

## Dynamical Correlation in ${}^6\text{Li}$ and Inelastic Electron Scattering

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A simple nucleon-nucleon correlation function has been invoked in the independent-particle shell-model (IPSM) wave functions of  ${}^6\text{Li}$ . This has been assumed due to a substantial residual two-body interaction in addition to the IPSM Hamiltonian of  ${}^6\text{Li}$ . Using the variational principle, the excitation energy of the first excited state of  ${}^6\text{Li}$  has been obtained in good agreement with the experimental value. The nuclear form factor for the inelastic electron scattering and the reduced transition probability have been calculated and compared with experimental data and other calculated results.

### 1. INTRODUCTION

It has been recommended rather strongly on several occasions that some sort of clustering representation, in the case of light nuclei in general and  ${}^6\text{Li}$  in particular, is relatively suitable for investigations where electromagnetic forces are particularly dominant.<sup>1</sup> A considerable amount of work has been published in recent years in support of an  $\alpha$ -deuteron representation of  ${}^6\text{Li}$  for a number of investigations on its structure. Almost all the workers used some form of the independent-particle shell-model (IPSM) wave functions for internal as well as relative motions of the individual clusters. In fact, these wave functions of  ${}^6\text{Li}$  can be shown to correspond to the single-particle oscillator shell-model wave function.<sup>2</sup> It may be remarked that a number of nuclear processes involving  ${}^6\text{Li}$  seem to have been given a fairly good quantitative description within the framework of the cluster representation. Nevertheless, several studies on the structure of this nucleus have shown that many kinds of experimental data cannot consistently be explained by the simple IPSM wave functions. Among those anomalies one typical example is electron scattering from  ${}^6\text{Li}$ . If we try to explain its form factors as obtained from electron scattering in the conventional IPSM, it is necessary to choose a somewhat larger value of the harmonic-oscillator parameter for  ${}^6\text{Li}$  than for other  $1p$ -shell nuclei.<sup>3</sup> However, its large value does not always consistently explain other experiments. Other attempts to describe  ${}^6\text{Li}$  by the harmonic-oscillator model have also been made—for example, the modified harmonic oscillator<sup>3,4</sup> and the mixing of higher configurations<sup>5-8</sup> to the lowest state of  ${}^6\text{Li}$ . In other words, its form factors for electron scattering may be understood reflecting the clustering character of nucleons in  ${}^6\text{Li}$ . Alternatively, many authors have suggested that electron scattering should provide a useful tool for the investigation of the correlation struc-

ture of the nucleus in question. Consequently many workers have directed their attention towards the study of the correlation functions in the IPSM wave function of  ${}^6\text{Li}$ .<sup>9-15</sup> In the present work it is shown how the approach described in a previous paper<sup>15</sup> can be applied to the calculation of correlation corrections to the inelastic scattering of electrons by  ${}^6\text{Li}$ .

### 2. WAVE FUNCTION OF ${}^6\text{Li}$

The formalism used here should be valid generally. We have, however, chosen the  ${}^6\text{Li}$  nucleus for the present consideration. We assume this nucleus to be described as a double closed shell consisting of four particles plus two valence particles. Of course, the core nucleons occupy the  $1s$  state and the valence nucleons are in  $1p$  states. In this procedure the core particles are treated in the IPSM moving in the simple harmonic-oscillator type of potential well, whereas the extra core particles are treated in some more detail which rests upon certain physical assumption about the nature of the nuclear wave function. The total wave function of  ${}^6\text{Li}$  is chosen to have pure  $L$ - $S$  coupling, in the form

$$\Psi_{LM} = (1/N_L)\Psi_{LM}^{(0)}(1 - C_L r/r_0), \quad (1)$$

where  $r$  is the spatial distance between the  $p$  particles, the coefficient  $C$  is the correlation parameter yet to be determined,  $r_0$  is some suitable unit of length, and  $\Psi_{LM}^{(0)}$  is the IPSM wave function of  ${}^6\text{Li}$  given by

$$\Psi_{LM}^{(0)} = (1s)^4(1p)^2. \quad (2)$$

The renormalization constant  $N_L$  is given by

$$N_L^2 = 1 + 2C_L \langle r/r_0 \rangle_L + C_L^2 \langle r^2/r_0^2 \rangle_L, \quad (3)$$

where the matrix elements  $\langle r/r_0 \rangle$  and  $\langle r^2/r_0^2 \rangle_L$  are evaluated with the uncorrelated wave function  $\Psi_{LM}^{(0)}$ .

The correlation function in (1) arises from the fact that the basic assumption of IPSM—that the two-body nucleon-nucleon forces can be complete-

ly represented by an average single-particle central potential—is not particularly valid in case of  ${}^6\text{Li}$ . Instead we expect to have a substantial residual two-body interaction which leads to the inclusion of the nucleon-nucleon correlation function in the total wave function of  ${}^6\text{Li}$ . In order to invoke the short-range nucleon correlation in the nuclear wave function most of the workers have used the Jastrow-type correlation function<sup>16</sup> arising from the repulsive, short-range part of the two-nucleon interaction. It may be pointed out that the short-range correlation is reasonably important in the  $\pi$ -meson absorption because the absorption process occurs in the short range, its drastic role may give rather wrong tendency in the electron scattering because the electromagnetic interaction is long-range interaction. Cheon<sup>10</sup> remarked that the calculations for the nuclear charge form factors, taking into account the short-range correlations, may not be considered good enough because they cannot produce the quadrupole moment even if the theoretical curve of charge form factor gives a good agreement with the experimental data on  ${}^6\text{Li}$ . Particularly in the case of  ${}^6\text{Li}$ , where the valence nucleons—which mainly contribute to the inelastic electron scattering cross section—are relatively less tightly bound, the invoking of the short-range correlation only may not give very realistic results. We note that in the  ${}^6\text{Li}(p, 2p)$  reaction the angular correlation function of the final protons for the case of knocking out weakly bound nucleons differs sharply from the corresponding data for  ${}^7\text{Li}$  and heavier nuclei.<sup>17</sup> The momentum distribution of the outer nucleons in  ${}^6\text{Li}$  turns out to be softer than in the neighboring nuclei. This does not imply that there should not be any nucleon correlation in  ${}^6\text{Li}$  while studying the electromagnetic phenomena. It may be timely remarked here that electromagnetic properties of a nucleon inside a nucleus may be seriously affected by the presence of other nucleons in its vicinity. It is thus quite appropriate to invoke some mild type of dynamical correlation like one in Eq. (1) at least in the valence nucleons. The matrix element for an  $E2$  ( $1^+ \rightarrow 3^+$ ) transition in  ${}^6\text{Li}$  calculated by using the wave function (1) is considerably enhanced over the single-particle value, and is relatively in closer agreement with the one measured in inelastic electron scattering experiment.<sup>18</sup>

The neglect of the dynamical correlation between the core and valence nucleons in Eq. (1) is justifiable because of the substantial isolation of the  $\alpha$  core and the valence  $n$ - $p$  system. The significant isolation of the two subsystems in the  ${}^6\text{Li}$  nucleus can be backed up by many experimental data and theoretical results; for example, the small value of the threshold for the  ${}^6\text{Li} \rightarrow \alpha + d$  breakup, the

large value of its nuclear radius,<sup>19</sup> etc. The Pauli correlations arising simply from the particle statistics are introduced to the core and the valence nucleons separately through the use of a properly antisymmetrized nuclear wave function of  ${}^6\text{Li}$ . Making the assumption that the functional forms of the nucleon-nucleon correlations are the same in the ground and the first excited state of  ${}^6\text{Li}$  we are able to calculate its charge form factor for the inelastic electron scattering using the ordinary IPSM wave functions as the basis. Using the method of second quantization<sup>20</sup> the IPSM wave function  $\Psi_{LM}^{(0)}$  of the target nucleus is given by linear combinations of functions of the following form:

$$\Psi_{LM}^{(0)} = p_a^\dagger p_b^\dagger |0\rangle, \quad (4)$$

where  $p_a^\dagger$  creates a particle in a  $1p$  harmonic-oscillator state with orbital, spin, and isospin magnetic quantum numbers designated by the subscript  $a$ . The vacuum state  $|0\rangle$  corresponds to the double-closed-shell core consisting of four  $1s$  particles in the IPSM. To designate the ground state and the first excited state we assume the following quantum numbers:

	$J$	$L$	$S$	$T$
$\Psi_g = \text{ground state}$	1	0	1	0
$\Psi_e = \text{first excited state}$	3	2	1	0.

Since the double-closed shell of four  $1s$  particles constitute an inert core, in this transition the change from a triplet  $S$  state to a triplet  $D$  state takes place only through the  $n$ - $p$  system of the valence nucleons.

### 3. CORRELATION COEFFICIENTS

The correlation coefficients  $C_g$  and  $C_e$  belonging to  $\Psi_g$  and  $\Psi_e$ , respectively, may be treated in the following ways: (i) set both equal to 0, in which case the wave functions (1) are reduced to the pure IPSM wave functions (2); (ii) adjust them so as to obtain the best fit of the experimental inelastic electron scattering form factor of  ${}^6\text{Li}$ ; (iii) obtain them by using the variational principle for the ground-state and first-excited-state energies of  ${}^6\text{Li}$  (the other parameters are adjusted by fitting them to the experimental value of the excitation energy of the first excited state of  ${}^6\text{Li}$  and to the observed inelastic electron scattering form factor); (iv) use the same criterion as (iii) except that the harmonic-oscillator length parameters are varied separately for the ground- and excited-state wave functions while making the fits to the excitation-energy and electron scattering data.

In order to evaluate  $C_g$  and  $C_e$  using the variational principle we assume the Hamiltonian of the

nuclear wave function of  ${}^6\text{Li}$  as

$$H = H_0 + \sum_{i < j} V_{ij}, \quad (5)$$

where  $H_0$  is the harmonic-oscillator Hamiltonian. The sum is over all nucleon pairs in  $V_{ij}$  which is the residual two-body interaction. In this nucleus we expect to have a substantial residual two-body interaction, which will be responsible for the nucleon-nucleon correlation in the harmonic-oscillator type of IPSM wave function. The basic assumption for the single-particle harmonic-oscillator-type wave function is that the two-body nucleon-nucleon forces can be completely represented by a single-particle potential. This assumption is particularly unlikely to be valid for  ${}^6\text{Li}$ .<sup>3,5</sup> For the two-body interaction we take the form

$$V_{12} = \left(\frac{3}{4}V_t + \frac{1}{4}V_s\right) + \left(\frac{1}{4}V_t - \frac{1}{4}V_s\right)\vec{\sigma}_1 \cdot \vec{\sigma}_2, \quad (6)$$

and

$$V_{t,s} = -V_{0t,0s} \left[ a - b \left( \frac{r}{r_0} \right)^{2\nu} \right] e^{-r^2/r_0^2}, \quad (7)$$

where  $r = |\vec{r}_i - \vec{r}_j|$ , the subscripts  $t$  and  $s$  denote the spin triplet and singlet states, respectively, and  $a$ ,  $b$ ,  $\nu$ ,  $r_0$ , and  $V_0$  are adjustable parameters. Since we are concerned only with the triplet spin state we shall drop the suffixes  $t$  and  $s$  from now on. Here we are using a simple type of two-body interaction which has its dependence on the relative coordinates and spins of the particles involved. Other terms, like tensor forces and those depending on the spin orbital orientation are not considered. Using the effective Hamiltonian formalism<sup>21</sup> we can calculate the so-called correlation energy terms for the ground and the excited states of  ${}^6\text{Li}$ . In the lowest order, the wave functions  $\Psi_g$  and  $\Psi_e$  are degenerate, and hence the excitation energy  $W$  is obtained entirely from the difference in the correlation energies of the two states, i.e.,

$$\begin{aligned} \Delta W &= \langle \Psi_e | V_{12} | \Psi_e \rangle - \langle \Psi_g | V_{12} | \Psi_g \rangle, \\ &= W_e - W_g. \end{aligned} \quad (8)$$

For the potential (6) the relevant matrix elements can be evaluated using Racah algebra<sup>22</sup> and Talmi integrals.<sup>23</sup> The minimum energies  $W_g$  and  $W_e$  are obtained from the condition

$$\left( \frac{\partial W(C)}{\partial C} \right)_{g,e} = 0. \quad (9)$$

Since Eq. (9) is quadratic in  $C_{g,e}$  we choose those roots for the  $C$ 's which give better agreement with the electron scattering data. The same criterion is used when  $C$ 's are adjusted instead of evaluating them by the variational principle. Having determined the relevant values of  $C$ 's we substituted

them back into  $W_g$  and  $W_e$  to get the minimum energy  $W$ . The correlation coefficients  $C$ 's as obtained from the variational principle give the energy spacing between the ground and the first excited state of  ${}^6\text{Li}$  in agreement with the mean experimental value within 10% of error. In order to get the best fit with the electron scattering data the parameters in the potentials given by (6) and (7) have been varied in the following manner:

$$\begin{aligned} a &= 0, \quad \nu = -1.0 \text{ to } 1.0 \text{ with } \Delta\nu = 0.5, \quad b = -1.0, \\ a &= 1.0, \quad \nu = -1.0 \text{ to } 1.0 \text{ with } \Delta\nu = 0.5, \\ b &= 0.05 \text{ to } 0.4 \text{ with } \Delta b = 0.05. \end{aligned}$$

In both cases  $r_0 = 1.36$  fm and  $V_0$  is adjusted to get correct  $\Delta W$  and best fit with the electron scattering. It is found that the best fit is obtained with the potential having a hard core defined by the parameters

$$a = 1.0, \quad b \neq 0, \quad \text{and } \nu = -1.$$

In Table I four sets of parameters are given which give the best agreement with the form factor of  ${}^6\text{Li}$  as observed by inelastic electron scattering.

#### 4. ELECTRON SCATTERING

For inelastic scattering the form factor in Born approximation is given by<sup>24</sup>

$$|F_{\text{in}}(q)|^2 = \frac{1}{2J+1} \sum_M |F_{\text{in}}^{MM'}(q)|^2, \quad (10)$$

where  $q$  is the momentum transfer,  $J$  is the total spin of the ground state of the nucleus, and

$$F_{\text{in}}^{MM'}(q) = \frac{1}{Z} \sum_{i=1}^Z \int \Psi_e^{M'} \Psi_g^M e^{i\vec{q} \cdot \vec{r}_i} d\vec{r}_1 \dots d\vec{r}_A. \quad (11)$$

Working again through Racah algebra and radial integrals, we obtain

$$|F_{\text{in}}|^2 = \frac{|F_0|^2}{N_g^2 N_e^2} \left( 1 + \frac{C_g + C_e}{r_0 \alpha} F_1 + \frac{C_g C_e}{r_0^2 \alpha^2} F_2 \right)^2. \quad (12)$$

Cazzola and Foglia<sup>25</sup> originally obtained the first term  $|F_0|^2$  which is readily available by setting  $\alpha_g = \alpha_e$  in the following expression:

$$|F_0|^2 = \frac{7}{2430} \left( \frac{\alpha_g \alpha_e}{\alpha^2} \right)^{10} \frac{q^4}{\alpha^4} e^{-q^2/2\alpha^2}, \quad (13)$$

where

$$\alpha^2 = \frac{1}{2} (\alpha_g^2 + \alpha_e^2). \quad (14)$$

The other terms in (12) are given by

$$\begin{aligned} F_1 &= \left[ 8 + \frac{1}{5} \sum_{n=1}^{\infty} \frac{4n^3 + 20n^2 + 33n + 20}{2^{n-1}} \right. \\ &\quad \left. \times {}_1F_1 \left( -n; \frac{7}{2}; \frac{q^2}{8\alpha^2} \right) \right] \frac{e^{q^2/8\alpha^2}}{24\sqrt{2}}, \end{aligned} \quad (15)$$

where  ${}_1F_1$  is the confluent hypergeometric series, and

$$F_2 = 6 - q^2/4\alpha^2. \quad (16)$$

In  ${}^6\text{Li}$  the two effects due to the finite size of protons and to the motion of the c.m. tend to cancel almost exactly each other, the correction terms due to those effects do not appear explicitly in Eqs. (12)–(16). By setting  $C_g = C_e = 0$ , the form factor (12) is reduced to (13), which is the same as for the simple IPSM in the standard harmonic-oscillator potential, with no effects due to the residual potential on the single-particle Hamiltonian or to the nucleon-correlation on the IPSM wave functions.

### 5. REDUCED TRANSITION PROBABILITY

We have also calculated the electric quadrupole transition probability using the correlated wave functions of the type (1) for the ground and the

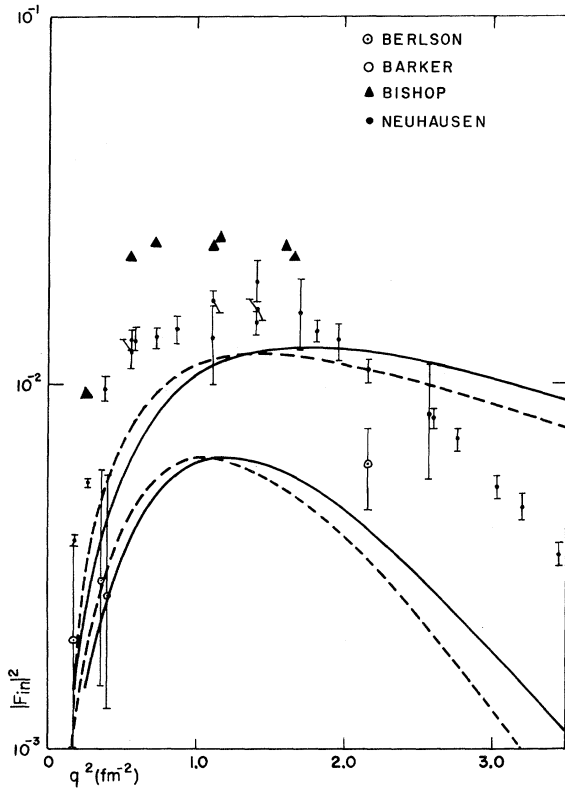


FIG. 1. The lower two curves belong to the simple harmonic type of wave functions with no nucleon correlation. The upper two curves belong to the correlated wave functions in which the correlation coefficient  $C_L$  are adjusted to get the best form factor. The parameters are  $\alpha_g = \alpha_e = 0.515 \text{ fm}^{-1}$ ,  $r_0 = 1.36 \text{ fm}$ ,  $C_g = -0.32$ ,  $C_e = -0.31$  for the broken curves and  $\alpha_g = \alpha_e = 0.544 \text{ fm}^{-1}$ ,  $r_0 = 1.36 \text{ fm}$ ,  $C_g = C_e = -0.31$  for the solid curves.

first excited states of  ${}^6\text{Li}$ . It is sometimes of interest to calculate the so-called quantity reduced transition probability denoted by  $B(E\lambda; J \rightarrow J')$  and defined by<sup>26</sup>

$$B(E\lambda\uparrow) = B(E\lambda; J \rightarrow J') = \frac{1}{2J+1} \sum_{MM'} |\langle J' | Q_{\lambda}^{\mu} | J \rangle|^2, \quad (17)$$

where in this case, i.e., for the quadrupole transition the value of  $\lambda$  is 2. The reduced transition probability  $B(E2\uparrow)$  can be readily derived from the form factor  $|F_{in}(q)|^2$  as obtained from the inelastic electron scattering according to the relation

$$B(E2\uparrow) = \frac{225}{4\pi} Z^2 \lim_{q \rightarrow 0} \frac{|F_{in}(q)|^2}{q^4}. \quad (18)$$

In the case of  ${}^6\text{Li}$ , using Eqs. (12)–(16) and (18), we get

$$B(E2\uparrow) = \frac{B_0}{N_g^2 N_e^2} \left[ 1 + 2.194 \frac{C_g + C_e}{r_0 \alpha} + \frac{C_g C_e}{r_0^2 \alpha^2} \right]^2, \quad (19)$$

where the term  $B_0$  denotes  $B_0(E2\uparrow)$  for the simple

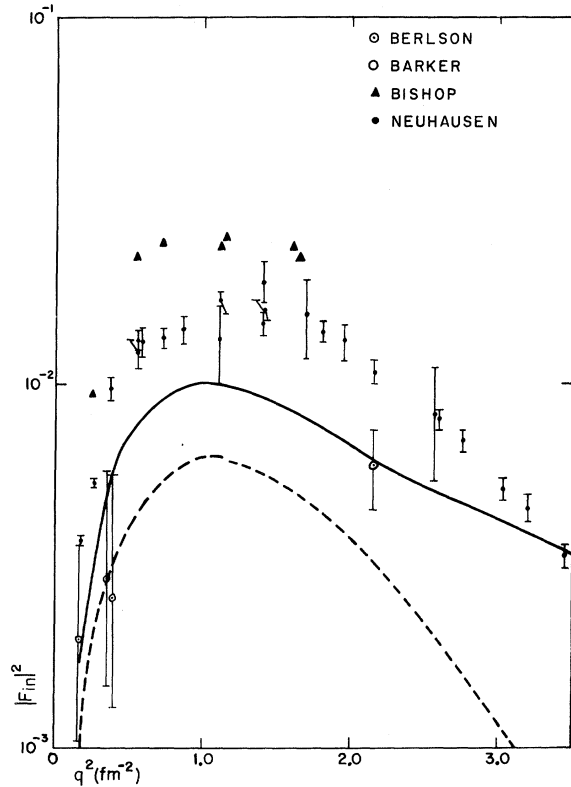


FIG. 2. The solid and broken curves belong to the correlated and uncorrelated IPSM wave functions, respectively. The  $C$ 's are obtained from the variational principle. The parameters used are:  $\alpha_g = \alpha_e = 0.53 \text{ fm}^{-1}$ ,  $r_0 = 1.36 \text{ fm}$ ,  $V_0 = 130.0 \text{ MeV}$ ,  $a = 1.0$ ,  $b = 0.25$ ,  $\nu = -1.0$ ,  $C_g = -0.375$ ,  $C_e = -0.365$ .

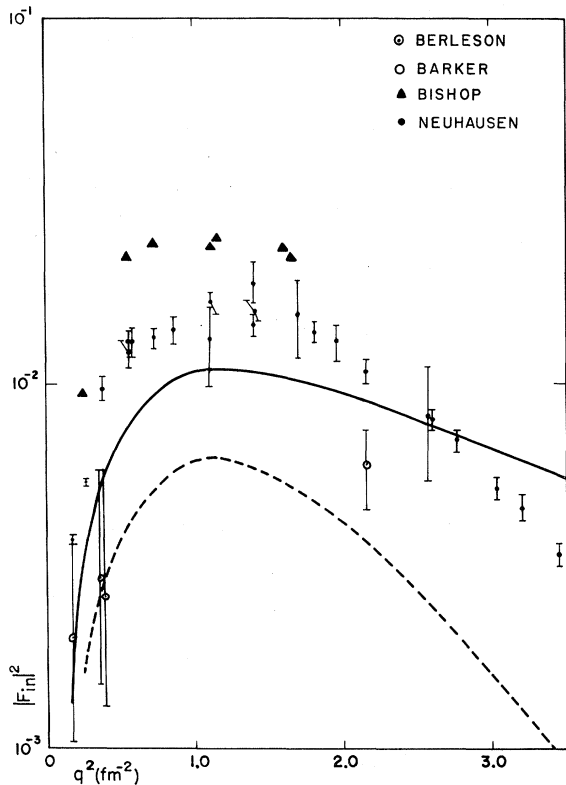


FIG. 3. The solid and broken curves belong to the correlated and uncorrelated wave functions, respectively. The  $C$ 's are the variational parameters and  $\alpha$ 's are varied for the ground- and excited-states independently.  $\alpha_g = 0.52 \text{ fm}^{-1}$ ,  $\alpha_e = 0.51 \text{ fm}^{-1}$ ,  $r_0 = 1.36 \text{ fm}$ ,  $V_0 = 150.0 \text{ MeV}$ ,  $a = 1.0$ ,  $b = 0.20$ ,  $\nu = -1.0$ ,  $C_g = -0.422$ ,  $C_e = -0.375$ .

IPSM with no residual potential and no nucleon-correlation effects, and is given by

$$B_0 = 0.464/\alpha^4.$$

## 6. RESULTS AND DISCUSSION

We have attempted to fit the inelastic electron scattering data by varying parameters  $\alpha$  and  $V_0$  in the ranges

TABLE I. The Roman numerals refer to different cases of treating the correlation coefficients in Sec. 3. Other entries are defined in the text.

Case	$\alpha_g$ ( $\text{fm}^{-1}$ )	$\alpha_e$ ( $\text{fm}^{-1}$ )	$V_0$ (MeV)	$b$	$C_g$	$C_e$	$\Delta W$ (MeV)
iiA	0.515	0.515			0.320	0.310	
iiB	0.544	0.544			0.310	0.310	
iiiA	0.551	0.551	200	0.20	0.445	0.412	2.28
iiiB	0.530	0.530	130	0.25	0.375	0.365	2.20
ivA	0.520	0.510	150	0.20	0.422	0.375	2.32

$$0.50 \text{ fm}^{-1} \leq \alpha \leq 0.60 \text{ fm}^{-1} \text{ with } \alpha_e \leq \alpha_g,$$

$$10.0 \text{ MeV} \leq V_0 \leq 200 \text{ MeV}.$$

Within this range the five sets of parameters are reported in Table I which give the best possible fit to the scattering data. In addition to that, the last three sets of parameters which have been obtained by using the variational principle give the excitation energy of the first excited state of  ${}^6\text{Li}$  within 6% of the experimental value.<sup>27</sup> The Orsay<sup>18</sup> scattering data are consistently higher than those of Neuhausen<sup>28</sup> (see Figs. 1-3). (The experimental points are taken from Refs. 18 and 28, and Burleson and Hofstadter,<sup>29</sup> and Barber *et al.*<sup>30</sup>)

The same sets of parameters have also been used in calculating the reduced transition probabilities which are given in Table II. The other calculated values of  $B(E2\uparrow)$  have been reported as 10.96 and 12.98  $\text{fm}^4$  by Bouten, Bouten, and van Leuven<sup>8</sup> using Volkov<sup>31</sup> and Brink<sup>32</sup> interactions, respectively, as against  $30 \pm 3 \text{ fm}^4$  and  $25.1 \pm 2 \text{ fm}^4$ , the experimentally observed values of Refs. 18 and 28, respectively. Comparing the reduced  $E2$  transition probability for the first excited state of  ${}^6\text{Li}$  obtained from the present calculations with the experimental ones we find that there is a disagreement by a factor of about 2. We can, however, get the value of  $B(E2\uparrow)$  equal to 25 or 30  $\text{fm}^4$  from these calculations but the parameters used in obtaining those values do not give even a good agreement of the inelastic form factors with the experiments, as shown in Figs. 1-3. Nevertheless, we can get the correct excitation energy of the first excited state.

From Figs. 1-3 it is evident that all the curves for the correlated form factors are in consistent shape with the data. By varying the parameters the maxima of the curves are shifted more or less horizontally but no substantial increase in the magnitude of the form factor is obtained.

## 7. CONCLUSION

We have used the Born approximation for the

TABLE II. The reduced electric transition probabilities  $B(E2\uparrow)$  in  ${}^6\text{Li}$  are given for the sets of parameters given in Table I.  $B_0$  and  $B_c$  refer to the reduced transitions calculated with the uncorrelated and correlated wave functions, respectively. Their units are  $\text{fm}^4$ .

Case	$B_0$	$B_c$	Method
iiA	6.61	12.02	By adjusting $C_g$ and $C_e$ to obtain best form factor
iiB	5.29	8.48	
iiiA	5.02	11.17	Variational method with $\alpha_g = \alpha_e$
iiiB	5.88	12.57	Variational method with $\alpha_g = \alpha_e$
iv	6.60	14.54	Variational method but $\alpha_g \neq \alpha_e$

evaluation of the form factor of  ${}^6\text{Li}$  for the inelastic electron scattering. Within the limitation of this technique, we find a significant improvement over the electron scattering results calculated in the simple IPSM. The amount of the nucleon-nucleon correction depends on the correlation coefficients which in turn depend on the potential and wave-function parameters. In general we obtain about more than 100% correction with the correlated wave functions to the IPSM uncorrelated results. The reduced electric quadrupole transition probability to the first excited state is enhanced considerably using the correlated wave function. Nevertheless there remains some discrepancy be-

tween the results of these calculations and experimental form factors and the reduced transition probability. However, it is evident that some nucleon-nucleon correlation in the IPSM wave functions of  ${}^6\text{Li}$  is important. This may be simulated to lead to some clustering effect in  ${}^6\text{Li}$ . Other evidence, such as that based on the  $(p, 2p)$  and  $(p, pd)$  reactions<sup>17, 33</sup> appears to lead to the belief that the two  $p$  nucleons may even be correlated in a deuteronlike cluster. If that is the case, only a single-particle central potential is not sufficient to describe the behavior of  ${}^6\text{Li}$ . A residual two-body interaction with a core is necessary.

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