*Work supported in part by the U. S. Atomic Energy Commision.

†Present address: Gulf General Atomic, San Diego, California.

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Isomer Ratio for (b, xn, yp, γ) Reactions

G. Liggett and D. Sperber*
Rensselaer Polytechnic Institute, Troy, New York 12181

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A method for the calculation of isomer ratios in reactions in which the compound nucleus decays by the emission of a few particles followed by γ emission is presented. It is shown that the calculated value of the isomer ratio is sensitive to the spin cutoff parameter, the amount of quadrupole admixture in the γ-ray cascade, and the presence of discrete levels. In most cases good agreement between experimentally measured and calculated values of isomer ratios is obtained.

I. INTRODUCTION

In this paper a new method for the calculation of isomer ratios for reactions in which a compound nucleus decays emitting several particles and γ rays is presented. The method allows for more accurate evaluation of isomer ratios and a more meaningful interpretation of these ratios.

Huizenga and Vandenbosch[1,2] were the first to realize that the isomer ratio depends not only on the properties of the initial state, the ground state, and isomeric state, but also on the properties of the intermediate states. Consequently they suggested a method for the calculation of isomer ratios. To simplify the computation, Huizenga and Vandenbosch[1,2] suggested an approximate method for the calculation of isomer ratios. The Huizenga-Vandenbosch approximation is based on the following assumptions: (i) The isomeric and ground state are populated only in the last state of the γ-ray cascade. (ii) In this final step, transitions with smaller spin change predominate. (iii) The number of γ rays in the cascade is introduced as a free parameter. (iv) In the γ-ray cascade only dipole radiation is considered except in the final transition. (v) The energy dependence of the density of levels is neglected. (vi) The effects of all discrete states, except the isomeric and ground state, are not considered. (vii) For each step in the evaporation, only the dominant decay is considered and competing decays are neglected. Yet despite its limitations the Huizenga-Vandenbosch method serves as a very good approximation.

Recently studies of isomer ratios were reported[3-6] which exclude some of the limitations imposed by the Huizenga-Vandenbosch[1,2] approximation. In particular, one of the present authors[6] developed a method for the evaluation of isomer ratios for (n, γ) reactions which eliminates most of these assumptions. This method was successfully applied[7] to the analysis of isomer ratios in such reactions. The method discussed in this paper is a generalization of the previous work.[6] The present method avoids all the limitations implied by the Huizenga-Vandenbosch[1,2] approximation and is applicable to the most complex reactions. First, the cross section for the formation of the compound nucleus as a function of spin and energy is calculated. Second, the cross section for the formation of the isomeric and ground state are evaluated. All possible decay routes which link the states of the originally formed compound nucleus with the isomeric or ground state are included. All decay rates are calculated as rigorously and as accurately as present information about nuclear structure allows.

II. THEORY

In this section the theory for calculating isomer ratios is outlined. It has been shown, in a previous paper,[6] that isomer ratios for the (n,γ) reaction depend on a function closely related to the function representing the population of states. Also, it has been shown[8] that for (α, xn) reactions all observable quantities can be evaluated from the knowledge of functions closely related to the population of states after the emission of a specified number of neutrons. In a similar way it can be seen that the isomer ratios for complex reactions can be determined from the knowledge of a function closely related to the population of states of all intermediate nuclei leading to the isomeric and ground state. The present method is general and allows all possible decays. However, since for the reactions under consideration, neutron, proton, and γ decay are the dominant decays, other decays such as α decay are neglected. In the present study, only neutron decay, proton decay, and γ decay are considered. Let $\eta(A, Z, E, J; t)$ be the population of states of a nucleus with A nucleons, Z protons at an excitation E with a spin J at time t after the formation of the compound nucleus. Let the original compound nucleus have A_0 nucleons and Z_0 protons and let x be the maximum allowed number of neutrons in the cascade and y the maximum allowed number of protons in this cascade; then the function $\eta(A, Z, E, J; t)$ satisfies the following integrodifferential equation:

$$\begin{aligned}\frac{\partial\eta(A,Z,E,J;t)}{\partial t} &= (1-\delta_{Z,Z_0})(1-\delta_{A,A_0})\Big[\sum_{J'}\int\eta(A+1,Z,E',J';t)S^n(E',J';E,J)dE' \\ &+\sum_{J'}\int\eta(A+1,Z+1,E',J';t)S^p(E',J';E,J)dE'\Big] \\ &-(1-\delta_{A_0-x-y,A})(1-\delta_{Z_0-y,Z})\eta(A,Z,E,J;t)\sum_{J'}\int[S^n(E,J;E',J')+S^p(E,J;E',J')]dE' \\ &+\sum_{J'}\int\eta(A,Z,E',J';t)S^\gamma(E',J';E,J)dE'-\eta(A,Z,E,J;t)\sum_{J'}\int S^\gamma(E,J;E',J')dE' \,. \qquad (1)\end{aligned}$$

All integrals in Eq. (1) are finite, the limits of integration depend on the various binding energies for simplicity; the limits are not written explicitly. In Eq. (1) $S^n(E,J;E',J')$, $S^p(E,J;E',J')$, and $S^\gamma(E,J;E',J')$ are the neutron decay rate, the proton decay rate, and the γ-ray decay rates, respectively. The function $X(A, Z, E, J)$, closely related to the occupation of levels $\eta(A, Z, E, J; t)$, is defined by

$$X(A,Z,E,J)=\int_0^\infty\eta(A,Z,E,J;t)\Gamma(A,Z,E,J)dt \,. \qquad (2)$$

In Eq. (2), $\Gamma(A, Z, E, J)$ is the total width of a nucleus with A nucleons, Z protons characterized by an energy E and a spin J. Integrating Eq. (1) over time, one obtains integral equations for the functions $X(A, Z, E, J)$. Similar equations were derived for more special cases where γ emission only is allowed[6] or where γ and neutron emission only are allowed.[8] There is a basic difference between the two previous cases and the present case. For the two previous cases, after integration over time, uncoupled integral equations were obtained. It can be seen by inspection that integration of Eq. (1) over time yields coupled integral equations. The solution of such coupled integral equations requires more sophisticated methods than the methods which were used for the solution of the uncoupled equations. Numerical methods for the solution of these equations were developed.

Following the method suggested previously for the calculation of isomer ratios in (n,γ) reactions, the isomer ratio is written as the ratio between two functions $P(E_G, J_G)$ and $P(E_{ISO}, J_{ISO})$ representing the probability of populating the ground state and the isomeric state, respectively. However, under the present circumstances the expressions for these functions take more complicated forms. These functions include terms which are due to contributions in which neutrons, protons, and γ rays are emitted in different order. Some of the terms contribute little to the evaluation of the functions $P(E_G, J_G)$ and $P(E_{ISO}, J_{ISO})$. To indicate the structure of these terms the leading term, $P(xn; yp; \gamma; E_G, J_G)$, for a reaction in which predominently neutron emission is followed by proton emission which in turn is followed by γ emission, is written explicitly as

$$P(x,n;y,p\gamma;E_GJ_G)=\sum_{\text{all }J}\int\Big[X(A_0,Z_0;E_{0,0},J_{0,0})\Big(\prod_{r=0}^{x-1}S^n(E_{r,0},J_{r,0};E_{r+1,0},J_{r+1,0})dE_{r,0}\Big)$$
$$\times\Big(\prod_{s=0}^{y-1}S^p(E_{x,s},J_{x,s};E_{x,s+1},J_{x,s+1})dE_{x,s}\Big)S^\gamma(E_{x,y},J_{x,y};E_G,J_G)dE_{x,y}\Big]\;. \quad (3)$$

In Eq. (3), $E_{p,q}$ and $J_{p,q}$ refer to the energy and spin of a nucleus after the emission of p neutrons and q neutrons.

At this point it is appropriate to briefly discuss the forms for the decay rates appearing as kernels in Eq. (1) and the population of state in the original compound nucleus at $t=0$. Both quantities, of physical significance, are required for the solution of the above integral equation and the determination of the isomer ratio.

First, an expression is obtained for the nucleon decay rate or probability of nucleon emission per unit energy per unit time. This nucleon decay rate is evaluated by breaking it into a sum of terms,[8] each term $S^n(E,J;E',J';l,j)$ (here the superscript n stands for a nucleon and may be either a proton or a neutron) corresponding to the emission of a nucleon with a specified orbital angular momentum l and total angular momentum j:

$$S^n(E,J;E',J')=\sum_{j=|J-J'|}^{J+J'}\;\sum_{l=j-\frac12}^{j+\frac12}S^n(E,J;E',J';l,j)\;. \quad (4)$$

The expression for $S^n(E,J;E',J';l,j)$ is obtained by applying the reciprocity theorem for nuclear reactions. In the present treatment, the reciprocity theorem is applied to each channel with specified l and j, so that each $S^n(E,J;E',J';l,j)$ is treated individually instead of applying the theorem to $S^n(E,J;E',J')$ as a whole; this is in the spirit of the reciprocity theorem. This approach yields

$$S^n(E,J;E'J';l,j)=\frac{\sigma(E-E',l,j;E',J';E,J)}{8h^2R^2\pi^4}$$
$$\times\left\{\left[1-\left(\frac{l-\frac12}{l_0}\right)^2\right]^{1/2}-\left[1-\left(\frac{l+\frac12}{l_0}\right)^2\right]^{1/2}\right\}\frac{\rho(E',J')}{\rho(E,J)} \quad (5)$$

In Eq. (5), R is the nuclear radius and the critical angular momentum l_0 is

$$l_0=R_0[2m(E-E')]^{1/2}/\hbar\;. \quad (6)$$

In Eq. (6), $\rho(E,J)$ represents the nuclear density of levels. Also, in the same equation $\sigma(E-E',l,j;E',J';E,J)$ is the inverse cross section for exciting a nucleus at an energy E' and spin J' to a state with an energy E and spin J by the absorption of a projectile of an energy $E-E'$, orbital angular momentum l, and total angular momentum j. This cross section for the inverse process is written as

$$\sigma(E-E',l,j;E',J';E,J)=(2j+1)\pi\lambda^2T_{lj}(E-E')\;. \quad (7)$$

In Eq. (7) the transmission coefficients $T_{lj}(E-E')$ calculated using an optical potential depend on the nucleon energy $(E-E')$, the orbital momentum l, and the total angular momentum j.

The γ decay rates are written as a sum of two terms: the first corresponds to electric transition, the second corresponds to magnetic transitions. Each of these two terms is written as a sum of terms corresponding to a radiation of a specified multipolarity l. Thus for electric transitions one obtains[8]

$$S^\gamma(E,J;E',J')=\frac{8\pi e^2}{\hbar c}\sum_{l=|J-J'|}^{J+J'}S_E^\gamma(E,J;E',J';l)\,. \quad (8)$$

Here

$$S_E^\gamma(E,J;E',J';l)=\frac{(l+1)}{[l(2l+1)!!]^2}\left(\frac{E-E'}{\hbar c}\right)^{2l+1}\rho(E',J')\times\sum_{MM'}|\langle JM|Q^l_{M-M'}|J'M'\rangle|^2\;. \quad (9)$$

As stated previously the integral equations can be solved only when the population of states for the original compound nucleus is known. This population for a specified energy E' and spin J' is proportional to the cross section $\sigma(\epsilon;E,J;E_0,J_0)$ of forming a nucleus with an energy E_0 and spin J_0 by bombarding a target with an energy E and spin J by particles of energy ϵ. This cross section is

$$\sigma(\epsilon=E_0-E;E,J;E_0,J_0)=\pi\lambda^2\sum_{j=|J_0-J|}^{J_0+J}\;\sum_{l=|j-\frac12|}^{j+\frac12}(2j+1)T_{lj}(\epsilon)\,. \quad (10)$$

So far, the population to the ground state and the isomeric state by states in the continuum have been considered. However, both states are also populated by discrete higher states, these latter states themselves are fed by states in the continuum or by other discrete states. These discrete states have spins which are closer either to the spin of the ground state or to the spin of the isomeric state and therefore decay to one of these states. Consequently, these states affect the isomer ratio. Pönitz[4] showed that the discrete levels can not be neglected in evaluating the isomer ratio for (n, γ) reactions. Watson and Medicus[9] showed that these levels affect the isomer ratio for $(\gamma, 2n)$ reactions. For more complex reactions, as the one considered in the present paper, the discrete levels which affect the isomer ratio most significantly are the discrete levels in the final nucleus following the emission of x neutrons and y protons. Therefore, only these discrete levels are included in the present treatment. This inclusion of the discrete levels requires the knowledge of the decay rate from the states in the continuum to the discrete levels and the knowledge of the decay rates between the discrete levels themselves. For γ decay rates between states in the continuum and discrete states the form suggested by Eqs. (8) and (9) is used except that the density of final states is eliminated. In particular the partial width for a decay from a state in the continuum at an energy E to a discrete state at an energy E' is given by

$$S^{\gamma}(E, J; E', J'; l) = \frac{l(l+1)}{[l(2l+1)!!]^2}\left(\frac{E-E'}{\hbar c}\right)^{2l+1} \times \sum_{MM'} |\langle JM|Q^{l}_{M-M'}|J'M'\rangle|^2 . \quad (11)$$

The dipole transitions have been calculated assuming a single-particle transition. The strength of the quadrupole transition appears as an adjustable parameter. Experimentally determined values are used for decay rates between discrete levels.

III. ISOMER-RATIO CALCULATIONS

Before applying the theory to the evaluation of isomer ratios it is worthwhile to discuss the parameters which enter the theory and how they are obtained.

The calculated values of the isomer ratios depend on a number of parameters. It seems natural that one would like to adopt as many parameters as possible from other sources to reduce the number of adjustable parameters. Neutron transmission coefficients are calculated as suggested by Auerbach and Perey[10] or Mani, Melkanoff, and Iori.[11] Proton transmission coefficients are calculated as suggested by Mani, Melkanoff, and Iori.[12] α transmission coefficients are obtained from the work of Huizenga and Iso.[13] Values of binding energies are taken from the work of Everling *et al.*[14]

The density of levels $\rho(E,J)$ appears in the expression for decay rates in the present study. The following form is used for this density of levels[15-18]:

$$\rho(E, J) = \rho(E)(2J+1)e^{-J(J+1)/2\sigma^2} . \quad (12)$$

The energy-dependent part of the density of levels and the parameters appearing in this part of the density of levels is obtained from the work of Gilbert and Cameron.[19]

The two parameters varied in the present calculation are the spin cutoff parameter σ and the amount of quadrupole admixture. It is impossible to gain confidence in the extracted values of these two parameters by comparing measured and calculated values of isomer ratios unless one knows the sensitivity of the calculated values to these parameters. The effect of the variation of the spin cutoff parameter σ and amount of quadrupole admixture are studied for the reaction[20,21] $^{82}Se(n, 2n)^{81}Se$ for 14-MeV incidence neutrons. Figure 1 shows the variation of the isomer ratio as a function of the spin cutoff parameter σ. In Fig. 1 there are two curves, one for dipole radiation, the other for quadrupole radiation.

In many calculations of isomer ratios the effect of all discrete levels except the isomeric and ground state are neglected. One expects that other discrete levels affect the isomer ratio. The effect of the presence of discrete levels on the isomer ratio is demonstrated by studying the reaction[22] $^{113}In(n, 2n)^{112}In$. The nucleus[23] ^{112}In has three known discrete levels: an isomeric state of spin 4, a ground state of spin 1, and another excited state with spin 7 decaying to the isomeric state. The isomer ratio is calculated in two different ways; one in which the discrete level is considered and another in which only the isomeric and ground state are considered. A comparison between these calculations is shown in Fig. 2. From Fig. 2 one can see that the isomer ratio is very sensitive to the presence of the state with spin 7.

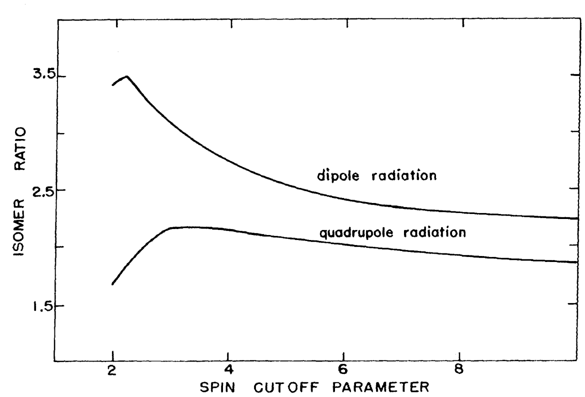

FIG. 1. The isomer ratio for the reaction $^{82}Se(n, 2n)^{81}Se$ as a function of σ.

Most reactions discussed in this paper were also studied using the Huizenga-Vandenbosch (HV)[1,2] method. Before analyzing isomer ratios using the present more rigorous method one would like to convince oneself of the need of such a more accurate theory. For this purpose results of a calculation based on the two different models are presented in Table I. These results indicate the requirement for a more rigorous calculation.

IV. COMPARISON BETWEEN THEORY AND EXPERIMENT

Calculated values of isomer ratios are compared with experimentally measured values of these ratios. Isomer ratios were compared for a 14-MeV-neutron-induced reaction and for a charged-particle-induced reaction with variable energy. In the calculated values of isomer ratios two parameters are varied: (a) the spin cutoff parameter σ and (b) the percentage of quadrupole radiation.

Isomer ratios were calculated for over 20 reactions induced by 14-MeV neutrons. For all but three reactions good agreement between theory and experiment is obtained. The uncertainty in the measured value of the isomer ratio allows for the existence of a range of spin cutoff parameters and a range of quadrupole admixture which yields agreement between theory and experiment. It should be noted that in some previous papers claims have been made to determine the spin cutoff parameter exactly by fitting isomer ratios. These claims, however, are based on the assumption that one disregards the uncertainty in the measured values of the isomer ratio and neglects quadrupole radiation. In the present paper the opposite point of view is adopted; both the uncertainty in the measured value of the isomer ratio and quadrupole admixture are included. This, however, does not allow an exact determination of the spin cutoff parameters or of the amount of quadrupole admixture. Only ranges for the values for both parameters can be obtained for which agreement between theory and experiment exists.

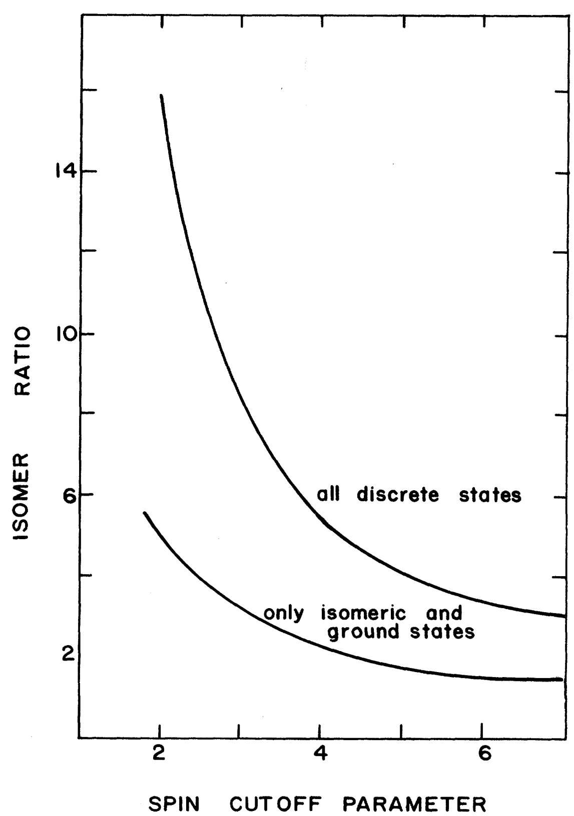

FIG. 2. The isomer ratio for the reaction $^{113}In(n, 2n)^{112}In$ as a function of σ.

The present analysis shows that the calculated values of isomer ratios can, to a good approximation, be divided into two groups. The first group consists of the isomer ratios which are less sensitive to the amount of quadrupole admixture. The second group contains the isomer ratios which are more sensitive to the amount of quadrupole admixture. To demonstrate this division into distinct groups, extreme cases of the two groups are considered. An example of the first group is the reaction[24-26] $^{90}Zr(n, 2n)^{89}Zr$. The experimental value of the isomer ratio is[25] between 0.24 and 0.36. This experimental result can be fitted with a value of σ such that $3.7 < \sigma < 4.6$ independent of quadrupole admixture. Therefore, for this group of cases a more accurately measured value of the isomer ratio holds the promise for an accurate determination of the spin cutoff parameter regardless of the amount of quadrupole admixture. For the residual nucleus ^{89}Zr the discrete levels have been determined.[27] The levels at 1.11 and 1.64 MeV have the biggest influence on the calculated isomer ratio. An example of the second group is the reaction[28,29] $^{115}In(n, 2n)^{114}In$. For this reaction the experimentally measured value for the isomer ratio is between 3.8 and 5.8. This isomer ratio can be fitted by $4.2 < \sigma < 5.4$ for pure dipole radia-

TABLE I. Comparison of theoretical isomer ratios.

$^{88}Sr(n, 2n)^{87}Sr$			$^{107}Ag(n, 2n)^{106}Ag$		
σ	HV [a]	paper [b]	σ	HV [a]	paper [b]
3.0	0.50	0.31	3.0	1.25	0.80
4.0	0.28	0.25	4.0	3.3	0.95
5.0	0.20	0.17	5.0	3.8	1.35

[a] See Ref. 24.
[b] Present calculation.

TABLE II. Values of spin cutoff parameters as determined from isomer ratios in $(n, 2n)$ reactions.

Reaction	Experimental value of isomer ratio	Spin cutoff parameter
$^{74}Se(n,2n)^{73}Se$	0.22 ± 0.08 [a]	3.0 ± 0.3
$^{90}Zr(n,2n)^{89}Zr$	0.30 ± 0.06 [b]	4.2 ± 0.6
$^{92}Mo(n,2n)^{91}Mo$	0.27 ± 0.08 [c]	3.8 ± 0.6
$^{113}In(n,2n)^{112}In$	5.0 ± 1.0 [d]	4.5 ± 0.8
$^{115}In(n,2n)^{114}In$	4.8 ± 1.0 [e]	3.8 ± 0.4
$^{121}Sb(n,2n)^{120}Sb$	0.6 ± 0.1 [f]	5.8 ± 0.7

[a] See Ref. 21.
[b] See Refs. 24, 25, and 26.
[c] See Refs. 24, 27, and 28.
[d] See Refs. 23 and 28.
[e] See Refs. 27 and 28.
[f] M. Borman, F. Preyer, V. Seebeck, and W. Voigts, Z. Naturforsh. 21, 988 (1966).

tion and $3.4 < \sigma < 4.2$ for pure quadrupole radiation. Therefore, for this reaction an exact experimental measurement of the isomer ratio is not sufficient for determination of the spin cutoff parameter. For this type of reaction the spin cutoff parameter can be determined only if the amount of quadrupole admixture is known exactly or vice versa provided that the isomer ratio is measured as accurately as possible. However, despite the above limitations all the spin cutoff parameters fall well within the expected range. This is best demonstrated in Table II. The residual nucleus ^{114}I has a discrete level above the isomeric state.[30] This level with a spin of 8 affects the isomer ratio very considerably. The ground state has a spin of 1, the isomeric state a spin of 5.

In Table II the values of spin cutoff parameters are listed as extracted from the analysis of isomer ratios. Cases have been chosen for which the measured value of the isomer ratio is relatively well known within a narrow limit. It is obvious that the narrower the limits of the experimental value of the isomer ratio the greater the possibility of an accurate determination of the spin cutoff parameter.

There are a number of charged-particle reactions for which the isomer ratio has been measured for a wide range of energy. A comparison between experimental and theoretical results for these reactions is a better test for the theory than $(n, 2n)$ reactions since, when the isomer ratio is measured as a function of energy, there are more experimental values to compare with theory.

For three reactions which lead to the nucleus ^{58}Co the isomer ratio has been measured[31-37] $^{59}Co(n, 2n)^{58}Co$, $^{58}Ni(n,p)^{58}Co$, and $^{55}Mn(\alpha, n)^{58}Co$. A comparison of the analysis of such reactions is especially meaningful, since for all reactions the isomeric state and ground state are the same. Furthermore, the amount of quadrupole admixture in the final residual nucleus is the same for all cases. Therefore, a comparison between the analysis of isomer ratios for such reactions is a more severe test for the consistency of the theory. Best agreement between theory and experiment is obtained with $\sigma = 3$ and 45% quadrupole admixture. A comparison between theory and experiment is shown in Figs. 3 and 4. Very little is known about the discrete levels other than the isomeric and ground state in ^{58}Co so that in this reaction the discrete levels are not considered. For the reaction $^{59}Co(n, 2n)^{58}Co$ a comparison between theory and experiment is difficult, since the experimental data show basically no systematics. For the (α, n)

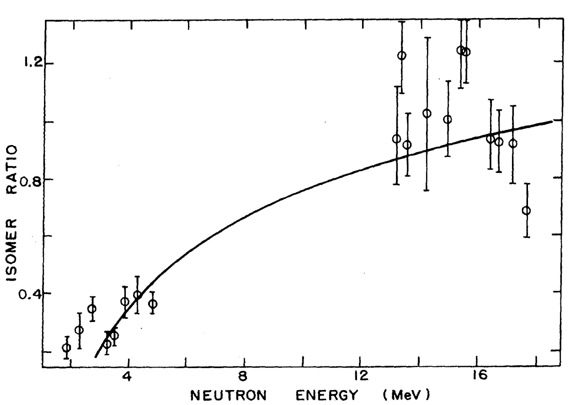

FIG. 3. The isomer ratio for the reaction $^{58}Ni(n,p)^{58}Co$ as a function of the energy of the incident neutron. The isomer ratio was calculated for the parameters $\sigma = 3$ with 45% quadrupole radiation. The vertical lines are the experimental results of Decowski *et al*. (see Ref. 32).

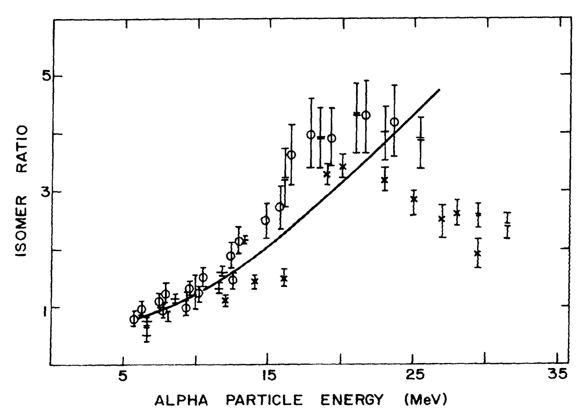

FIG. 4. The isomer ratio for the reaction $^{55}Mn(\alpha,n)$-^{58}Co as a function of the energy of the incident α particle. The isomer ratio was calculated for the parameters $\sigma = 3$ with 45% quadrupole radiation. The experimental results of Iwata (see Ref. 35), ⫩ ; Wing and Haskin (see Ref. 41), ϕ ; Matsuo *et al*. (see Ref. 34), ⚹ ; and Keedy *et al*. (see Ref. 36), +; are shown.

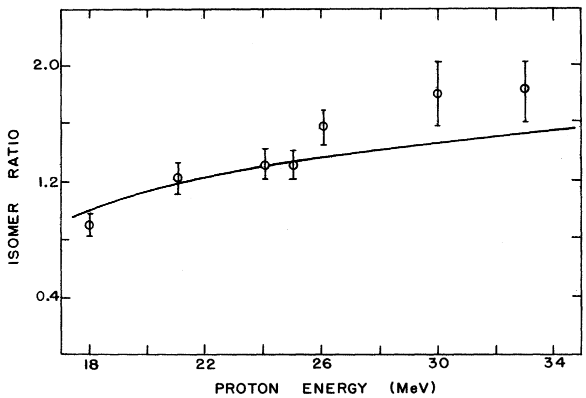

FIG. 5. The isomer ratio for the reaction $^{88}Sr(p, 2n)$-^{87}Y as a function of the energy of the incident proton. The isomer ratio was calculated for the parameters $\sigma = 5$ with 26% quadrupole radiation. The vertical lines are the experimental results of Sachdev (see Ref. 39).

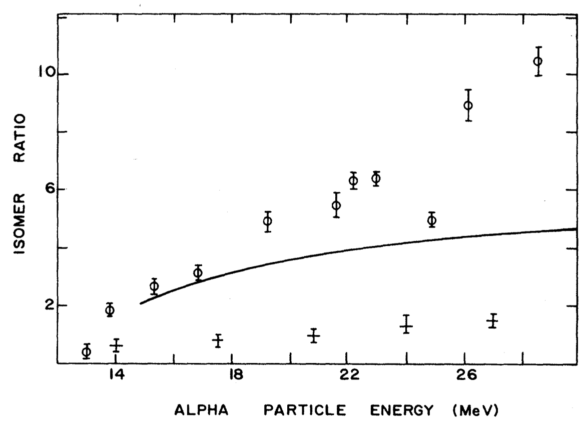

FIG. 6. The isomer ratio for the reaction $^{85}Rb(\alpha, 2n)$-^{87}Y as a function of the energy of the incident α particle. The isomer ratio was calculated for the parameters $\sigma = 5$ with 26% quadrupole radiation. The experimental results of Iwata (see Ref. 35), + ; and Vandenbosch, Haskin, and Norman (see Ref. 40), ϕ ; are shown.

reaction good agreement is obtained for low-energy α particles; however, the agreement gets worse for α particles incident with energies about 25 MeV. Direct reactions may be responsible for the continued rise in the measured isomer ratio.[38, 39]

For two reactions ^{87}Y is the residual nucleus. Isomer ratios have been measured[40] for $^{88}Sr(p, 2n)$-^{87}Y and[41] $^{85}Rb(\alpha, 2n)^{87}Y$. Best results for analyzing the isomer ratio for the $(p, 2n)$ reaction are obtained for $\sigma = 3$ and 26% quadrupole radiation. A comparison between theory and experiment for this reaction is seen in Fig. 5. These parameters are used to calculate the isomer ratio for the $(\alpha, 2n)$ reaction. Again very little is known about other discrete levels other than the isomeric and ground state. Therefore, the effect of the other discrete levels on the isomer ratio is neglected. For this case the theoretical results fall between the experimental results obtained by different groups as can be seen in Fig. 6.

V. DISCUSSION

The present study shows that a meaningful analysis for the interpretation of isomer ratios requires rigorous tools, such as developed in this paper. It is shown that isomer ratios are very sensitive to the presence of discrete levels, so these levels have to be included in the evaluation of isomer ratios.

Exact determination of the spin cutoff parameters from isomer ratios in $(n, 2n)$ reactions is not possible, mainly because of the large uncertainty in the measured values of the isomer ratios. In many cases even exact knowledge of the isomer ratio is not sufficient to determine the cutoff parameter. For these latter cases the isomer ratio is very sensitive to the amount of quadrupole admixture. For a reaction for which the isomer ratio is known over a wide range of incident particle energy both the amount of quadrupole admixture and the spin cutoff parameter σ can be better determined.

The spin cutoff parameters σ determined by the present analysis of isomer ratios is consistent with a rigid-body moment of inertia for the nucleus. It is premature to draw a conclusion about detailed structure of highly excited states from the amount of quadrupole admixture. A similar conclusion was drawn from the analysis of isomer ratios in (n, γ) reactions.[7]

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Multiple-Scattering Calculation of π-Nucleus Scattering near the 3-3 Resonance

Ryoichi Seki
San Fernando Valley State College, Northridge, California 91324
(Received 27 May 1970)

The π-nucleus interaction near the π-nucleon 3-3 resonance $\Delta(1236)$ is examined by use of multiple-scattering theory. π-nucleus total cross sections are calculated and are found to be in agreement with experimental data. Multiple-scattering effects are found to be important for this agreement.

I. INTRODUCTION

The π-nucleus interaction at low and intermediate energies (the pion lab energy T_π, say, less than 500 MeV) has very different features from the nucleon-nucleus interaction in the same energy domain. First, because of the π-nucleon 3-3 resonance $\Delta(1236)$ at $T_\pi = 195$ MeV,[1] the p-wave contribution is dominant over the energy range and is appreciable even near the elastic threshold. Second, because of pion absorption by constituent nucleons of the nucleus and because of the three different charge states of the pion, various inelastic channels are open even at the elastic threshold, and yield various nuclear final states.[2]

Some experimental data on intermediate-energy π-nucleus scattering have been accumulated in the last decade. They are not, however, extensive enough to perform detailed phase-shift analyses; perhaps with exceptions of π-He^4 for 20 MeV $< T_\pi <$ 100 MeV[3] and π-C^{12} for 70 MeV $< T_\pi <$ 280 MeV.[4] Among them, the total cross section seems to be most extensively measured. In Fig. 1 we summarized the total cross sections for π-He^4, π-Be^9, π-C^{12}, and π-O^{16} scattering below 1.3 BeV together with the symmetric part of the π-nucleon total cross section.[5] In all cross sections we observe a large bump near 200 MeV and a rather flat part