*Research supported in part by the U. S. Atomic Energy Commission.

[1]M. M. Lévy, Phys. Rev. 88, 725 (1952).

[2]A. B. Volkov, Nucl. Phys. 74, 33 (1965); I. R. Afnan and Y. C. Tang, Phys. Rev. 175, 1337 (1968); Phys. Rev. C 1, 750(E) (1970).

[3]R. Jastrow, Phys. Rev. 79, 389 (1950); 81, 165 (1951).

[4]G. Breit and W. G. Bouricius, Phys. Rev. 75, 1029 (1949); H. Feshback and E. Lomon, Phys. Rev. 102, 891 (1956).

[5]R. V. Reid, Jr., Ann. Phys. (N.Y.) 50, 411 (1968); T. Hamada and I. D. Johnston, Nucl. Phys. 34, 382 (1962); K. E. Lassila, M. H. Hull, Jr., H. M. Ruppel, F. A. McDonald, and G. Breit, Phys. Rev. 126, 881 (1962); D. P. Saylor, R. A. Bryan, and R. E. Marshak, Phys. Rev. Letters 5, 266 (1960); H. Feshbach and E. Lomon, *ibid.* 6, 635 (1961).

[6]L. D. Faddeev, Zh. Eksperim. i Teor. Fiz. 39, 1459 (1960) [transl.: Soviet Phys. – JETP 12, 1014 (1961)].

[7]E. O. Alt, P. Grassberger, and W. Sandhas, Nucl. Phys. B2, 167 (1967); D. D. Brayshaw, Phys. Rev. 176, 1855 (1968).

[8]If this were not so, one could prove in the usual fashion that the t matrix would approach the potential as the energy approached infinity. However, we show later in this paper that the BCM t matrix must increase at least linearly with the energy in this limit.

[9]Y. E. Kim and A. Tubis, Phys. Rev. C 1, 414 (1970).

[10]See, for example, L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1949), or any equivalent textbook on elementary quantum mechanics.

[11]D. D. Brayshaw, Phys. Rev. 167, 1505 (1968).

[12]We shall regard the fact that Eq. (1) has a unique solution as well established by many other sources. In our derivation we assume for convenience that our fixed value of s is not an eigenvalue.

[13]The functions $h_l^+(x)$ are sometimes called spherical Hankel functions of the first kind, and are often denoted by $h_l(x)$ in the literature.

[14]M. M. Hoenig and E. L. Lomon, Ann. Phys. (N.Y.) 36, 363 (1966). This involves the combination of a δ function and its derivative, implying that the pseudopotential is separable in momentum space. Usually separable potentials imply separable t matrices; the fallacy here is due to the fact that the BCM t matrix is not the solution of the LS equation with this potential. Indeed, the solution does not even exist because of lack of convergence.

[15]Due to the factorization of the t matrix at a bound-state pole, the off-shell residue is completely determined if the half-on-shell residue is known.

[16]See, for example, M. L. Goldberger and K. M. Watson, *Collision Theory* (John Wiley & Sons, Inc., New York, 1964).

[17]R. P. Boas, *Entire Functions* (Academic Press Inc., New York, 1954).

[18]E. L. Lomon and M. McMillan, Ann. Phys. (N.Y.) 23, 439 (1963).

[19]The virtues of separable approximations in three-body calculations are by now well documented; for a review see D. D. Brayshaw, Phys. Rev. 182, 1658 (1969).

[20]D. D. Brayshaw and B. Buck, Phys. Rev. Letters 24, 733 (1970).

[21]J. M. Blatt and L. C. Biedenharn, Phys. Rev. 86, 399 (1952).

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Binding Energy of a Λ Particle in Nuclear Matter

B. Ram and W. Williams

Physics Department, New Mexico State University, Las Cruces, New Mexico 88001

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The results of a complete calculation of the binding energy D of a Λ particle in nuclear matter using the method of the independent-pair approximation, which systematically take into account the second- and third-order Born corrections, are presented. It is found that these corrections are small and that the Born series converges rapidly. A comparison of our results with those obtained using other methods based on the Brueckner theory shows that they are identical.

I. INTRODUCTION

Calculations of the binding energy D of a Λ particle in nuclear matter have lately increased in tempo,[1-7] for they provide information about the Λ-nucleon interaction in angular momentum states higher than zero and the possible presence of noncentral components.[8] The calculations which have been performed using central Λ-N potentials with hard cores have primarily used two approaches – the variational approach of Jastrow[9] and various versions[10] of the Brueckner-Bethe theory. Both of these approaches give results for D which are much larger than the experimental estimates of about 30 MeV.[11] Calculations using the Jastrow method give values for D about 20 MeV higher than those obtained by methods based on the Brueckner theory.

The disparity of about 20 MeV between the two approaches was first noticed by Ram and James[12]

when they calculated D for the central, spin-dependent Λ-N potential H of Herndon and Tang[13] in the so-called independent-pair-approximation (IPA) version of the Brueckner theory and in the Jastrow method using a simple form of the Λ-N correlation function. They obtained $D = 46.4$ MeV in the IPA and $D = 62.3$ MeV in the Jastrow method. An independent Jastrow calculation by Mueller and Clark,[14] who used a more sophisticated form for the Λ-N correlation function, gave the value $D = 64.0$ MeV. Subsequent calculations of D by Bodmer and Rote,[6] and Dabrowski and Hassan,[5] using other versions of the Brueckner theory, gave results about the same as those obtained in the method of the IPA. However, these authors criticized the IPA on the grounds that the calculation by Ram and James[12] and the previous calculations by other workers[15] contained only the first-order term in the attractive potential tail while an estimate by Ranft[16] indicated that higher-order Born terms were large. In a recent letter,[2] the present authors have shown that the estimates of second-order Born corrections by Ranft[16] are incorrect and that this correction in S waves for each of the three potentials H, E, and E′ of Herndon and Tang[13] is indeed small.

In these calculations, however, the numerical value of the parameter N, which appears in the approximate Bethe-Goldstone function of Downs and Ware[15] (see Sec. II), was either taken to be the same as given by them or was obtained by linearly extrapolating their values for different hard-core radii. Thus the value of N used for a given Λ-N potential was not an accurate representation, since it is a function of the attractive potential tail for which D is to be calculated. The optimum value of N for each individual attractive potential should be obtained by the procedure outlined in the original paper of Downs and Ware.[15] The purpose of this paper is to report the results of a complete calculation of D in the method of the IPA for the potentials H, E, and E′ of Herndon and Tang and an old potential DW of Downs and Ware, which remedies this fault and in which the first-, second-, and third-order Born terms are systematically taken into account. Our results show that higher-order corrections are small and that the Born series converges rapidly. Finally, a comparison of our results with those obtained in other versions of the Brueckner theory shows that they are identical, thus justifying the assumptions made in the IPA.

In Sec. II we briefly outline[17] the method of the independent-pair approximation. The results are given in Sec. III. Section IV is devoted to concluding remarks.

II. METHOD OF IPA

In the method of IPA the binding energy D is given by[15]

$$D = -\frac{4}{(2\pi)^3}\left(\frac{M_N^*}{\mu^*}\right)^3 \int_0^{\beta=\mu^* k_F/M_N^*} d^3k \int e^{-i\vec{k}\cdot\vec{r}} [V_C(r) + V_A(r)]\,\psi_{BG}(\vec{k},\vec{r})\,d^3r\,, \tag{1}$$

where V_C is the hard-core part of the Λ-N potential (this is the same for both spin states) and

$$V_A(r) = \tfrac{1}{4}V_A^s(r) + \tfrac{3}{4}V_A^t(r)\,, \tag{2}$$

where s and t refer to the singlet and triplet states. The vectors $\vec{r}$ and $\vec{k}$ denote the relative coordinate and momentum of the Λ-N pair, and $\psi_{BG}(\vec{k},\vec{r})$ is the solution to the Bethe-Goldstone equation[18] for the relative motion of a Λ-N pair in nuclear matter. However, the Bethe-Goldstone equation has only been solved in S wave for a pure hard-core potential, and the solution is[15]

$$R_{BG}^0(k,r) = \frac{A^0(k)}{kr}\left\{\sin k(r-c) + \frac{1}{\pi}\int_0^r \sin k(r-r')\left[\frac{\sin k_F(r'+c)}{(r'+c)} - \frac{\sin k_F(r'-c)}{(r'-c)}\right]dr'\right\} \tag{3a}$$

with

$$A^0(k) = \left[\cos kc + \frac{1}{\pi}\left(\sin kc\{\mathrm{Ci}[c(k_F+k)] - \mathrm{Ci}[c(k_F-k)]\} - \cos kc\{\mathrm{Si}[c(k_F+k)] + \mathrm{Si}[c(k_F-k)]\}\right)\right]^{-1}. \tag{3b}$$

An approximate form for (3), which is more appropriate for practical use, has been suggested by Downs and Ware.[15]

$$R^0(k,r) = N[1 - e^{-2(r-c)/a}]\,j_0(kr) \tag{4}$$

in which values of the parameters N and a depend on the potential $V_A(r)$. Optimum values of a have been determined by Downs and Ware for hard-core radii $c = 0.4$ and 0.6 fm so that the function (4) provides a good representation of (3). (In our calculations, we have used the same values of a as theirs for the potentials H and DW which have $c = 0.6$ and 0.4 fm, respectively, but for the potentials E and E′ which have $c = 0.45$ fm we have used a value of $a = 1.05/k_F$ which is obtained by linearly extrapolating their values.) The

value of the parameter N for each potential $V_A(r)$ is then determined[15] by requiring that the value of the attractive contribution D_A^0 calculated with (4) equal that calculated with (3). The values of N thus obtained are given in Table I for the respective potentials.

Once the values of N and a are determined by the above procedure, the function (4) is generalized to

$$\varphi_{BG}^{core}(\vec{k},\vec{r}) \equiv \varphi_{BG}(\vec{k},\vec{r}) = f(r)e^{i\vec{k}\cdot\vec{r}}, \tag{5a}$$

where

$$f(r) = N(1-e^{-2(r-c)/a}), \tag{5b}$$

to represent the full solution to the Bethe-Goldstone equation for a pure hard-core potential. Substitution of (5) for $\psi_{BG}(\vec{k},\vec{r})$ in (1) then gives the first-order contributions D_C and D_A to the Λ binding energy. The second- and third-order contributions can be evaluated by using the perturbation expansion[16]

$$\psi_{BG}(\vec{k},\vec{r}) = \varphi_{BG}(\vec{k},\vec{r}) + \frac{\mu^*}{\hbar^2}\int G(\vec{r},\vec{r}')V_A(r')\varphi_{BG}(\vec{k},\vec{r}')d^3r' + \left(\frac{\mu^*}{\hbar^2}\right)^2\int G(\vec{r},\vec{r}')[V_C(r')+V_A(r')]G(\vec{r}',\vec{r}'')V_A(r'')\varphi_{BG}(\vec{k},\vec{r}'')d^3r'\,d^3r'', \tag{6}$$

where the Green's function G has the same property as the Bethe-Goldstone function, i.e., it vanishes for $r<c$, and is given by

$$G = \frac{1}{(2\pi)^3}\int \frac{|\psi\rangle\langle\psi|\,d^3k'}{k^2-k'^2}. \tag{7}$$

The functions $|\psi\rangle$ form an orthogonal, complete set of eigenfunctions of the equation

$$(\nabla^2+k^2)\psi(\vec{r}) = 0, \quad r>c, \tag{8}$$

with the boundary condition $\psi(|\vec{r}|=c)=0$.

Substitution of the expression (6) for $\psi_{BG}(\vec{k},\vec{r})$ in (1) gives for the binding energy D

$$D = (D_C+D_A)+(D_{CA}+D_{AA})+(D_{CCA}+D_{CAA}+D_{ACA}+D_{AAA}) = D_I+D_{II}+D_{III}, \tag{9}$$

where D_{CA}, D_{AA}, D_{CCA}, etc. represent in (1) terms involving $V_C\times V_A$, $V_A\times V_A$, $V_C\times V_C\times V_A$, etc. Using the property that the Bethe-Goldstone function has a discontinuous derivative at the hard-core, Ranft[16] showed that the overlap terms D_{CCA} and D_{ACA} vanish. We give explicit expressions for the rest of the terms in (9) in the Appendix.

III. RESULTS

We have evaluated various contributions to D for the Λ-N central potentials H, E, and E′ of Herndon and Tang,[13] which have been obtained from studies of light hypernuclei and Λ-p scattering, and for an old potential DW of Downs and Ware.[15] The potentials of Herndon and Tang are spin dependent, and are of the form

$$V_{s,t}(r) = \infty \quad r\le c$$
$$= -V_{s,t}\exp[-3.5412(r-c)/b^0] \quad r>c. \tag{10}$$

The potential DW is also of the form (10) except that it is spin independent. For each potential the averaged value $\overline{V} = \frac{1}{4}V_s + \frac{3}{4}V_t$, the hard-core radius c, and the total intrinsic range $r_{int} = b^0 + 2c$ are listed in Table I with the corresponding values of the parameters N and a which appear in the approximate Bethe-Goldstone function (4) of Downs and Ware. The values of N and a were obtained as described in Sec. II.

Various contributions to D appear in Tables II–V. These have been calculated by using the expressions given in the Appendix. The contributions D_C^I for $c = 0.4$ fm and $c = 0.6$ fm were first calculated

TABLE I. Parameters of the Λ-N potentials and the respective values of N and a in the function (4).

Potential designation	$\overline{V}$ (MeV)	c (fm)	r_{int} (fm)	N	a (fm)
H	685.95	0.6	2.1	1.116	$1.2/k_F$
E	414.5	0.45	2.0	1.07	$1.05/k_F$
E′	398.9	0.45	2.0	1.07	$1.05/k_F$
DW	330.9	0.4	1.9	1.054	$1.0/k_F$

TABLE II. Various contributions to D_C in MeV for different values of the hard-core radius c.

c (fm)	D_C^0	D_C^1	D_C^2	D_C
0.4	−54.7	−2.6	−0.02	−57.3
0.45	−64.8	−3.6	−0.04	−68.5
0.6	−102.9	−8.1	−0.19	−111.2

in Ref. 15. From the results given in Table III one can see that the contribution to D_A from all partial waves $l > 2$ is only about 0.2 MeV.

Table IV lists the second-order contributions D_{CA}^l and D_{AA}^l for $l = 0$ and 1. We have also calculated these terms for $l = 2$. In each case the magnitude of $(D_{CA}^2 + D_{AA}^2)$ is less than 10^{-3} MeV. This means that in calculating D_{II}^l one need not go any higher than $l = 1$. Also note that the value of D_{CA}^1 (D_{AA}^1) is smaller by two orders of magnitude than the corresponding value of D_{CA}^0 (D_{AA}^0).

In Table V we have given the third-order contributions for $l = 0$. We have not calculated these corrections for $l = 1$, since the integrals[19] involved become quite complicated. However, we expect these to be negligibly small on the reasonable assumption that the ratio between the contributions D_{CAA}^1 (D_{AAA}^1) and $D^0{}_{CAA}$ (D_{AAA}^0) would be of the same order of magnitude as between D_{CA}^1 (D_{AA}^1) and D_{CA}^0 (D_{AA}^0) for the second-order contributions. It is apparent from Tables IV and V that the S-wave third-order contributions D_{CAA}^0 (D_{AAA}^0) are one order of magnitude smaller than the second-order contributions D_{CA}^0 (D_{AA}^0). It is not unreasonable to expect at least the same ratio between the S-wave fourth-order and third-order contributions. Thus it would be safe to say that our values for D (second column of Table VI) may be slightly overestimated so as to affect only the last figure.

In Table VI we have compiled our results together with results obtained by other authors both in the Brueckner theory and in the Jastrow method. For each potential (except DW) there appear in each column two values for the binding energy D. The upper values are for the potential suppressed by 40% in P waves; lower values are for the potential without suppression. The values given in Tables II–V for various contributions are for potentials without suppression. A glance across this table clearly shows that the results obtained in all versions of the Brueckner theory are identical[20]; however, they are considerably lower than those obtained in the Jastrow method, the difference being about 20 MeV for the potential H and about 10 MeV for the potentials E and E′.

TABLE III. Various contributions to D_A and the values of $D_I = D_C + D_A$ in MeV for specified Λ-N potentials.

Potential	D_A^0	D_A^1	D_A^2	D_A	$D_I = D_C + D_A$
H	138.6	31.6	2.6	172.9	61.7
E	105.0	24.9	2.5	132.6	64.1
E′	101.0	24.0	2.4	127.6	59.1
DW	77.3	17.4	1.7	96.5	39.2

TABLE IV. Various contributions to D_{II} in MeV for specified Λ-N potentials.

Potential	D_{CA}^0	D_{CA}^1	D_{AA}^0	D_{AA}^1	D_{II}
H	−29.72	−0.17	27.14	0.04	−2.71
E	−16.69	−0.05	15.61	0.02	−1.11
E′	−16.06	−0.05	14.45	0.02	−1.64
DW	−11.46	−0.03	9.11	0.01	−2.37

IV. CONCLUDING REMARKS

In the previous section we presented the results of a complete calculation of the binding energy D in the method of IPA for the spin-dependent, central Λ-N potentials of Herndon and Tang[13] and for a spin-independent potential DW of Downs and Ware.[15] These results conclusively show that the higher-order corrections to the binding energy are small and that the Born series converges rapidly; and also, that the results in the IPA are the same as those obtained in other methods[5,6] based on the Brueckner theory, thus justifying the assumptions made in the IPA.[21]

All our results presented in Sec. III are for the Λ-particle effective mass $M_\Lambda^* = M_\Lambda$. We have also made calculations for $M_\Lambda^* = 0.9M_\Lambda$; however, the results obtained with $M_\Lambda^* = 0.9M_\Lambda$ are about 3–6 MeV lower than those given in the second column of Table VI with $M_\Lambda^* = M_\Lambda$. If it is granted that the method employed by Dabrowski and Hassan[5] or that by Rote and Bodmer is very accurate and extremely reliable, then the fact that our results with $M_\Lambda^* = M_\Lambda$ are identical to theirs also indicates that the value of the Λ-particle effective mass is most likely to equal M_Λ rather than $0.9M_\Lambda$, as was first suggested by Dabrowski and Kohler.[22] In other words, future calculations of D in the method of the IPA

TABLE V. Various contributions to D_{III} in MeV for specified Λ-N potentials.

Potential	D_{CAA}^0	D_{AAA}^0	D_{III}
H	−6.03	4.76	−1.27
E	−2.62	2.05	−0.57
E′	−2.43	1.83	−0.60
DW	−1.42	0.95	−0.47

TABLE VI. Results for the binding energy D in MeV obtained by different authors.

Potential	Our results	Results of Dabrowski and Hassan (Ref. 5)	Results of Rote and Bodmer (Ref. 6)	Results in the Jastrow method (Ref. 14)
H	45.1	45.4	45.6	62–64
	57.7	56.7	56.8	72
E	52.5	52.4	51.9	63
	62.4	61.1	61.7	69
E′	47.3	47.2	47.1	57
	56.9	55.6	56.0	62
DW	36.4	36.3		42.0

including higher-order corrections should use the value $M_\Lambda^* = M_\Lambda$.

The fact still remains that the calculated values of D using the Brueckner theory for all realistic potentials of Herndon and Tang (the potential DW is an old potential and does not satisfy the present data on Λ-p scattering and binding energies of light hypernuclei) are much larger than the empirical estimates of about 30 MeV. The rearrangement correction to the binding energy D has been estimated[5] to be about −10% of D. Even after this correction is taken into account, a difference of about 10 MeV persists between the calculated value and the empirical estimate of D for the best potential H. (Out of all potentials, the calculated value of D is lowest for H.) Various mechanisms have been suggested in the past to account for this difference. The potential H already includes a 40% P-wave suppression. However, this suppression was based on the Λ-p scattering data obtained before 1968. Future data on Λ-p elastic scattering at higher energies, which are highly desirable, may require a larger P-wave suppression, which will lower the theoretical value of D, since D is quite sensitive to this reduction in P waves. The effect of tensor suppression has been estimated[23] to be about minus 2–3 MeV. Recent quantitative estimates[1,24,25] of the Λ-Σ conversion effect (or the so-called isospin suppression effect) have shown that it could further reduce the calculated value of D by as much as 10–20 MeV. Thus it would seem that inclusion of all these effects could bring at least those values of D which have been obtained in the Brueckner theory close to the experimental estimate. However, we feel that quantitative analyses of all these effects are only of academic interest until an explanation of the discrepancy (which is as much as about 20 MeV for the best potential H) between the results obtained in the two approaches – the Brueckner approach and that of Jastrow – is found, especially since any conclusion regarding the inclusion of "genuine"[26] three-body ΛNN forces in the calculations of D depends upon this explanation and the "true" theoretical value of D.

APPENDIX

In this Appendix we give explicit expressions for the various terms which appear in (9). For the first three partial-wave contributions D_C^l,

$$D_C^0 = -\frac{8}{\pi}\left(\frac{M_N^*}{\mu^*}\right)^3\left(\frac{\hbar^2}{2\mu^*}\right)\int_0^{\beta=\mu^* k_F/M_N^*} A^0(k)\sin(kc)k\,dk\,, \tag{A1}$$

$$D_C^1 = -\left(\frac{8}{5\pi}\right)\left(\frac{\hbar^2 k_F^2}{2\mu^*}\right)\left(\frac{\mu^*}{M_N^*}\right)^2(k_F c)^3\left[1-\frac{3}{7}\left(\frac{\mu^*}{M_N^*}\right)^2(k_F c)^2\right], \tag{A2}$$

$$D_C^2 = -\left(\frac{8}{63\pi}\right)\left(\frac{\hbar^2 k_F^2}{2\mu^*}\right)\left(\frac{\mu^*}{M_N^*}\right)^4(k_F c)^5\,. \tag{A3}$$

The total contribution D_A from all partial waves can be obtained by integrating expression (1) with only the attractive part $V_A(r)$ of the potential and using the function (4) in place of $\psi_{BG}(\vec{k},\vec{r})$. However, the contribution D_A^l from individual partial waves is given by

$$D_A^l = -\frac{4(4\pi)^2}{(2\pi)^3}\left(\frac{M_N^*}{\mu^*}\right)^3(2l+1)\int_0^\beta k^2 dk\int r^2 dr\, j_l(kr) f(r) V_A(r)\,. \tag{A4}$$

The second-order terms D^l_{CA} and D^l_{AA} for $l=0$, 1, and 2 are given by

$$D^l_{CA}=-\frac{4}{(2\pi)^6}(4\pi)^4\left(\frac{M^*_N}{\mu^*}\right)^3\left(\frac{c}{2}\right)(2l+1)\int_0^\beta k^2dk\, j_l(kc)\int_{k_F}^\infty \frac{k'^2dk'}{k^2-k'^2}$$
$$\times(k'c)j_{l-1}(k'c,\varphi_l^{k'})\int r'^2dr'\, j_l(k'r',\varphi_l^{k'})V_A(r')f(r')j_l(kr'), \tag{A5}$$

$$D^l_{AA}=-\frac{4}{(2\pi)^6}(4\pi)^4\left(\frac{M^*_N}{\mu^*}\right)^3\left(\frac{\mu^*}{\hbar^2}\right)(2l+1)\int_0^\beta k^2dk\int r^2dr\, j_l(kr)V_A(r)$$
$$\times\int_{k_F}^\infty \frac{k'^2dk'}{k^2-k'^2}\, j_l(k'r,\varphi_l^{k'})\int r'^2dr' j_l(k'r',\varphi_l^{k'})V_A(r')f(r')j_l(kr'), \tag{A6}$$

where the functions $j_l(kr,\varphi_l^k)$ are spherical Bessel functions $j_l(kr)$ in which the argument (kr) of the trigonometric functions sine and cosine has been replaced by $(kr-\varphi_l^k)$; for instance,

$$j_0(kr,\varphi_0^k)=\frac{\sin(kr-\varphi_0^k)}{kr},$$

$$j_1(kr,\varphi_1^k)=\frac{\sin(kr-\varphi_1^k)}{(kr)^2}-\frac{\cos(kr-\varphi_1^k)}{(kr)}$$

and,

$$j_{-1}(kc,\phi_0^k)\equiv\frac{1}{kc}. \tag{A7}$$

The functions φ_l^k are determined by the following condition:

$$j_l(kc,\varphi_l^k)=0. \tag{A8}$$

The third-order contributions for $l=0$ are

$$D^0_{CAA}=-\frac{4}{(2\pi)^9}(4\pi)^6\left(\frac{M^*_N}{\mu^*}\right)^3\left(\frac{\mu^*}{2\hbar^2}\right)\int_0^\beta kdk\sin(kc)\int_{k_F}^\infty\frac{k'^2dk'}{k^2-k'^2}\int r'^2dr' j_0(k'r',\phi_0^{k'})$$
$$\times V_A(r')\int_{k_F}^\infty\frac{k''^2dk''}{k^2-k''^2}j_0(k''r',\phi_0^{k''})\int r''^2dr'' j_0(k''r'',\phi_0^{k''})V_A(r'')j_0(kr'')f(r''), \tag{A9}$$

$$D^0_{AAA}=-\frac{4}{(2\pi)^9}(4\pi)^6\left(\frac{M^*_N}{\mu^*}\right)^3\left(\frac{\mu^*}{\hbar^2}\right)^2\int_0^\beta k^2dk\int r^2dr\, j_0(kr)V_A(r)\int_{k_F}^\infty\frac{k'^2dk'}{k^2-k'^2}j_0(k'r,\phi_0^{k'})\int r'^2dr' j_0(k'r',\phi_0^{k'})V_A(r')$$
$$\times\int_{k_F}^\infty\frac{k''^2dk''}{k^2-k''^2}j_0(k''r',\phi_0^{k''})\int r''^2dr'' j_0(k''r'',\phi_0^{k''})V_A(r'')j_0(kr'')f(r''). \tag{A10}$$

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[1] A. R. Bodmer and D. M. Rote, in *Proceedings of the International Conference on Hypernuclear Physics, Argonne National Laboratory, May, 1969,* edited by A. R. Bodmer and L. G. Hyman (Argonne National Laboratory, Argonne, Illinois, 1969), p. 521, and references cited therein.

[2] B. Ram and W. Williams, Phys. Letters 31B, 49 (1970).

[3] J. R. James and B. Ram, Nucl. Phys. A140, 688 (1970).

[4] R. K. Bhaduri and J. Law, Nucl. Phys. A140, 214 (1970).

[5] J. Dabrowski and M. Y. M. Hassan, Phys. Rev. C 1, 1883 (1970).

[6] D. M. Rote and A. R. Bodmer, Nucl. Phys. A148, 97 (1970).

[7] J. W. Clark and G. Mueller, Nuovo Cimento 64B, 217 (1969).

[8] In fact, these calculations also help to fix, in an indirect manner, the parameters, spin-dependence, and shape of the effective *S*-wave central Λ-N potential (see Refs. 3, 5, 6, 24, and 25).

[9] R. Jastrow, Phys. Rev. 98, 1479 (1955).

[10]See, for example, L. C. Gomes, J. D. Walecka, and V. F. Weisskopf, Ann. Phys. (N.Y.) 3, 241 (1958); G. Dahll, E. Østgaard, and B. Brandow, Nucl. Phys. A124, 481 (1969).
[11]See, for example, Ref. 1.
[12]B. Ram and J. R. James, Phys. Letters 28B, 372 (1969); 29B, 227 (1969).
[13]R. C. Herndon and Y. C. Tang, Phys. Rev. 159, 853 (1967).
[14]G. Mueller and J. W. Clark, Nucl. Phys. B7, 227 (1968).
[15]B. W. Downs and W. E. Ware, Phys. Rev. 133, B133 (1964).
[16]G. Ranft, Nuovo Cimento 45B, 138 (1966).
[17]For details, the reader should refer to Refs. 15 and 16.
[18]H. A. Bethe and J. Goldstone, Proc. Roy. Soc. (London) A238, 551 (1957).
[19]All numerical integrations involved in our calculations were done on the New Mexico State University computer IBM 360, Model 50, using the method of eight-point Gaussian quadrature.
[20]In all our calculations we have used the following numerical values: $k_F = 1.366$ fm^{-1}, $M_N^* = 0.735\, M_N$, and $M_\Lambda^* = M_\Lambda$. The values of D given in column 3 as the results of Dabrowski and Hassan are obtained by multiplying their results in Ref. 5 by $(1.366/1.35)^3$, which is the ratio of the densities involved. This is done for the sake of comparison.
[21]The IPA differs from the other two methods [Dabrowski and Hassan (Ref. 5) and Rote and Bodmer (Ref. 6)] in the assumed forms of the single-particle energies and in the procedures employed in solving the g matrix. A discussion concerning the differences in the IPA and the method employed by Dabrowski and Hassan is given in the Appendix of their paper (Ref. 5). Since their method is essentially the same as the reference-spectrum method (RSM) used by Rote and Bodmer (Ref. 6), their discussion equally applies to the differences between the IPA and RSM. Dabrowski and Hassan (Ref. 5) have obtained exact results for D for the potential DW using their form of the single-particle energies, as well as the one assumed in IPA. They obtain nearly identical results. This means that if in the IPA the g matrix is solved exactly via the Bethe-Goldstone equation, which is done here by an explicit perturbation expansion (both Dabrowski and Hassan and Rote and Bodmer solve the g matrix by an iterative procedure), then one would expect the IPA results to be identical to those obtained by these authors. This expectation is well fulfilled by our results. The combined effect, in the IPA, of a different assumed form for the single-particle spectra and a different procedure in solving the g matrix is not to alter the total outcome for D but is only to distribute differently (than in the other two methods) various contributions, hard core and attractive.
[22]J. Dabrowski and H. S. Kohler, Phys. Rev. 136, B162 (1964).
[23]J. Dabrowski and M. Y. M. Hassan, Phys. Letters 31B, 103 (1970); A. R. Bodmer and D. M. Rote, to be published; G. F. Goodfellow and Y. Nogami, Nucl. Phys. B18, 182 (1970).
[24]Y. Nogami and E. Satoh, Nucl. Phys. B19, 93 (1970).
[25]E. Satoh and Y. Nogami, to be published.
[26]For a review of three-body ΛNN forces, see Nogami, Ref. 1, p. 244.

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Connection Between Symplectic, Quasispin, and Generalized Bogoliubov Groups

A. Bose
Université de Montréal, Montréal, Canada
and
A. Navon
Université du Québec, Trois-Rivières, Canada
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A new derivation of the quasispin groups used in nuclear spectroscopy is given. It is shown that these groups arise as the maximum commuting subgroups of the Flowers symplectic groups inside groups of generalized Bogoliubov transformations. These transformations are defined to be the most general transformations which conserve the anticommutation relations of the fermion operators of a nuclear shell.

1. INTRODUCTION

Nuclear shell-model states in j-j coupling have been classified by Flowers[1] with the aid of symplectic groups in $2j+1$ dimensions. The symplectic group acting on states of identical nucleons furnishes the seniority quantum number, while the symplectic group acting on states of neutrons and protons provides the two quantum numbers of seniority and reduced isospin. These quantum numbers are very useful for labeling N-particle states and for calculating various nuclear matrix elements.[2] More recently, it was found that two smaller groups, of the type $SU(2)$ and $USp(4)$, provide the same quantum numbers as the initial symplectic groups but are more advantageous for some nu-