The kernel A differs from A_0 by a kernel of rank 1. If the largest eigenvalue of A_0 is nondegenerate, it can be removed by putting

$$(k'|\gamma|k) = (k'|\gamma_0|k) - \frac{\alpha_0\chi(k')(k^2 - k'^2)}{\int_0^\infty dk''(k|V|k'')\chi(k'')}, \tag{58}$$

where α_0 is the largest eigenvalue of A_0 and $\chi(k)$ is the corresponding eigenfunction, i.e.,

$$A_0\chi = \alpha_0\chi . \tag{59}$$

This is the optimum choice for γ if uniform convergence of the series (45) is desired.

V. CONCLUSIONS

The preceding analysis can be trivially extended to a finite number of coupled channels.

The unmodified Sasakawa series converges for local potentials of arbitrary strength. This feature does not obtain for nonlocal potentials or the modified series. But under very general conditions the Sasakawa kernel A is a Hilbert-Schmidt kernel, and both $(r'|A|r'')$ and $(k'|A|k'')$ are continuous functions of their arguments. Thus, the integral equation

$$A\Omega = A\Omega_0 + A(A\Omega) \tag{60}$$

can be solved numerically to arbitrary accuracy both in the radial representation and in the momentum representation.

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Neutron-Proton Interaction in Odd-Odd Deformed Nuclei*

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Calculations of the energy splittings between the parallel and antiparallel coupling states of odd neutron and odd proton in deformed nuclei have been made for both zero-range and finite-range nuclear forces. The energy splittings for both types of forces agree with the corresponding experimental energies. Calculations have also been made for the odd-even shift in $K=0$ bands. Finite-range interactions improve the agreement between experiment and theory for the odd-even shift, but the results are still unsatisfactory. It is shown that a tensor force is important in reproducing the experimental odd-even shift, especially in the case where $\Sigma_n + \Sigma_p = 1$.

INTRODUCTION

One of the most important problems in nuclear physics is the determination of the effective residual interaction. In general, the nuclear force has many facets and plays involved roles in the nucleus. Nevertheless, sometimes we can see certain characteristic parts of the nuclear force by using the appropriate phenomena. For example, the spectroscopy of odd-odd nuclei gives us detailed information about the neutron-proton interaction. Many authors have investigated the neutron-proton interaction in the framework of the spherical shell model.[1] The Nordheim rule,[2] which was proposed for the spherical odd-odd nuclei in order to predict ground-state spins, has been extended to deformed odd-odd nuclei by Gallagher and Moszkowski.[3] In the deformed odd-odd nucleus, there are twofold degenerate states in which the coupling of the spins of neutron and proton is either parallel or antiparallel along the symmetry axis. After rotational energies are subtracted, the lowest-order term in the splitting energy between these intrinsic states is caused by the residual neutron-proton interaction. These splitting energies, then, are a probe for the residual neutron-proton force. Another even more sensitive probe is the odd-even energy shift [the difference in energy between the observed and normal $I(I+1)$ energies of even- and odd-spin members of $K=0$ bands].

Calculations of the parallel-antiparallel splitting energy and the odd-even energy shift have been made by de Pinho and Picard[4] and by Newby.[5] However, no effort was made to find a consistent interaction for the description of many different deformed odd-odd nuclei simultaneously. A more systematic approach for a number of odd-odd deformed nuclei was tried by Pyatov.[6] However, since a zero-range force was used, the exchange character of the nucleus could not be included and backward scattering, which is especially important in the odd-even shift for large $|\Omega_p|=|\Omega_n|$, might have been overestimated. In this paper, more detailed comparisons between experiment and theory are presented. Considerable additional recent[7-10] experimental data are compared with the theory which involves potentials including a variety of central force ranges and a tensor force.

THEORY

A reasonable model of the odd-odd axially symmetric deformed nucleus may be obtained by combining neutron and proton single-particle Hamiltonians with the rotational kinetic energy of the entire system and the residual interaction between the neutron and proton. This description may be represented by the following Hamiltonian:

$$H=H_R+H_n+H_p+H_{\mathrm{RPC}}+H_{\mathrm{int}}, \qquad (1)$$

where

$$H_R=\frac{\hbar^2}{2\mathfrak{I}}[\vec{\mathrm{I}}\cdot\vec{\mathrm{I}}-I_3{}^2], \qquad (2)$$

$\mathfrak{I}$ is the moment of inertia. $\vec{\mathrm{I}}$ is the total angular momentum operator and I_3 is the component along the symmetry axis. The Hamiltonian H_R has solutions of the form $\mathfrak{D}^I_{MK}$, which are the well-known rotational functions.[11] The second and third terms are the single-particle Hamiltonians in the deformed well for the neutron and proton, respectively.

The new Nilsson potential[12] used to generate the single-particle Hamiltonians for both neutrons and protons is

$$H_{n(p)}=-\frac{\hbar^2}{2m}\nabla^2+\frac{m\omega^2}{2}r^2+C\vec{\mathrm{L}}\cdot\vec{\mathrm{S}}+D(\vec{\mathrm{L}}^2-\langle\vec{\mathrm{L}}^2\rangle_{\mathrm{shell}})$$
$$-\delta\hbar\omega_0\tfrac{4}{3}(\tfrac{1}{5}\pi)^{1/2}r^2Y_{20}, \qquad (3)$$

where $\langle\vec{\mathrm{L}}^2\rangle_{\mathrm{shell}}$ is the average value of $\vec{\mathrm{L}}^2$ in one major shell. The single-particle wave functions obtained are

$$|\nu\Omega\rangle=\sum_j C^{\Omega}_{\nu j}|j\,\Omega\rangle, \qquad (4)$$

where $|j\,\Omega\rangle$ is the $j-j$ coupling harmonic-oscillator wave function and $C^{\Omega}_{\nu j}$ may be obtained by solving Eq. (3). The unperturbed Hamiltonian is defined as

$$H_0=H_R+H_n+H_p, \qquad (5)$$

and the eigenfunction of H_0 is given as

$$\varphi_{IMK}=\mathfrak{D}^I_{MK}|\nu_n\Omega_n\rangle|\nu_p\Omega_p\rangle. \qquad (6)$$

This expression must be symmetrized under a rotation of π about an axis perpendicular to the symmetry axis. The resulting expression for the normalized wave functions is

$$\psi_{IMK}=\left(\frac{2I+1}{16\pi^2}\right)^{1/2}[\mathfrak{D}^I_{MK}|\nu_n\Omega_n\rangle|\nu_p\Omega_p\rangle$$
$$+(-)^{I-K}\mathfrak{D}^I_{M-K}|\nu_n\tilde{\Omega}_n\rangle|\nu_p\tilde{\Omega}_p\rangle], \qquad (7)$$

where $|\nu\tilde{\Omega}\rangle$, obtained by rotating by π about the y axis, is

$$|\nu\tilde{\Omega}\rangle=R_y(\pi)|\nu\Omega\rangle=\sum_j C^{\Omega}_{\nu j}(-)^{j-\Omega}|\nu-\Omega\rangle. \qquad (8)$$

If $C^{\Omega}_{\nu j}$ is real, $|\nu\tilde{\Omega}\rangle$ corresponds to the time reversed state of $|\nu\Omega\rangle$. If the time reversed state of $|\nu\tilde{\Omega}\rangle$ is defined as

$$|\nu\tilde{\Omega}\rangle=(-)^{\Omega-1/2}|\nu-\Omega\rangle, \qquad (9)$$

then $C^{-\Omega}_{\nu j}$ is automatically related to $C^{\Omega}_{\nu j}$ by

$$C^{-\Omega}_{\nu j}=(-)^{j-1/2}C^{\Omega}_{\nu j}. \qquad (10)$$

It should be pointed out that the quantum number K is related to $\Omega(n)$ and $\Omega(p)$ by the equation

$$K=\Omega(n)+\Omega(p). \qquad (11)$$

Since $\Omega(n)$ and $\Omega(p)$ can be either positive or negative, we see that for a given configuration two values of K are possible, namely

$$K_1=\|\Omega_p|-|\Omega_n\| \qquad (12)$$

and

$$K_2=|\Omega_p|+|\Omega_n|. \qquad (13)$$

Thus, each intrinsic configuration gives rise to a pair of states with superimposed rotational bands. These pairs of states, called Gallagher-Moskowski pairs, are the primary object of this study. The unperturbed energy of these states is given by

$$E(K)=\frac{\hbar^2}{2\mathfrak{I}}[I(I+1)-K^2]+E_n+E_p. \qquad (14)$$

In the total Hamiltonian H_{RPC}, the rotational particle coupling and the particle-particle coupling in rotating nuclei is defined as

$$H_{\mathrm{RPC}}=-\frac{\hbar^2}{\mathscr{g}}[I_1(j_{n1}+j_{p1})+I_2(j_{n2}+j_{p2})-j_{n1}j_{p1}-j_{n2}j_{p2}]\,, \tag{15}$$

where j_{ni} and j_{pi} are the angular momentum operators of the neutron and proton, respectively, in the frame of the body-fixed coordinate system. The term $(\hbar^2/2\mathscr{g})(j_{n1}{}^2+j_{n2}{}^2+j_{p1}{}^2+j_{p2}{}^2)$ should be added to Eq. (15), but it involves only single-particle operators, and therefore its contribution to $E(K)$ can be absorbed by the constants E_n and E_p in Eq. (14). If we neglect configuration mixing, H_{RPC} does not contribute to the energy except in the case where $|\Omega_n|=\frac{1}{2}$, $|\Omega_p|=\frac{1}{2}$, and $K=0$. In this case the diagonal matrix element simplifies to the form,

$$\langle\psi_{IM0}|H_{\mathrm{RPC}}|\psi_{IM0}\rangle=-(-)^I\frac{\hbar^2}{2\mathscr{g}}a_n a_p\,, \tag{16}$$

where a_n and a_p are the decoupling parameters for the Nilsson orbits, defined by

$$a_{n(p)}=\sum_j(-)^{j-1/2}|C^{\,\Omega}_{\nu j}|^2(j+\tfrac{1}{2})\,. \tag{17}$$

The last term in Eq. (1), H_{int} is the residual interaction which couples the last odd neutron and proton. The matrix element of the residual interaction is calculated in the j-j coupling basis:

$$\langle\nu_n\Omega_n\nu_p\Omega_p|V|\nu_n\Omega_n\nu_p\Omega_p\rangle=\sum_{j_nj_pj_n'j_p'}C^{\,\Omega_n}_{\nu_nj_n}C^{\,\Omega_p}_{\nu_pj_p}C^{\,\Omega_n}_{\nu_nj_n'}C^{\,\Omega_p}_{\nu_pj_p'}\langle j_n\Omega_nj_p\Omega_p|V|j_n'\Omega_nj_p'\Omega_p\rangle\,. \tag{18}$$

The interactions are represented in terms of a tensor expansion, where the central potential is

$$V_C=\sum_k(2k+1)\mathfrak{V}^{(k)}(r_n,r_p)\Big[[(V_W+V_B/2)+(V_H+V_M/2)P_H][C^{(k)}(\hat{r}_n)C^{(k)}(\hat{r}_p)] +\Big((V_B/2+V_M/2P_H)\sum_T(-)^{k+1-T}\{[\sigma_n\times C^{(k)}(\hat{r}_n)]^{(T)}[\sigma_p\times C^{(k)}(\hat{r}_p)]^{(T)}\}\Big)\Big]\,, \tag{19}$$

and $C^{(k)}_q(\hat{r})$ is related to the usual normalized spherical harmonic $Y_{kq}(\hat{r})$ by

$$C^{(k)}_q(\hat{r})=\left(\frac{4\pi}{2k+1}\right)^{1/2}Y_{kq}(\hat{r})\,. \tag{20}$$

We used the Heisenberg exchange operator, P_H instead of the Majorana exchange operator because the j-j coupling scheme was used as a basis.

In a similar fashion, tensor forces can be expanded.

$$S_{12}V_T(\vec{r}_{pn})=(\tfrac{2}{3})^{1/2}\sum_{k_nk_p}i^{k_n+k_p}(2k_n+1)(2k_p+1)\langle k_nk_p00|20\rangle \times W(k_n1k_p1;T2)\mathfrak{V}^{(k_nk_p,2)}(r_n,r_p)\Big([\sigma_n\times C^{(k_n)}(\hat{r}_n)]^{(T)}[\sigma_p\times C^{(k_p)}(\hat{r}_p)]^{(T)}\Big)\,. \tag{21}$$

The first term of the central-force interaction does not contribute to the splitting energy and the odd-even shift. This is shown simply in what follows. In general the result of sign changes in omega (Ω) can be written

$$\langle j-\Omega|\hat{O}^{\lambda}_0|j'-\Omega\rangle=(-)^{j+\lambda-j'}\langle j\Omega|\hat{O}^{\lambda}_0|j'\Omega\rangle\,. \tag{22}$$

If the phase $(-)^{j-j'}$, which comes from Eq. (10), is taken into account, an operator of even rank does not contribute to the splitting energy between the parallel and antiparallel states. The odd-even shift is caused by the cross term in the energy matrix between the first and second parts of Eq. (7); that is, the symmetrized term with respect to the π rotation about the y axis. This odd-even shift caused by the residual interaction is nonzero only if $K=0$ and is then given as:

$$B=\langle\nu_n\bar{\Omega}_n\nu_p\bar{\Omega}_p|V|\nu_n\Omega_n\nu_p\Omega_p\rangle\,. \tag{23}$$

Like the splitting energy difference, the odd-even shift will vanish for the first term in Eq. (19). The proof of this is illustrated by the following property of the matrix elements under time reversal:

$$\langle j_n\bar{\Omega}_n|\hat{O}^{\lambda}_{\mu}|j_n'\Omega_n\rangle=-(-)^T\langle j_n'\bar{\Omega}_n|\hat{O}^{\lambda}_{\mu}|j_n\Omega_n\rangle\,, \tag{24}$$

where T is the sign of the tensor operator $\hat{O}^{\lambda}_{\mu}$ under time reversal. The first part of the central-force expansion involving terms in $C^{(k)}$ will have positive sign under time reversal, while terms in $[\sigma\times C^{(k)}(\hat{r})]^{(T)}$ will be negative. Under exchange of j and j', the terms containing $C^{(k)}$ will cancel while the terms with $[\sigma\times C^{(k)}]^{(T)}$ will not. The integration over the radial coordinates r_n and r_p was performed using the formula of Horie-Sasaki.[13] In the limit of the zero-range force, $\mathfrak{V}^{(k)}(r_n,r_p)$ has the following form:

$$\mathfrak{V}^{(k)}(r_n,r_p)=\frac{\delta(r_p-r_n)}{r_nr_p}\,, \tag{25}$$

which is independent of the transferred orbital angular momentum k. Therefore, the calculation leads to a much simplified result. Furthermore, since the Majorana operator does not cause any change in the zero-range force, the Majorana and

Heisenberg parts give the same results as the Wigner and Bartlett parts, respectively. The zero-range force which involves only s-state interactions in the relative coordinates $(\vec{r}_p - \vec{r}_n)$ can therefore not include a tensor force. A simple form of this interaction is given by

$$H_{\rm int} = -4\pi g \delta(\vec{r}_p - \vec{r}_n)[1 - \alpha + \alpha \vec{\sigma}_n \cdot \vec{\sigma}_p]. \tag{26}$$

The energy shift (including the splitting energy and the odd-even shift) is simply

$$\Delta E = E(K_1) - E(K_2) = -2\alpha\{A_{pn}[1 + (-)^I \pi_p \pi_n \delta_{K,0}] + B_{pn}\}, \tag{27}$$

where A_{pn} and B_{pn} are given by Pyatov[6] and K_1 and K_2 are as defined in Eqs. (12) and (13). ΔE is the energy shift contributed by the residual interaction only and π_p and π_n are the parities of the states.

CALCULATIONS

First the splitting energy between the parallel and antiparallel coupled states will be calculated using a zero-range force. Since the Majorana operator is the identity operator in the zero-range interaction, the contribution of the Majorana force to the splitting energy and to the odd-even shift vanishes for the same reason as the Wigner force. Furthermore, $\vec{\sigma}_n \cdot \vec{\sigma}_p V(\vec{r}) P_M$, which is from the Heisenberg part of the potential, gives the same results as $\vec{\sigma}_n \cdot \vec{\sigma}_p V(\vec{r})$ which comes from the Bartlett part of the potential. Therefore, the residual interaction is characterized by only one parameter αW, where

$$W = g\left(\frac{2\nu^3}{\pi}\right)^{1/2} \tag{28}$$

and ν is a quantity which appears in the expressions for the radial wave function. Because the radial integration is simple using the zero-range force, it is possible to employ more suitable wave functions which result from the solution of a Wood-Saxon potential.[14] In order to make a satisfactory comparison, it is necessary to compare the calculated values with the experimental values of the splitting energies from which the kinetic terms of Eq. (2), the experimental odd-even shift, and the Coriolis coupling term of Eq. (16) have been subtracted. A least-squares fit to the splitting energies alone is obtained for $\alpha W = 0.85$ MeV with a deviation of 41 keV. The resulting energies are shown in column A of Tables I and II. In column B, the energies for the value of αW originally chosen by Pyatov (0.24 MeV) are also shown for comparison. It is surprising that this simple calculation reproduces the experimental energy splittings so well. Even some of the discrepancies are understandable. For example, it is reasonable that the zero-range force should overestimate the energy

TABLE I. Theoretical and experimental splitting energies. The calculations A, B, C, and D (in keV) are described in the text.

Nucleus	Configuration Proton	Neutron	K^π $\Sigma_n+\Sigma_p=0$	$\Sigma_n+\Sigma_p=1$	Theory A	B	C	D	Experiment (keV)
^{158}Tb	411↑	400↑	1^+	2^+	136	38	206	197	136
	411↑	402↓	3^+	0^+	−186	−53	−180	−204	−111
	411↑	521↑	0^-	3^-	174	49	113	132	132
^{164}Ho	523↑	400↑	3^-	4^-	73	21	101	99	102
	523↑	402↓	5^-	2^-	−82	−23	−117	−115	−85
	523↑	521↑	2^+	5^+	160	45	102	70	171
	523↑	523↓	6^+	1^+	−248	−70	−158	−158	−144
	523↑	642↑	1^-	6^-	74	21	110	115	67
^{166}Ho	523↑	633↑	0^-	7^-	135	38	168	165	91
	523↑	521↓	4^+	3^+	−147	−42	−118	−95	−171
^{168}Tm	411↓	521↓	0^-	1^-	154	44	103	127	191
	411↓	512↑	3^-	2^-	−255	−72	−158	−164	−234
	411↓	633↑	4^+	3^+	−87	−25	−112	−115	−157
^{170}Tm	411↓	521↓	0^-	1^-	154	44	103	128	187
	411↓	512↑	3^-	2^-	−255	−72	−158	−164	−232
^{174}Lu	404↓	521↑	5^-	2^-	−77	−22	−72	−72	−90
	404↓	521↓	3^-	4^-	71	20	79	76	80
	404↓	512↑	6^-	1^-	−106	−30	−134	−130	−110
^{176}Lu	404↓	514↓	0^-	7^-	194	55	185	189	240
	404↓	510↑	4^-	3^-	−93	−26	−125	−120	−118

splitting in the case of ^{164}Ho [523↓] [523↑], because the wave functions for both neutron and proton have the same asymptotic quantum numbers and an unreasonably high overlap in the zero-range approximation. If the same force strength is applied to the calculation of the odd-even shift, the agreement with experiment is not good. The calculated values, in general, are larger than the experimental odd-even shifts. As Newby has discussed, the odd-even shift is caused by the backward scattering parts of the nuclear force, especially when $|\Omega|$ is large. For this reason the δ force is likely to overestimate the odd-even shift. If one tries to fit the odd-even shift by decreasing the strength of the interaction, then the energy splittings are seriously underestimated. Furthermore, in those cases where the wrong sign is found for the odd-even shift, no satisfactory solution is available. It is clear then that the zero-range force is not suitable for the calculation of the odd-even shift, although it may be quite successfully applied to the calculation of the energy splitting.

In view of this difficulty of the zero-range force, a finite-range calculation was also performed. A Gaussian form of finite range was used:

$$V(\vec{r}_{pn}) = e^{-\vec{r}_{pn}^{\,2}/r_0^2}. \qquad (29)$$

The dependence of the matrix elements on the force range is illustrated in Fig. 1. It is convenient in comparing the matrix elements to divide them by $\sqrt{\pi^3}r_0^3$, where r_0 is the force-range parameter appearing in Eq. (29). The matrix elements of the Gaussian force are calculated for $r_0 = 0.2$, 0.8, 1.6, and 2.2 F using the harmonic-oscillator wave function. Figure 1 shows the matrix element of $\vec{\sigma}_n \cdot \vec{\sigma}_p V(\vec{r}_{pn})$ for energy shifts ΔE (the solid line) and odd-even shifts B (dotted line).

While $|\Delta E|$ changes smoothly and gradually, $|B|$ decreases rapidly as the force range increases especially for $|\Omega| = \frac{7}{2}$. This indicates that as far as the Bartlett interaction is concerned, the odd-even shift B is much more sensitive to the force range r_0 than the splitting energy.

The matrix elements of $P_M V(\vec{r}_{pn})$ are sensitive to the force range in the region $r_0 = 0.2 \sim 0.8$ but not for $r_0 > 0.8$. However, this contribution to the splitting energy and odd-even shift is not very large with the exception of the configurations [523↑] [523↓] and [523↑] [633↓]. Therefore, the splitting energy and odd-even shift are not good probes to determine the Majorana force. In the limit of $r_0 \to 0$, $\vec{\sigma}_n \cdot \vec{\sigma}_p P_M V(\vec{r}_{pn})$ is the same as $\vec{\sigma}_n \cdot \vec{\sigma}_p V(\vec{r}_{pn})$. This effect decreases as the range r_0 increases in general.

The results of the calculations for a range of 1.4 F are shown in column C of Tables I and II, as an example. The strengths were chosen to give a least-squares fit to the splitting energies and the odd-even shifts, with a resulting deviation of 55 keV. The finite-range interaction improves the agreement between experiment and calculations very little as far as the energy splitting is concerned. This is understandable because the Bartlett force is not sensitive to the range of the interaction, and the effect of the Majorana force is small even in the finite range. Furthermore, $\sigma_n \sigma_p P_M$ has the same tendency for all matrix elements. On the other hand, when the odd-even shifts are fitted simultaneously with the splitting energies, a reasonable agreement between experiment and theory is obtained. This is quite in contrast with the situation for zero range where simultaneous fitting with both splitting energies and odd-even shift is very unsatisfactory. The standard deviation from experiment can be decreased from 100 keV in the case of the zero-range interaction to 55 keV for the finite range. The force parameters used are tabulated in Table III. The magnitude but not the sign of the disagreement in the odd-even shift of the configuration [404↓] [514↓] may be improved by the finite-range interaction.

The importance of the tensor force in calculating

TABLE II. Theoretical and experimental odd-even energy shifts. The calculations A, B, C, and D (in keV) are described in the text.

Nucleus	Configuration Proton	Neutron	$\Sigma_1+\Sigma_2$	Parity	Theory A	B	C	D	Experiment (keV)
^{158}Tb	411↑	521↑	0	−	−202	−57	−79	−74	8
	411↑	402↓	1	+	16	4	3	−34	−32
^{166}Ho	523↑	633↑	0	−	−182	−51	−8	−13	−32
^{168}Tm	411↓	521↓	0	−	−193	−55	−97	−75	−27
^{170}Tm	411↓	521↓	0	−	−193	−55	−97	−76	−38
^{172}Lu	404↓	633↓	1	+	32	9	1	−24	−56
^{176}Lu	404↓	514↓	0	−	−211	−60	18	19	69

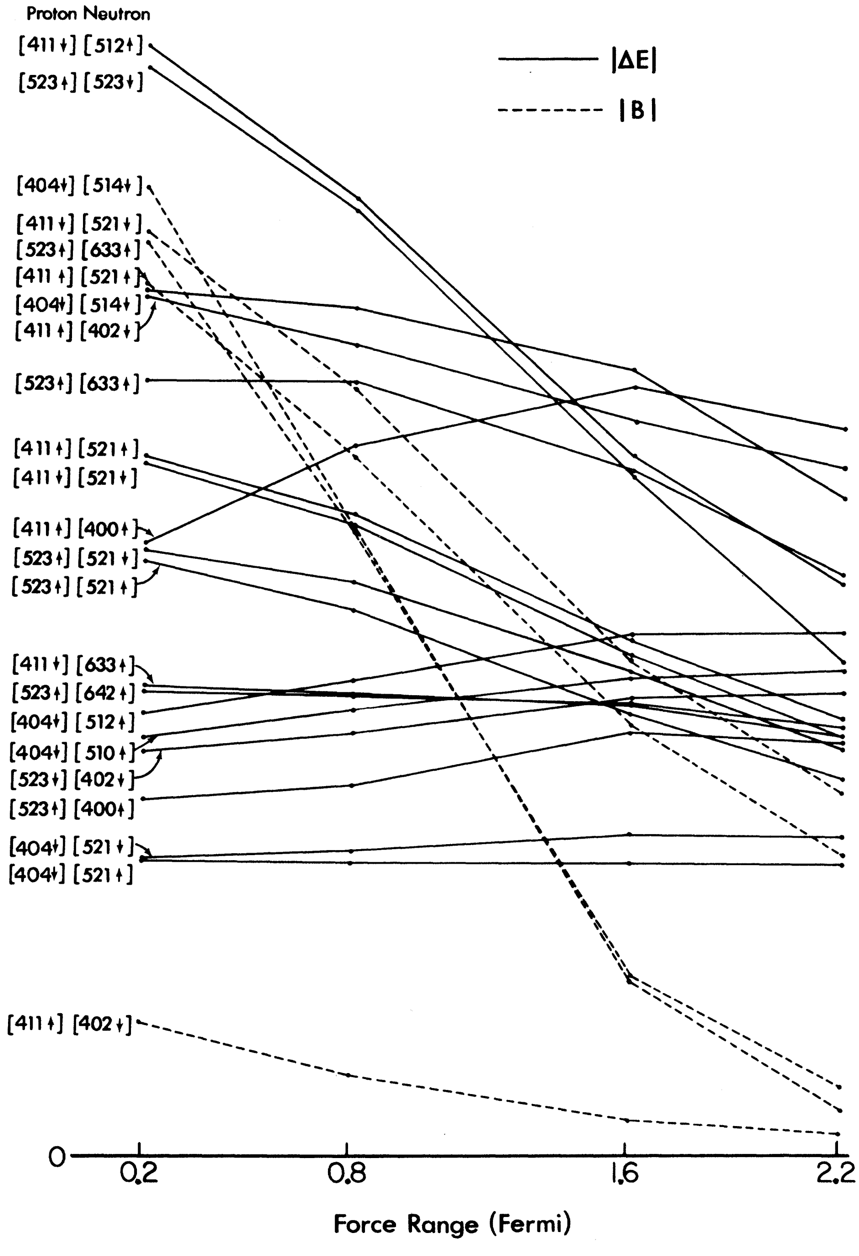

FIG. 1. Dependence of the matrix elements of the residual interaction on the range. The dashed lines represent the odd-even shifts and the solid lines represent the splitting energies.

the odd-even shift has been discussed by Newby.[5] The tensor force is of particular importance in the case of parallel intrinsic spins for which the transition to the time-reversed state is forbidden for a central force. Indeed using a central force, the calculated values for the odd-even shift for the configurations [404↓] [633↑] and [411↑] [402↓], are small and have the opposite sign to that of the experiment. The results of the tensor-force calculation are shown in column D of Tables I and II. The strengths were again chosen to give a least-squares fit to both the splitting energies and the odd-even shifts. The force strengths are shown in Table III. The standard deviation using the tensor force improves from 55 to 52 keV. Although the signs of the odd-even shifts of the configurations

TABLE III. The strength of the nuclear force in case of finite range ($r_0 = 1.4$).

	Bartlett	Majorana	Heigenberg	Tensor
Theory C	−71.2	−13.1	33.1	...
Theory D	−63.0	−1.4	16.0	−79.2

[404↓] [633↑] and [411↑] [402↓] are corrected, not all comparisons of odd-even shifts are improved by introducing a tensor force. In particular we cannot explain the sign of the odd-even shift of the configuration [411↑] [521↑].

CONCLUSION

The zero-range force can fit the splitting energy between parallel and the antiparallel states in deformed odd-odd nuclei as well as the finite-range force. However, it cannot satisfactorily explain the odd-even shift in $K = 0$ rotational bands because it overestimates the backward scattering part of the nuclear force. The tensor force is needed to interpret the odd-even shift for configurations in which the matrix element is forbidden in Newby's selection rule and gives the correct sign while the central force does not. Therefore, experimental data on the odd-even shifts in odd-odd nuclei provide an excellent testing ground for studying the tensor force.

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