistically expect it to provide an arbitrarily precise representation of the p-p data.

Perhaps the most reasonable means of overcoming this difficulty would be to introduce an estimate of the errors inherent in the theoretical model. Then, if a given phase shift, as calculated from the theory, is uncertain by an amount ϵ_{th} while the experimental error is ϵ_x, it is easy to show that the appropriate error to be used in determining the χ^2 is given by

$$\epsilon^2=\epsilon_{th}^2+\epsilon_x^2. \tag{3}$$

If our previous discussion of the difficulties we encountered in trying to simultaneously fit the $T=0$ and $T=1$ phase shifts is valid, we would expect to find that $\epsilon_{th}<\epsilon_x$ for the $T=0$ states whereas $\epsilon_{th}>\epsilon_x$ for the $T=1$ states. This would result in our fits to the $T=0$ phase shifts being limited in precision essentially by the experimental uncertainties while the $T=1$ fits are largely limited by the approximations in the model.

The essential difficulty now is to provide reasonable estimates of the theoretical errors. This is a problem which is by no means trivial if one is after a very rigorous result. We have satisfied ourselves with defining a prescription which, we feel, does provide a reasonable estimate. In performing our initial searches on what we believe is our most physically realistic model (see Sec. IV), we allowed ourselves to vary all coupling constants, including that of the pion. We obtained a "best" fit with a value of $g_\pi^2 = 13.5$ as opposed to a value of 14.8 which is obtained from π-N scattering data.[10] If our model were exact, we would expect that our best fit should occur for a coupling constant very close to the experimentally determined value. That this is not the case is a reflection of the approximate nature of our model and the difference between 13.5 and 14.8 is, in some sense, a measure of the errors inherent in the model. Thus, if we consider the change in the phase shifts as we change the pion coupling constant from 13.5 to 14.8, we can expect this to provide an estimate of the uncertainties in the phase shifts due to the approximations in our model. These "theoretical" errors were compounded with the experimental errors, as described previously, and have been used in the studies to be described in Sec. IV.

IV. SCALAR MESONS

A weakness in all previous OBEP models was the essential use of scalar mesons ($J=0$) whose existence had not been established experimentally.[11] In particular an $I=0$, $J=0$ (scalar-isoscalar) meson which is needed in all models to cancel the large static contributions of the ω vector meson and to provide a residual attractive interaction in the middle region has been elusive. Recently, however, experimentalists have reached a consensus that scalar mesons do indeed exist.[12] The ϵ, ($I=0$), at long last appears firmly established as a broad S-wave resonance or enhancement in π-π scattering data. The exact width of this resonance is still uncertain and it appears that there are four different possible π-π phase-shift solutions which are consistent with existing data. The δ, ($I=1$), is now quite well established, having a mass of about 960 MeV.[13]

It is our purpose here to present a model which incorporates only the ϵ and δ scalar mesons and which embodies the broad mass nature of the ϵ. We represent the ϵ's potential by

$$J(r) = \int_{\mu_l}^{\infty} \rho(\mu) Y_\mu(r) d\mu \,. \tag{4}$$

This is an obvious adaption of the well known ansatz of Charap and Fubini[14] and its Fourier transform has been used in many recent works.

Because the available π-π data are too imprecise to accurately determine the distribution function $\rho(\mu)$, we select its form in an empirical manner by setting the requirements that it vanish at threshold, $\mu = \mu_l = 2m_\pi$, and reach a maximum value in the vicinity of the resonance position, $\mu = \mu_r$. A form such that the integral in Eq. (4) can be done analytically is convenient for calculations, especially in treating the nonstatic contributions of the ϵ which involve derivatives of $J(r)$.[1]

A function which satisfies these requirements is

$$\rho(\mu) = N[ze^{-(z-1)}]^\nu \,, \tag{5}$$

where $z = (\mu - \mu_l)/(\mu_r - \mu_l)$ and ν is a width parameter. When this expression is inserted in Eq. (4), we obtain

$$J(r) = \frac{e^{-\mu_l r}}{r}\left[\frac{1+(\mu_r - \mu_l)r}{\nu}\right]^{-(\nu+1)} \,, \tag{6}$$

where we have chosen N such that $\int \rho(\mu) d\mu = 1$. Note that the larger the value of ν the narrower the mass distribution and, when $\nu \to \infty$, $J(r)$ becomes a simple Yukawa function corresponding to a meson of mass μ_r. We use this limit for the δ meson as well as the π, η, ω, and ρ mesons.

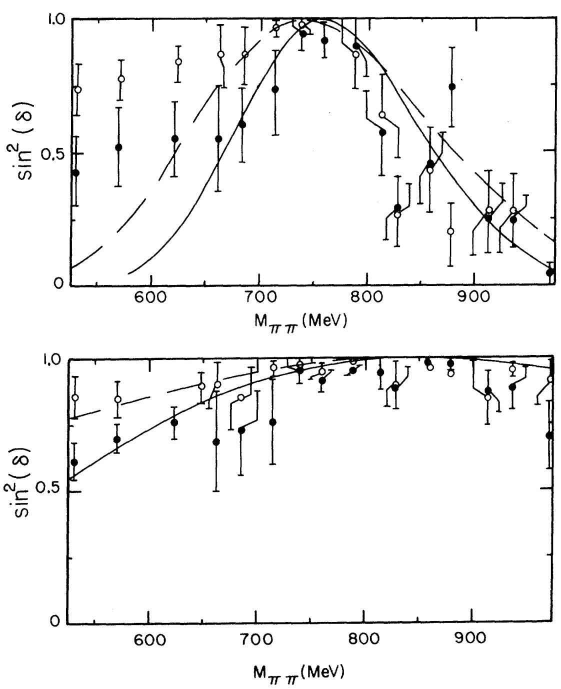

FIG. 1. The first graph shows fits to the up-up solution (open circles, dashed lines, $\nu = 17.4$, $\mu_r = 740$ MeV) and the down-up solution (solid circles, solid line, $\nu = 32.7$, $\mu_r = 760$ MeV) of Smith and Manning. The lower graph shows fits to the up-down (open circles, dashed line, $\nu = 0.8$, $\mu_r = 900$ MeV) and down-down (solid circles, solid line, $\nu = 2.2$, $\mu_r = 850$ MeV) solutions of the same authors.

In practice, we do not use Eq. (6) as it stands, but rather, we choose $J(r)$ to be a superposition of well-regulated potentials[1,2] which have been derived from a generalized field theory.[6] This procedure has recently been interpreted as equivalent to including realistic meson-nucleon form factors.[7] For the broad mass meson, we assume that the corresponding regulator masses are also characterized by the mass distribution given in Eq. (5). To this end, we replace the regulator mass, Λ, of previous works, by a regularization parameter, λ, defined by $\Lambda = \lambda\mu$.

To determine a reasonable range for μ_r and ν we note that the S-wave π-π scattering amplitude can be written approximately as

$$\int \frac{\rho(\mu)}{\mu^2 - k^2 - i\epsilon}\, d\mu \sim \frac{\sin\delta\, e^{i\delta}}{k}. \qquad (7)$$

Taking the imaginary part of each side we get $\rho(\mu) \sim \sin^2\delta$. We therefore fit $\rho(\mu)$ to $\sin^2\delta$ for the various proposed phase-shift solutions.

A fit to one such set of phase shifts[15] is shown in Fig. 1. We found that the proposed phase-shift solutions fall into two distinct classes. One has a resonance position between 700 and 800 MeV and a width from 100 to 400 MeV. The other is an extremely broad solution with a resonance mass about 900 MeV and a width of 1000 MeV or more. Consequently, we chose five different "standard" cases for our consideration. Three of them have resonance masses of 750 MeV and widths of 100, 200, and 300 MeV respectively, while the other two have a resonance mass of 900 MeV and widths of 1000 and 1500 MeV.

We began our test by readjusting the parameters of one of the Ueda-Green[2] models to optimize the fit to the most recent N-N[4,5] phase shifts. The coupling constant and regularization parameter of each of our five test cases were then adjusted to fit the potential due to the two Ueda-Green[2] isoscalars. The fit was excellent at short distances, indicating that the ficticious σ in conjunction with a sharp ϵ simulate the short range effects of a broad-mass ϵ. Finally, a search was performed on all the parameters of the distributed mass mod-

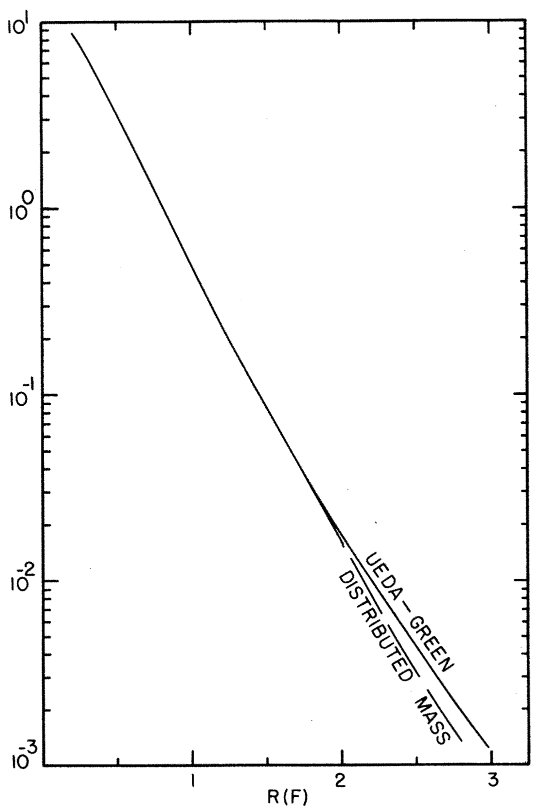

FIG. 2. The well-regulated Yukawa function, $J(r)$, for the sum of the two scalar-isoscalar mesons of Ref. 2 (solid line) and for a single distributed mass meson (dashed line).

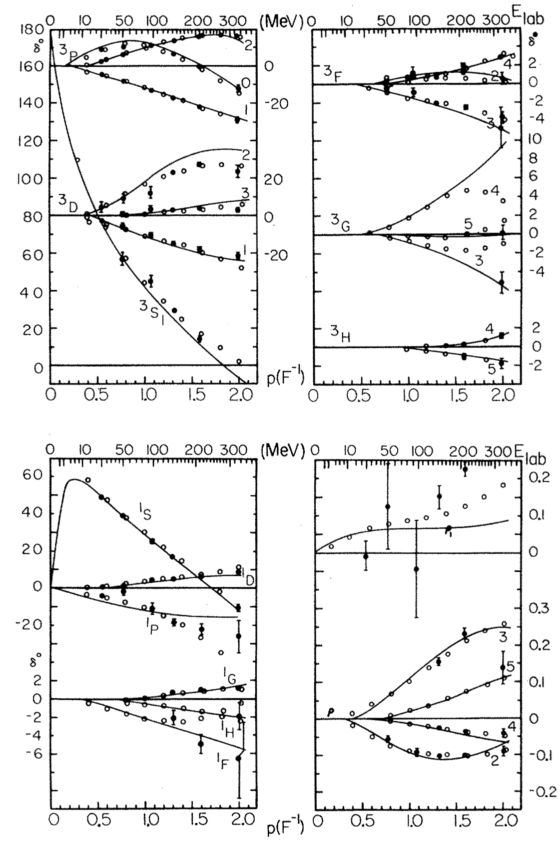

FIG. 3. Fits to the N-N phase shifts for the distributed scalar model. Solid circles are the single energy solutions of MacGregor, Arndt, and Wright and the open circles are the energy-dependent solutions of Breit *et al.* (Ref. 4). Theoretical error bars are not shown.

TABLE I. Potential parameters for all models. DMS: distributed mass scalar; L^2: L^2 force; CT: contact term; RCD: realistic charge dependent; PCD: phenomenological charge dependent. DMS′ and PCD′ are fit to observable data; others are fit to S, P, and D phase shifts at six energies. The distributed mass ϵ meson has a monopole form factor with parameter λ. Each discrete mass meson has a dipole form factor with its parameter fixed at twice the nucleon mass.

Meson	Mass (MeV)	DMS	L^2 [a]	CT [b]	RCD n-p	RCD p-p	PCD n-p	PCD p-p	DMS′	PCD′ n-p	PCD′ p-p
π	138.7	14.26	13.70	12.93	12.97	13.36	10.57	13.38	13.97	10.97	12.87
η	548.7	2.53	3.38	6.87	5.59	5.59	−26.3	6.50	2.48	−25.16	6.69
ω	782.8	10.00	10.5	10.3	15.0	9.57	15.4	8.15	10.00	15.37	8.15
ρ	763.0	0.583	0.957	0.942	1.00	1.14	0.411	1.45	0.69	0.595	1.44
δ	963.0	1.39	1.75	2.89	1.43	1.43	1.69	−1.67	1.39	2.20	−1.84
ϵ	782.8	13.9	14.6	13.3	19.3	13.8	20.3	14.5	13.53	19.68	14.35
$(f/g)_\rho$		5.18	3.74	3.71	3.45	3.48	5.82	3.30	4.87	4.43	3.33
ν		3.80	3.88	3.80	3.84		3.84		3.70	3.84	
λ		1.50	1.58	1.50	1.58		1.58		1.50	1.58	

[a] Also uses $g_L{}^2 = 0.511$.

[b] Also uses $g_c{}^2 = 83.6$.

el to minimize the χ^2 with respect to the N-N phase shifts. This procedure clearly distinguished the five different cases. First of all, the two extremely broad cases had a significantly higher χ^2 than any of the narrow width cases. Furthermore, among the three narrow resonance cases, the χ^2 decreased by approximately 20% with each 100-MeV increase in the width. Our results, then, tend to favor a resonance mass between 700 and 800 MeV with a width of 300 MeV or more.

Very recently it has been suggested that the π-π phase-shift analyses may suffer from an even greater uncertainty than has been believed.[16] Thus, it may be fruitful to allow ourselves greater flexibility in characterizing the ϵ mass distribution than that permitted by π-π phase-shift analyses. We therefore sought a model in which the parameters of the mass distribution were to be determined solely by searching on N-N phase shifts such as have been given by Seamon *et al.*[4] and MacGregor, Arndt, and Wright.[5] Since the phase shifts turn out not to be very sensitive to the precise position of the resonance (as long as it is between 700 and 800 MeV), we chose to impose the five-vector constraint, originally suggested by Green,[17] by setting the resonance mass of the ϵ equal to the mass of the ω.

That our suspicions that the weak light-mass σ used earlier[1,2] primarily described the effects of the instability of the ϵ are plausible can be seen in Fig. 2. Here we have plotted $J(r)$ for a broad-mass scalar and for the sum of the weakly coupled σ and strongly coupled ϵ in the Ueda-Green III[2] model. As can be seen, this combination of two stable mesons can provide an excellent approximation to a single strongly coupled distributed mass ϵ.

Our final fits to the N-N phase shifts are shown in Fig. 3 and the corresponding potential parameters and deuteron parameters are given in Tables I and II. Deuteron wave functions are shown in Fig. 4. In performing the parameter search, we have used the compound errors described in Sec. III with the exception of the $^3S_1$ states, where experimental errors were used as a stronger constraint.

V. L^2 FORCE

In a previous paper, Ueda and Green[7] attempted to remove the discrepancies at high energies by introducing a phenomenological L^2 force in the following form:

$$V_L = -g_L{}^2(\vec{\tau}_1 \cdot \vec{\tau}_2)[\tfrac{1}{2}(1+P)]L^2 J(r), \qquad (8)$$

where P is the parity operator and $J(r)$ is the generalized Yukawa function corresponding to a quadrupole form factor.[2] The presence of the term in square brackets allows this force to act only in even-L states so that the P waves are not adversely by its presence. The isospin dependence is included to reverse the sign of the potential between spin-singlet and spin-triplet states. With

TABLE II. Deuteron parameters obtained from the various models. All models have had the coupling constant of the δ meson adjusted to give the correct binding energy.

E_B (MeV)	DMS	L^2	CT	PCD	Expt.
	2.224	2.224	2.224	2.224	2.2245
ρ (F)	1.85	1.83	1.77	1.80	1.82 ± 0.05
% D	4.6	4.0	4.9	4.1	···
Q (F^{-2})	0.275	0.262	0.258	0.240	0.282
μ (μ_N)	0.855	0.861	0.857	0.849	0.8574

this form it is possible to provide some extra attraction in the $^1D_2$ state, while at the same time providing the needed repulsion in the $^3D_2$ state. Furthermore, the S, P, and F waves will not be affected – the S waves, since $L^2=0$ for these; and the P and F waves, because of the presence of the projection operator. The coupling constant, $g_L{}^2$, is chosen sufficiently small and the "meson mass" in $J(r)$ made sufficiently large that we do not expect this additional term to have any appreciable effect on the higher partial waves.

With this additional freedom, Ueda and Green were able to provide precise fits to the $^1P_1$, $^1D_2$, and $^3D_2$ phase shifts, even at high energies. Unfortunately, no attempt was made to constrain the fits to the very low-energy data, with the result that the deuteron parameters are quite imprecise. In the present work, the model has been derived by adding the potential given by Eq. (8) to the model of Sec. IV. The parameters have been adjusted to yield a much improved fit to the low-energy data. We note that due to the central nature of the L^2 terms (as opposed to a tensor type term, which mixes different partial waves), this model does not provide any significant improvement in our fit to the $J=1$ mixing parameters. The parameter values and the final fit for this model are given in Tables I and II and in Figs. 4 and 5.

VI. CONTACT TERM

When the prescription for determining theoretical errors is introduced, it is found that the resulting compounded errors are quite reasonable, in the sense that they conform to our *a priori* expectations. The lone exception is in the $^3S_1$ state, where the prescribed theoretical errors are significantly larger than we would have liked. Consequently, when a parameter search is performed we find that all the previous major discrepancies can be resolved at the expense of a drastic decrease in the $^3S_1$ phase shift. This may seem a rather large price to pay, since many-body problems are often rather sensitive to this phase shift. However, we do gain the purely statistical advantage that we can now introduce some small amount of phenomenology to correct only one phase shift rather than trying to juggle four at once.

The problem is now reduced to finding some way to increase the attraction in the $^3S_1$ state while holding all other phase shifts approximately constant. This can be accomplished most easily by introducing a very short-range force which acts only in spin triplet states. The short-range feature will prevent it from affecting anything but an S wave while the spin dependence is necessary to avoid destroying the quality of the $^1S_0$ fit. Ideally, this would take the form of a pure contact term (i.e., a δ function of the origin) but the necessity of carrying out numerical calculations essentially forbids this. Instead, we employ a contact term which is "well-regulated"[1,2]; i.e., it is modified by the introduction of a dipole form factor. A pure δ function may be written in the form

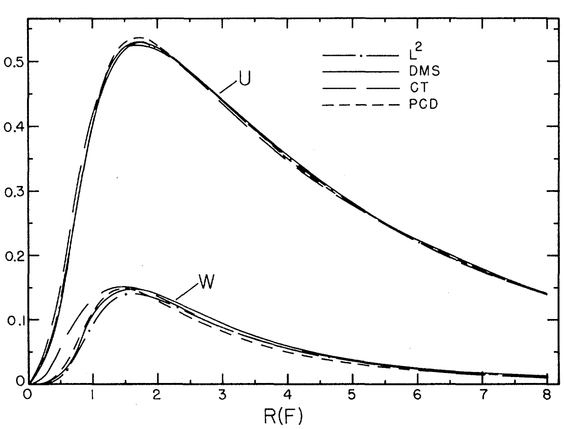

FIG. 4. Deuteron wave functions for all models.

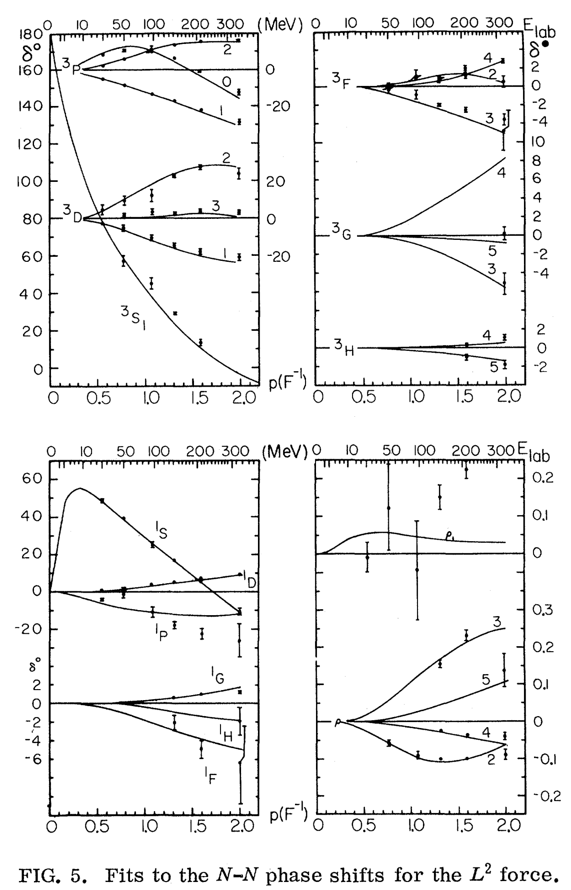

FIG. 5. Fits to the N–N phase shifts for the L^2 force. Experimental data points are those given by MacGregor, Arndt, and Wright.

$$\delta(\vec{r}) = (2\pi)^{-3}\int e^{i\vec{k}\cdot\vec{r}}\, d^3k\,. \tag{9}$$

We can generalize this by introducing one of the form factors described in Sec. II into this integral. That is, if we define:

$$\Delta(\vec{r}) = (2\pi)^{-3}\int e^{i\vec{k}\cdot\vec{r}}\, |F(k^2)|^2\, d^3k\,, \tag{10}$$

where, in our case, we choose $F(k^2)$ to be the dipole form factor, then we note that, in the limit as the regulator masses become infinitely large, the form factor approaches unity and $\Delta(\vec{r})$ approaches $\delta(\vec{r})$. If the regulator masses are chosen to be sufficiently large, then $\Delta(\vec{r})$ will provide a suitably accurate representation of a true contact term without introducing any undue numerical difficulties.

In our first attempt at introducing the contact term, we added a potential of the form

$$V_{\text{Contact}} = g_c^{\,2}\left(\frac{\hbar}{Mc}\right)^3 \Delta(\vec{r}) P_\sigma^{(+)} Mc^2\,, \tag{11}$$

where $P_\sigma^{(+)}$ is a projection operator onto the spin triplet states. The regulator masses were arbitrarily chosen to be $2M_n$ and $g_c^{\,2}$ was chosen so as to give the correct deuteron binding energy. Thus, at the expense of one adjustable parameter, the $^3S_1$ was corrected and, in addition, we were able to obtain a very precise fit to the deuteron parameters. Unfortunately, the introduction of an additional central force resulted in a drastic decrease in the $J=1$ mixing parameter, thus defeating one of the principal reasons for introducing the contact term.

The latter deficiency can be remedied quite simply. By deleting the simple spin dependence and replacing it with a tensor force, that is, by setting

$$V_{\text{Contact}} = g_c^{\,2}\left(\frac{\hbar}{Mc}\right)^3 \Delta(\vec{r})\, \hat{S}_{12}\, Mc^2\,. \tag{12}$$

We can again avoid any conflict with the singlet states while at the same time the $J=1$ mixing parameter can be increased to provide even better agreement with experiment.

The parameters for the latter fit are given in Tables I and II and the deuteron wave functions and phase shifts are plotted in Figs. 4 and 6. The only significant difference is the improved fit to ρ_1 in the tensor force model.

VII. CHARGE DEPENDENCE

As experimental accuracy increases, it is becoming apparent that charge independence is only approximate.[18] The most direct evidence for the violation of charge independence is found by comparing the $^1S_0$ scattering lengths for p-p and n-p scattering. Also Noyes[19] has found that the p-p and n-p effective ranges seem to differ by at least 10% for the $^1S_0$ state, although the evidence is not as strong as in the case of the scattering length[20] and the data are still being revised.[21] These effects need not be due to any fundamental breakdown of symmetry, but may be caused by such phenomena as[18,22,23] two-boson exchange, N-N pair formation, electromagnetic effects on the charged mesons, or nonpseudoscalar pion–physical-nucleon couplings. Also, the small-mass differences between the neutral and charged pions may cause relatively large charge-dependent effects.[18] Any theory of nuclear forces which fails to take these processes into account, must find some means to distinguish between p-p, n-p, and n-n scattering.

The N-N phase shifts used in our GOBEP analysis are determined solely from p-p and n-p scattering because of the relative inexactness of n-n experiments. We will thus be able to consider only the validity of charge *symmetry*, rather than

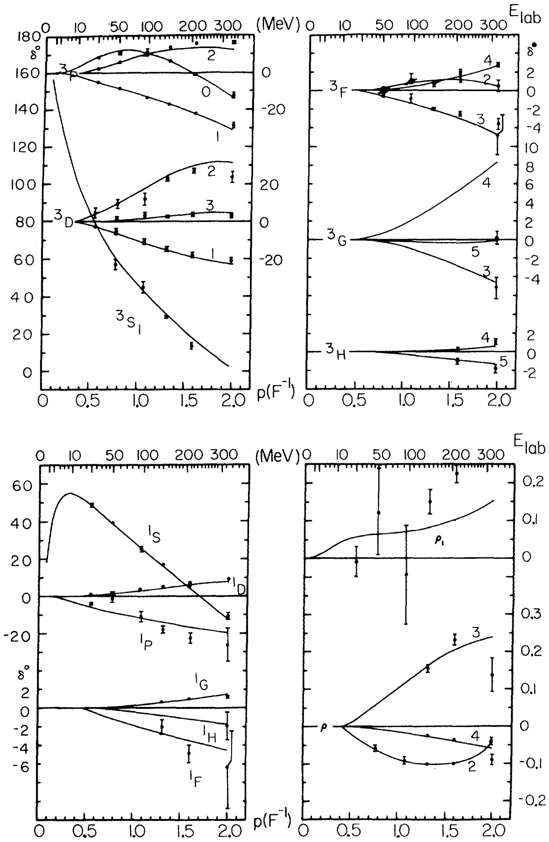

FIG. 6. Fits to the N-N phase shifts for the contact term model. Experimental data points are those given by MacGregor, Arndt, and Wright.

TABLE III. χ^2 and F for models fitted to phase shifts.

Model	χ^2	No. of data	χ^2/datum	Parameters	F
DMS	321	69	4.65	9	1.00
L^2	227	69	3.29	10	1.39
RCD	291	74	3.93	14	1.10
PCD	196	74	2.65	16	1.58

complete charge *independence*. The phase shifts for states with total isospin $T=0$ are determined from n-p data while those with $T=1$ result from p-p data. The only exception is the ${}^1S_0$, for which both an n-p and a p-p phase shift are given. Hence each experimental phase shift corresponds to either n-p or p-p scattering.

We can introduce charge dependence into a GOBEP model by fitting to the n-p and the p-p data separately. This procedure, of course, gives better agreement with experiment than charge-independent calculations due simply to its greater

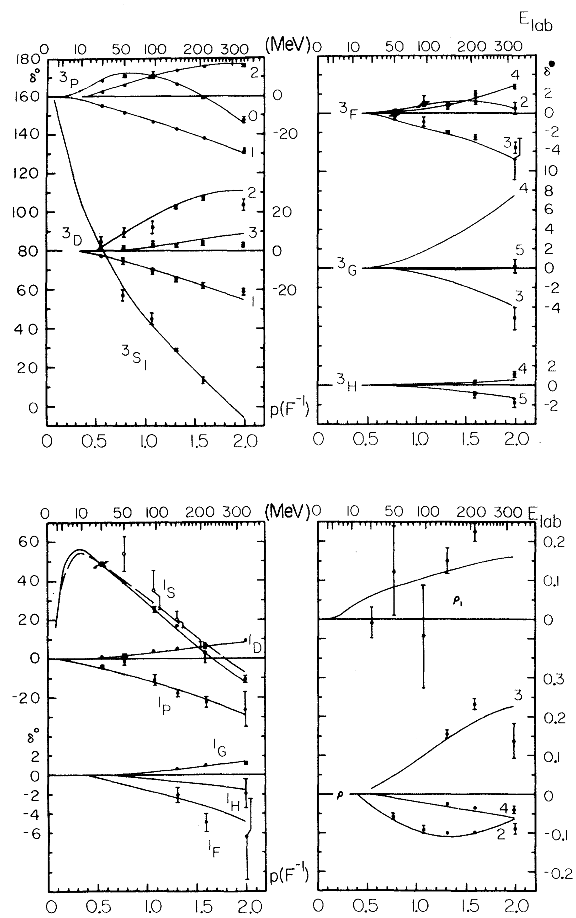

FIG. 7. Fits to the N-N phase shifts for the phenomenological charge-dependent model. Experimental data points are those given by MacGregor, Arndt, and Wright.

number of arbitrary parameters. The improved accuracy increases the usefulness of the calculated potentials in applications where n-p and p-p interactions may be treated individually. Also, the need for the "theoretical error bars" of Sec. III disappears, since n-p and p-p data are now put on an equal footing.

As a starting point, consider the distributed scalar model discussed in Sec. IV, for which a fit using the actual, rather than the "theoretical," error bars gives a χ^2 of 321 for 69 data. There is no reason to assume that the scalar-meson parameters ν and λ should depend on the charge state. Also, the coupling constants for the η and δ must change considerably to produce an appreciable change in the phase shifts. Thus holding ν, λ, g_η^2, and g_δ^2 constant, separate searches on n-p and p-p data give a total χ^2 of 291 for 74 data, where both the n-p and p-p ${}^1S_0$ phase shifts are used. The coupling constants are given in Table I. Since the χ^2 did not improve greatly, we must conclude that if the GOBEP model is essentially correct, then the effects of charge dependence are indeed slight.

Note that the pion coupling constant is greater for p-p than for n-p scattering. This conclusion was also reached by Seamon, Friedman, and Breit,[22] who calculated that $g_{pp}^2 - g_{np}^2 = 2.2 \pm 1.3$. We found, however, that an improvement in the χ^2 did not result unless coupling constants other than that of the pion were assumed to be charge dependent. Since the coupling constants for this model take on physically reasonable values, we shall refer to it as the realistic charge-dependent (RCD) model.

A much better fit to the experimental phase shifts can be achieved if g_η^2 and g_δ^2 are not fixed, but allowed the same freedom as the other coupling constants. The resulting χ^2 is 196, or 2.65 per datum, which is a definite improvement over both the distributed scalar model and the RCD model. However, the coupling constants take on the unusual values given in Table I. Thus they must be considered to be phenomenological parameters with no specific physical significance. It appears that g_η^2 and g_δ^2 assume negative values in order to compensate for approximations or missing ingredients in the model. We term this model the phenomenological charge-dependent (PCD) model. The phase shifts are shown in Fig. 7 and deuteron parameters and wave functions are given in Table II and Fig. 4.

VIII. DISCUSSION AND CONCLUSION

Our efforts in this paper have been in two different, although related, directions. Our first objective has been a further examination of the basic validity of the GOBEP model. One of the major criticisms[11] of past models has been their utilization of scalar mesons which had not been observed experimentally. Experiment, however, now supports the use of two of the scalar mesons – the ϵ and δ. However, the unestablished σ still remains in some models[1,2] as an *ad hoc* weakly coupled light scalar meson (m_σ = 416 MeV) which is used to provide an extra middle-range attractive force. Our basic model eliminates this σ and thus contains only mesons which are well-established experimentally. Furthermore, an allowance is made for the broad-mass nature of the ϵ in a realistic way. It should be noted that our ν value (3.8) corresponds to a very broad-mass distribution. A similar conclusion was reached by Furuichi, Kanada, and Watanabe.[24] This might in part reflect a phenomenological allowance for virtual excitation of the N^* and for uncorrelated 2π effects.

Our second objective has been to satisfy the very practical need for a precise description of the nuclear force in a form which is convenient for many-body calculations. Before we evaluate our results in this light, let us discuss the general question of comparison of a theoretical nuclear force model with experiment.

The values of χ^2 per datum provide a basis for comparing the fit of each model with the experimental phase shifts. However, such a comparison does not take into account the number of adjustable parameters in each model. Thus we cannot tell whether an improvement in χ^2 signals the discovery of a new ingredient in the nuclear force or whether it is merely a result of the flexibility in curve-fitting gained by an increase in the number of paremeters.

The F test may be used as a statistical test for approximately assessing the effect of a change in the number of parameters. For each model we find the χ^2 per degree of freedom by dividing the total χ^2 by the number of data less the number of adjustable parameters. The ratio of this quantity for one model to that of another is the F value for the models. An F of approximately unity indicates that the improvement in χ^2 is just what would be expected due to the extra number of parameters, provided that the experimental error bars have realistic values. If F differs from unity, the confidence level for assuming that the improvement is real, rather than statistical, can be found in standard tables. The F test may have applicability to our present study since the number of phase-shift "data" is of the same order as the number of adjustable constants required to individually parametrize the phase shifts as, for example, in energy-dependent phase-shift analyses.[4,5]

Taking the distributed scalar model as our basis and using actual, rather than "theoretical," error bars we can calculate the F value for our other models. Since each of these models results from the addition of a component of the nuclear force to the distributed scalar model, F is roughly a measure of how likely it is that the component is meaningful. The values of F, along with other information, are given in Table III. The contact term model is not included for reasons which will be explained.

Assuming that the F test is applicable to our phase-shift fits, the F value of 1.39 for the L^2 force indicates a much greater improvement than chance associates with one additional parameter.

From the two charge-dependent models we can see why the F test is a useful comparison tool. Although the χ^2 per data point is improved in the realistic charge-dependent model, the F value is close to unity. This suggests that the improved fit to the data probably results only from the increase in the number of parameters and not from any actual charge dependence in the phase shifts.

TABLE IV. χ^2 per datum determined both from phase shifts and from the second-derivative matrix.

	χ^2 per datum	
	Phase shifts	Observable data
Charge-dependent [a]	4.12	2.27
Charge-dependent	2.65	2.59
Distributed mass [a]	7.14	4.24
Ueda-Green I	8.1	4.44
Gersten-Thompson-Green [a,b]	...	4.58
Contact term	11.1	4.87
Distributed mass	4.65	5.31
L^2 force	3.29	5.43
Bryan-Scott	...	12.13
Chiang-Gleiser-Huq	>70	>50

[a] Fit to the observable data using the second-derivative matrix.

[b] Fit includes 425-MeV data.

However, the F test suggests that the phenomenological charge-dependent model shows an improvement which would be expected to result from chance alone less than 5% of the time. Thus, it appears that there is a significant extra ingredient of the nuclear force which is not included in our basic model and which depends upon the value of the total isospin quantum number.

Although the goodness-of-fit to the phase shifts gives a reasonable indication of the relative merit of each model, the procedure may not provide reliable information concerning how well the models predict observable data such as cross sections or polarizations. A better method would be to adjust the GOBEP parameters to fit directly to observable data. This, however, requires too much computer time to be practical in our case. Instead we use the method of fitting to the second-derivative matrix developed by Arndt and MacGregor[5,25] which is almost equivalent to fitting the experimental data.

Table IV gives the χ^2 per datum of several models determined both from phase shifts and the second-derivative matrix method. Included are models by Ueda and Green,[2] Bryan and Scott,[3] Gersten, Thompson and Green,[26] and Chiang, Gleiser, and Huq.[27] The entries from searches using the second-derivative matrix are indicated. A glance at the table reveals that there is little correlation between the two methods of determining χ^2. Of the cases which were fit to the phase shifts, only the phenomenological charge-dependent model had a χ^2 which did not change greatly when computed with respect to the observables. The Ueda-Green model, which was fitted to older phase-shift data, still gives a good fit to the observables. The model of Bryan and Scott was fitted to the phase shifts at three energies. A slight readjustment of the paremeters would no doubt lead to a much better fit to the observables.

The model of Chiang, Gleiser, and Huq is a phenomenological adaptation of an earlier model which was based on Sudarshan's[28] universal theory of primary interactions. In this recent form, the model has the same essential mesonic ingredients (π, ω, ρ, η, and an ϵ or σ) of almost all one-boson-exchange models (see Refs. 1 or 27 for 1962–1964 literature references). One coupling constant and a cutoff value are the only adjustable parameters. Although axial-vector mesons (A, D, and E) are included, their effects are for the most part screened by the hard-core cutoff at 0.4 F. As indicated by the values of χ^2 the model does not provide a quantitative description of the N-N interaction. Thus this recent model must be classed with earlier qualitative models which use three or less parameters by embodying relationships between coupling constants. Among these are the five-vector model of Green,[17] the modified five-vector models of Green and Sawada,[1,8] and the zero-parameter model of Sugawara and von Hippel.[29] Thus far realistic models which quantitatively characterize the experimental N-N data (including the deuteron properties) appear to require six or more adjustable parameters and hence might appropriately be referred to as semiempirical.

If we compare the parameters in Table I for the distributed mass scalar (DMS′) with DMS and PCD′ with PCD, one notes that only minor readjustments are needed to optimize fits to the observables. The changes in phase shifts on the figures are slight, although significant contributions to the phase shift χ^2 are made by phase shifts with small error bars. The effect is undoubtedly due to correlations in phenomenological phase-shift assignments which reflect the particular philosophy used in fitting the experimental data.

It should be mentioned that we have tried, briefly, to optimize the L^2, and the contact term models using the second-derivative matrix, but did not find a descending path in the χ^2 hyperspace near our previous solutions. We believe an extensive search would lead to parameters whose χ^2 is somewhat lower than DMS′. However, on the basis of our phase-shift search work, we suspect the gains will not be as large as that achieved by the PCD′ model.

The significance of the negative η coupling constant in the PCD′ is obscure. It may possibly relate to errors in n-p data or possibly be associated with a middle-range uncorrelated two- or three-pion effect not fully described by the broad ϵ. In general, changes in the η coupling constant have much smaller effects than changes in other coupling constants. The negative δ coupling constant obtained in fitting the p-p data is probably a reflection of a missing short-range ingredient.

In final conclusion, we note that from the viewpoint of observables the UG-I represents a good GOBEP model incorporating two scalar-isoscalar mesons with sharp masses at 416 and 1016 MeV. The model DMS′, which incorporates the broad mass ϵ and eliminates the *ad hoc* σ, is our most physical and precise model constrained by charge dependence. Our PCD′ is our most precise model. This model implies a small charge-dependent component. It is clear, however, that existing experimental data, especially for the n-p interactions, are now the limiting factor determining how strongly one can state conclusions concerning comparisons between various physical models. We have noted that optimization to phase shifts and optimization to the second-derivative matrix led to slightly different sets of model parameters but substantially different assignments of χ^2/datum. This suggests a definite need for further additions to and improvements in the N-N data and the accompanying refinements in the phase-shift assignments or second derivative matrix assignments. With such advances those pursuing theoretical models could determine more precisely the importance of effects which remain to be examined such as charge dependence, relativistic effects, effects of uncorrelated multimeson exchanges, virtual excitation of the N^*, recoil effects, and a whole gamut of additional effects related to the structure and interaction of nucleons.

We wish to thank M. Parkinson, J. S. Hornstein, R. H. Thompson, and D. L. Sellin for their helpful comments.

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*Work supported in part by the U. S. Air Force Office of Scientific Research Grant No. AFOSR-68-1397.

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