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[21]The IPA differs from the other two methods [Dabrowski and Hassan (Ref. 5) and Rote and Bodmer (Ref. 6)] in the assumed forms of the single-particle energies and in the procedures employed in solving the g matrix. A discussion concerning the differences in the IPA and the method employed by Dabrowski and Hassan is given in the Appendix of their paper (Ref. 5). Since their method is essentially the same as the reference-spectrum method (RSM) used by Rote and Bodmer (Ref. 6), their discussion equally applies to the differences between the IPA and RSM. Dabrowski and Hassan (Ref. 5) have obtained exact results for D for the potential DW using their form of the single-particle energies, as well as the one assumed in IPA. They obtain nearly identical results. This means that if in the IPA the g matrix is solved exactly via the Bethe-Goldstone equation, which is done here by an explicit perturbation expansion (both Dabrowski and Hassan and Rote and Bodmer solve the g matrix by an iterative procedure), then one would expect the IPA results to be identical to those obtained by these authors. This expectation is well fulfilled by our results. The combined effect, in the IPA, of a different assumed form for the single-particle spectra and a different procedure in solving the g matrix is not to alter the total outcome for D but is only to distribute differently (than in the other two methods) various contributions, hard core and attractive.

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Connection Between Symplectic, Quasispin, and Generalized Bogoliubov Groups

A. Bose
Université de Montréal, Montréal, Canada
and
A. Navon
Université du Québec, Trois-Rivières, Canada
(Received 31 August 1970)

A new derivation of the quasispin groups used in nuclear spectroscopy is given. It is shown that these groups arise as the maximum commuting subgroups of the Flowers symplectic groups inside groups of generalized Bogoliubov transformations. These transformations are defined to be the most general transformations which conserve the anticommutation relations of the fermion operators of a nuclear shell.

1. INTRODUCTION

Nuclear shell-model states in j-j coupling have been classified by Flowers[1] with the aid of symplectic groups in $2j+1$ dimensions. The symplectic group acting on states of identical nucleons furnishes the seniority quantum number, while the symplectic group acting on states of neutrons and protons provides the two quantum numbers of seniority and reduced isospin. These quantum numbers are very useful for labeling N-particle states and for calculating various nuclear matrix elements.[2] More recently, it was found that two smaller groups, of the type $SU(2)$ and $USp(4)$, provide the same quantum numbers as the initial symplectic groups but are more advantageous for some nuclear-spectroscopy calculations.[3,4] These two new groups, called quasispin groups, were shown by Helmers[5] to be generated by bilinear invariants of the symplectic groups.

The purpose of the present work is to show that the quasispin groups arise in fact in the frame of a more general structure. It is shown here that there exist larger groups which contain the symplectic groups and their corresponding quasispin groups as subgroups, such that a quasispin subgroup is the centralizer (i.e., the maximum commuting subgroup) of the symplectic subgroup into the larger group. It appears that these larger groups are groups of generalized Bogoliubov (g.B.) transformations, which were used by several authors to study pairing and self-consistent effects on the same footing.[6] The structure of g.B. groups was studied in detail in previous papers of the authors.[7,8] Here are given only the main properties of the g.B. Lie algebra and those needed for the proof of our argument.

In Sec. 2 the structure of the g.B. Lie algebra is described in terms of nucleon creation and annihilation operators of a j shell. In Sec. 3 we consider the symplectic Lie algebra as a subalgebra of the g.B. Lie algebra. In Sec. 4 we prove that the quasispin Lie algebra is the centralizer of the symplectic Lie algebra for the simple case of identical nucleons.

2. GENERALIZED BOGOLIUBOV LIE ALGEBRA

We consider the $(2j+1)$ single-nucleon states of a j shell. Instead of labeling them by the magnetic quantum number m, we use the index $i=m+j+1$ running from 1 to $2j+1$. The corresponding creation and annihilation operators $a_i^\dagger, a_i$ satisfy the usual fermion anticommutation relations

$$[a_i^\dagger, a_k^\dagger]_+ = [a_i, a_k]_+ = 0, \quad [a_i^\dagger, a_k]_+ = \delta_{ik}. \tag{1}$$

The anticommutator $[\ ,\ ]_+$ may be interpreted[7-9] as a bilinear symmetric form defined on the $2(2j+1)$-dimensional vector space spanned by the $a_i^\dagger, a_i$. Following this interpretation, linear transformations which conserve the anticommutation relations (1) are orthogonal and the group of g.B. transformations is therefore isomorphic to a $2(2j+1)$-dimensional orthogonal group. Its Lie algebra L is of the even orthogonal type D_{2j+1}.

Let us recall first some of the properties[10] of an algebra of type D_{2j+1}. Such an algebra is of dimension $(2j+1)(4j+1)$. It is usual to associate with it a set of $4j(2j+1)$ root vectors $\{\vec{\alpha}\}$, which can be expressed in terms of $(2j+1)$ orthonormal vectors $\vec{v}_i$ as follows:

$$\{\vec{\alpha}\} = \{\pm(\vec{v}_i + \vec{v}_k), \vec{v}_i - \vec{v}_k; \quad i \neq k = 1, \ldots, 2j+1\}. \tag{2}$$

The scalar products of the roots are then determined from the orthonormality relations of the vectors $\vec{v}_i$

$$(\vec{v}_i, \vec{v}_k) = \delta_{ik}. \tag{3}$$

The set $\{\vec{\alpha}\}$ may be partitioned in two equal parts: (i) the set of positive roots $\{+(\vec{v}_i+\vec{v}_k),\ \vec{v}_i-\vec{v}_k;\ i<k\}$, and (ii) the set of negative roots $\{-(\vec{v}_i+\vec{v}_k),\ -(\vec{v}_i-\vec{v}_k);\quad i<k\}$. Among the positive roots one can find $(2j+1)$ roots, called simple roots, such that any other positive root can be expressed as a sum of simple roots.[10] A possible choice of simple roots is

$$\vec{\alpha}_i = \vec{v}_i - \vec{v}_{i+1}, \quad i = 1, \ldots, 2j;$$

and

$$\vec{\alpha}_{2j+1} = \vec{v}_{2j} + \vec{v}_{2j+1}. \tag{4}$$

In a semisimple Lie algebra of rank n there exist n linearly independent commuting elements which span a subalgebra called the Cartan subalgebra.[10] It is usual to identify the Cartan subalgebra with the $(2j+1)$-dimensional space of the roots. Hence, one can associate to each root $\vec{\alpha}$ or vector $\vec{v}_i$, respectively, elements H_α or h_i from the Cartan subalgebra. The scalar product in the Cartan subalgebra is induced by that in the root space, and one has

$$(h_i, h_k) = \delta_{ik}. \tag{5}$$

The remaining elements of D_{2j+1} can be described as raising operators E_α and lowering operators $E_{-\alpha}$, each couple $\{E_\alpha, E_{-\alpha}\}$ corresponding to a positive root $\vec{\alpha}$. If we denote by H an arbitrary element of the Cartan subalgebra, the commutation relations in a semisimple Lie algebra read

$$[H, H_\alpha]=0, \quad [H, E_\alpha] = (H, H_\alpha) E_\alpha, \quad [E_\alpha, E_{-\alpha}] = H_\alpha,$$

$$[E_\alpha, E_\beta] = N_{\alpha\beta} E_{\alpha+\beta} \quad \text{if } \alpha+\beta \text{ is a root},$$

$$= 0 \quad \text{if } \alpha+\beta \text{ is not a root}. \tag{6}$$

For D_{2j+1} the scalar products (H, H_α) are determined from the expressions of the H_α's in terms of the h_i's [see (2)] and from the orthonormality relations (5). The coefficients $N_{\alpha\beta}$ are equal to +1 or −1 (the sign will be decided by inspection when necessary).

The elements $\{E_i, E_{-i}, H_i;\quad i=1, \ldots, 2j+1\}$ associated with the simple roots α_i are called canonical generators.[10] These $3(2j+1)$ elements generate by commutation all the $(2j+1)(4j+1)$ elements of D_{2j+1}. Most of the properties of D_{2j+1} we have described here are also valid for other types of

Lie algebras, and we shall need especially the canonical generators for the symplectic Lie algebra.

We can determine now the explicit structure of the g.B. Lie algebra, which we shall note L, and identify its elements with those of D_{2j+1}. Let us consider the $(2j+1)(4j+1)$ bilinear operators

$$a_i^\dagger a_k^\dagger\ (i<k),\quad a_i a_k\ (i<k),\quad a_i^\dagger a_k\ (i\neq k),\quad a_i^\dagger a_i-\tfrac{1}{2};\quad i,k=1,\ldots,2j+1. \tag{7}$$

They are linearly independent and closed under commutation and they satisfy the operator relation $O^t=-O$ characterizing orthogonal Lie algebras, where we take the transpose of a product of operators $a_i^\dagger, a_i$ as the product of the same operators written in reversed order, e.g. $(a_i^\dagger a_k)^t=a_k a_i^\dagger$. The expressions (7) form therefore a basis of L.

It is easy to verify by evaluating commutators that the operators $a_i^\dagger a_i-\frac{1}{2}$ span the Cartan subalgebra C of L and that the operators $a_i^\dagger a_k^\dagger, a_i a_k$ are raising and lowering operators. We can now set

$$h_i=a_i^\dagger a_i-\tfrac{1}{2},\quad i=1,\ldots,2j+1, \tag{8}$$

and establish the following correspondence between roots and raising and lowering operators of L

$$\vec{v}_i+\vec{v}_k\to a_i^\dagger a_k^\dagger,\quad -(\vec{v}_i+\vec{v}_k)\to a_k a_i,\quad \vec{v}_i-\vec{v}_k\to a_i^\dagger a_k. \tag{9}$$

Thus, the canonical generators of L can be written in terms of creation and annihilation operators as follows:

$$E_i=a_i^\dagger a_{i+1},\quad E_{-i}=a_{i+1}^\dagger a_i,\quad H_i=h_i-h_{i+1};\quad i=1,\ldots,2j, \tag{10}$$

$$E_{2j+1}=a_{2j}^\dagger a_{2j+1}^\dagger,\quad E_{-(2j+1)}=a_{2j+1}a_{2j},$$
$$H_{2j+1}=h_{2j}+h_{2j+1}.$$

3. SYMPLECTIC LIE ALGEBRA

The symplectic group for identical nucleons is formed by those unitary transformations of the j-shell states which conserve the pair $J=0$ operator

$$\sum_{m>0}(-1)^{j-m}a_m^\dagger a_{-m}^\dagger. \tag{11}$$

Its Lie algebra L' is of the symplectic type $C_{j+1/2}$, and Helmers[5] has shown that its elements are of the form

$$a_m^\dagger a_{m'}-(-1)^{m'-m}a_{-m'}^\dagger a_{-m}. \tag{12}$$

The simple roots $\vec{\alpha}_1',\ldots,\vec{\alpha}_{j+1/2}'$ of $C_{j+1/2}$ have the following scalar products[10]:

$$(\vec{\alpha}_i',\vec{\alpha}_i')=1,\quad(\vec{\alpha}_i',\alpha_{-i}')=-\tfrac{1}{2}\quad\text{for } i=1,\ldots,j-\tfrac{1}{2};$$
$$(\vec{\alpha}_{j+1/2}',\alpha_{j+1/2}')=2,\quad(\vec{\alpha}_{j+1/2}',\alpha_{j-1/2}')=-1;$$
$$(\vec{\alpha}_i',\vec{\alpha}_k')=0\quad\text{for } k=i,i\pm1. \tag{13}$$

We can obtain a set of $3(j+\frac{1}{2})$ canonical generators $\{E_i',E_{-i}',H_i'\}$ for L' by using the expression (12) and by requiring that the scalar products $(H_i',H_k')=(\vec{\alpha}_i',\alpha_k')$ in commutation relations $[H_i',E_k']=(H_i',H_k')E_k'$, have the same values as in (13). We get then by reusing the index $i=m+j+1$:

$$E_i'=a_i^\dagger a_{i+1}+a_{2j+1-i}^\dagger a_{2j+2-i},\quad E_{-i}'=(E_i')^\dagger,$$
$$H_i'=\tfrac{1}{2}(a_i^\dagger a_i-a_{i+1}^\dagger a_{i+1}+a_{2j+1-i}^\dagger a_{2j+1-i}-a_{2j+2-i}^\dagger a_{2j+2-i}),$$
$$i=1,\ldots,j-\tfrac{1}{2};$$
$$E_{j+1/2}'=a_{j+1/2}^\dagger a_{j+3/2},\quad E_{-(j+1/2)}'=(E_{j+1/2}')^\dagger,$$
$$H_{j+1/2}'=a_{j+1/2}^\dagger a_{j+1/2}-a_{j+3/2}^\dagger a_{j+3/2}. \tag{14}$$

The canonical generators of L' can be expressed in terms of the elements of L as follows:

$$E_i'=E_i+E_{2j+1-i},$$
$$H_i'=\tfrac{1}{2}(H_i+H_{2j+1-i})=\tfrac{1}{2}(h_i-h_{i+1}+h_{2j+1-i}-h_{2j+2-1})$$
$$E_{j+1/2}'=E_{j+1/2},\quad H_{j+1/2}'=H_{j+1/2}=h_{j+1/2}-h_{j+3/2}. \tag{15}$$

It is seen from the above equations that the Cartan subalgebra C' of L' is contained in the Cartan subalgebra C of L. It is also clear that L' is a subalgebra of L and that the scalar product in L' or in C' is determined by the scalar product in L.

4. DETERMINATION OF CENTRALIZER

We look now for the centralizer of L' in L, i.e., for the set of elements of L which commute with all the elements of L'. Since the canonical generators of L' generate by commutation and linear combination all the elements of L', it is sufficient to find the elements of L commuting with the canonical generators of L'.

We try first to find an element $\bar{H}$ of the Cartan subalgebra C of L which commutes with the canonical generators of L'. It must satisfy the relations

$$[\bar{H},E_i']=0\quad\text{for } i=1,\ldots,j+\tfrac{1}{2}. \tag{16}$$

By using (15) and (6), these relations become

$$(\bar{H},H_i)E_i+(\bar{H},H_{2j+1-i})E_{2j+1-i}=0,\quad i=1,\ldots,j-\tfrac{1}{2};$$
$$(\bar{H},H_{j+1/2})E_{j+1/2}=0. \tag{17a}$$

Expressing the H_i in terms of the h_i [cf. (4)] and noting that E_i and E_{2j+1-i} are linearly independent, $\bar{H}$ has now to satisfy the set of $2j$ equations

$$(\bar{H},h_i-h_{i+1})=0,\quad i=1,\ldots,2j. \tag{17b}$$

Since $\bar{H}$ can be written as a linear combination of the h_i, one finds that the only solution of (17), up to a constant factor, is

$$\bar{H}=\sum_{i=1}^{2j+1} h_i . \tag{18}$$

Thus, there is only one element of C which commutes with L'.

We now look for elements of L outside C which commute with L'. Our task is simplified by a result due to Morozov[11] which, in our case, asserts that there can be at most a three-dimensional simple subalgebra, i.e., two more operators $\bar{E}_+$ and $\bar{E}_-$, commuting with L'. These operators should satisfy the relations

$$[\bar{H},\bar{E}_\pm]=\pm 2\bar{E}_\pm , \tag{19a}$$

$$[\bar{E}_+,\bar{E}_-]=\bar{H} , \tag{19b}$$

$$[H_i',\bar{E}_\pm]=0 , \tag{19c}$$

$$[E_i',\bar{E}_\pm]=0 . \tag{19d}$$

The operator $\bar{E}_+$ can be written

$$\bar{E}_+=\sum_\alpha S_\alpha E_\alpha , \tag{20}$$

where S_α are scalars and E_α are raising operators of L. Introducing the expressions (18) and (20) for $\bar{H}$ and for $\bar{E}_+$ in (19a), we deduce that the roots $\vec{\alpha}$ appearing in the summation of (20) can only be of the form $\vec{v}_i+\vec{v}_k$. On the other hand, (19b) entails that

$$\sum_\alpha S_\alpha^2 H_\alpha=\bar{H}=\sum_{i=1}^{2j+1} h_i . \tag{21}$$

This last equation shows that in the summation there must be exactly $j+\frac{1}{2}$ roots $\vec{\alpha}$, no two of them having in common a vector $\vec{v}_i$, and also that

$$S_\alpha=\pm 1 . \tag{22}$$

Let us now replace in the $j+\frac{1}{2}$ equations (19c) the H_i' and $\bar{E}_+$ by their expressions in terms of the h_i and E_α, respectively, and apply the general commutation relations (6). We get then a set of scalar products equated to zero and, by using our previous results, we find that the roots $\vec{\alpha}$ are explicitly

$$\vec{v}_{j+1/2-l}+\vec{v}_{j+3/2+l} , \quad l=0,\ldots, j-\tfrac{1}{2} . \tag{23}$$

Hence, the raising operators E_α appearing in the development (20) of $\bar{E}$ are

$$a^\dagger_{j+1/2-l}a^\dagger_{j+3/2+l} , \quad l=0,\ldots,j-\tfrac{1}{2} . \tag{24}$$

It remains to find the signs of the coefficients S_α in (20). This is done by replacing in (19d) the E_i and $\bar{E}_+$ by their expressions in terms of the elements of L. By using once more the commutation relations (6) we find that

$$\bar{E}_+=\sum_{l=0}^{j-1/2}(-1)^{j+3/2+l}\, a^\dagger_{j+1/2-l}\, a^\dagger_{j+3/2+l} . \tag{25}$$

Indexing in terms of the magnetic quantum number m, the basis of the centralizer Lie algebra $\bar{L}$ now reads

$$\bar{H}=\sum_{m>0}(a^\dagger_m a_m-\tfrac{1}{2}) ,$$
$$\bar{E}_+=\sum_{m>0}(-1)^{j-m}a^\dagger_m a^\dagger_{-m} , \tag{26}$$
$$\bar{E}_-\equiv(\bar{E}_+)^+=\sum_{m>0}(-1)^{j-m}a_{-m}a_m .$$

We recognize the well-known basis of the quasispin Lie algebra.[3-5]

5. CONCLUDING REMARKS

We have calculated, for the simple case of identical nucleons, the centralizer of the symplectic Lie algebra into the generalized Bogoliubov Lie algebra, and we have found that it coincides with the familiar quasispin Lie algebra. A similar result can be obtained, with a little more work, for the case of neutrons and protons. In that case, the g.B. group and the quasispin group are, respectively, of the types $O(4(2j+1))$ and $USp(4)$. Some general insight can be gained by noting that both cases are related to the maximum subgroup chain decomposition[12]

$$O(st)\supset USp(s)\times USp(t) .$$

Several authors[13,14] have investigated the quasispin in L-S coupling. In that case, the seniority group and the quasigroup are both orthogonal. This fact is to be related to another maximum subgroup decomposition[12]

$$O(st)\supset O(s)\times O(t) .$$

An interesting point we would like to mention here is the connection between our work and a problem which has not yet received a satisfactory solution. We refer to the problem[15,16] of the determination of a fourth quantum number necessary to label the states of the quasispin group of the type $USp(4)$. This fourth quantum number corresponds to an operator of order not smaller than 4 in the $a_i^\dagger$ and a_i. We think that this problem could be tackled in a systematic way by extending our g.B. Lie algebra, which contains only bilinear tensors in $a_i^\dagger, a_i$, to the algebra which contains all the tensors in $a_i^\dagger, a_i$ (Clifford algebra). This, however, seems quite difficult and it probably involves lengthy calculations.

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Boundary-Condition-Model T Matrix

Michael G. Fuda
Department of Physics and Astronomy, State University of New York at Buffalo, Buffalo, New York 14214
(Received 20 July 1970)

A number of results on the boundary-condition-model (BCM) T matrix are developed. It is shown that the half-off shell T matrix is unique, but the fully off-shell T matrix is not. Further, it is shown that the ambiguity in the T matrix resides in that part of the complete T matrix which is identifiable as the T matrix for the pure BCM, where by pure BCM is meant no forces outside the boundary-condition radius. Three different formulas for the pure BCM T matrix are presented. The first is derived by using the relations that exist between the half-off-shell T matrix and the fully off-shell T matrix for well-behaved potentials, and is found to be separable. The second is taken from the work of Kim and Tubis. The third is derived from a pseudopotential constructed by Hoenig and Lomon. All three agree exactly half off shell, and satisfy the off-shell unitarity relation. Numerical comparisons are given which show that significant differences can occur in the fully off-shell T matrices. An integral- as well as a differential-equation approach are given for finding the contribution to the BCM T matrix from the forces outside the boundary-condition radius. Separable representations for the BCM T matrix are developed, and their usefulness in carrying out calculations on the three-nucleon system is discussed.

I. INTRODUCTION

In a boundary-condition model (BCM), part or all of the force between a pair of particles is represented by a logarithmic boundary condition on the Schrödinger wave function. The boundary condition may or may not be energy dependent. It appears that the BCM was first used to describe the two-nucleon interaction by Breit and Bouricius,[1] who showed that the low-energy $^1S_0$ scattering data could be fitted by a pure BCM (no outside forces). They considered both energy-independent and energy-dependent logarithmic derivatives. The pure BCM was extended to higher energies and to tensor forces by Feshbach and Lomon,[2] who found that they could obtain a reasonable fit to the scattering data up to 274 MeV if they allowed the $^1S_0$ core radius to change with energy.

A good fit to pp data was later obtained by using an energy-independent boundary condition in conjunction with a local potential outside the core. The local potentials were of two types: a purely phenomenological exponential potential,[3] and a meson-theoretic potential[4] which included one- and two-pion-exchange contributions. In recent years, the fits to the nucleon-nucleon scattering data have been improved and the effects of mesons other than π mesons have been incorporated into the model.[5] The analytic properties of the scattering amplitudes arising in the BCM have been studied and found to be similar to those of the more conventional potential models.[6] It is now clear that the BCM with outside forces taken from meson field theories leads to as reasonable a description of the two-nucleon system as conventional potential models, in that it fits the elastic scattering data and gives rise to scattering amplitudes with acceptable analytic properties. The situation with respect to the many-nucleon problem is not so clear.

The application of the pure BCM (no outside