Comments and Addenda

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Electromagnetic Form Factors and Radii of He^3†

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The electromagnetic form factors and radii of He^3 have been evaluated using the three-body wave function obtained from a perturbational solution of the trinucleon system with the Eikemeier-Hackenbroich potential. The effect of S'-state admixture and magnetic-exchange interactions are included in the calculation. Satisfactory fits to the form factors to large momentum transfer, including the correct prediction of the location of the diffraction minimum in the charge form factor, are found. Our calculations are in agreement with the latest Stanford e-He^3 results in suggesting that the magnetic radius is larger than the charge radius if suitable magnetic-exchange effects are considered.

1. INTRODUCTION

Recently, a group at Stanford has performed high-energy electron-He^3 scattering to determine the electromagnetic radii and form factors[1] to large momentum transfer of the nucleus. Two important features of their results are the presence of a diffraction minimum at $q^2 = 11.6\ \text{fm}^{-2}$ in the charge form factor and that the magnetic radius is larger than the charge radius. The structure observed in the form factor is not entirely unexpected; this had already been seen in e-He^4 scattering[2] at large q^2 and predicted, as a direct consequence of the repulsive core in the nuclear interaction, in He^3 and He^4 in our work[3] on the perturbational solution of the trinucleon system using the "realistic" Eikemeier-Hackenbroich potential.[4] The radius difference, however, disagrees with an earlier experiment[5] and with the result of Schiff.[6]

The theoretical analysis of the electromagnetic form factors of He^3 is beset by a number of problems, viz., (i) the question of admixtures of S', $T=\frac{3}{2}$, P, and D states in the ground state, (ii) lack of information about the neutron form factor, (iii) the importance of exchange currents, and (iv) the validity of a nonrelativistic treatment. In considering (i), it is usually assumed that only S and S' contributions are significant, the $T=\frac{3}{2}$ and P states occurring with negligible probability, while the S' interference term is adjusted to include the effect of the D states.[6-9] The neutron form factor is now accepted as nonzero and behaving like $0.021q^2$ for small momentum transfer.[10,11] Exchange charge effects are neglected but several previous analyses[12,13] of the magnetic form factor have shown that it is essential to include phenomenological magnetic interactions, especially the isovector term. It is expected from Cocho and Flores[14] that nonrelativistic expressions for the electromagnetic form factors are valid.

2. ELECTROMAGNETIC STRUCTURE OF He^3

Using the assumptions described in the previous section, we can write the form factors as

$$F_{\text{ch}} = [F_{\text{ch}}(p) + \tfrac{1}{2}F_{\text{ch}}(n)]F_{\text{B}} + \tfrac{1}{3}[F_{\text{ch}}(n) - F_{\text{ch}}(p)]F_2 , \tag{1}$$

$$\mu F_{\text{mag}} = \mu(n)F_{\text{mag}}(n)F_{\text{B}} + \tfrac{2}{3}[\mu(n)F_{\text{mag}}(n) + \mu(p)F_{\text{mag}}(p)]F_2 + [\mu - \mu(n)]F_{\text{XV}}. \tag{2}$$

F_{B} and F_2 are the contributions of the S and S' states; F_{ch}, $F_{\text{ch}}(n)$, and $F_{\text{ch}}(p)$ are the charge form factors of He^3, n, and p, respectively, while those terms with subscript "mag" are their magnetic counterparts. The μ's are the static magnetic moments and F_{XV} is the phenomenological magnetic isovector form factor. To evaluate F_{ch}, the charge form factors of the nucleons given in Ref. 11 are used. F_B, the body form factor, and F_2 are derived from our previous work.[3,9] The probability of the S' state, wherever included, is 2%.

It is immediately obvious from Fig. 1, which shows our predicted charge form factors for q^2 ranging up to 20 fm^{-2}, that the experimental results, for the whole range of q^2 displayed, can be fitted very satisfactorily by our perturbational wave function. A small admixture of S' state appears necessary in order to fit the minimum, but there appears no need for isoscalar and isovector charge-exchange terms.

Using the expansion for small q^2,

$$F(q^2) = 1 - \tfrac{1}{6} q^2 \langle r\rangle^2 + \cdots , \qquad (3)$$

[for $F_{ch}(n)$ and F_2, the expansion is $F(q^2) = \frac{1}{6} q^2 \langle r\rangle^2$] we find

$$\langle r_{ch}\rangle^2 = \langle r_B\rangle^2 + \langle r_{ch}(p)\rangle^2 - \langle r_{ch}(n)\rangle^2 + \tfrac{1}{3}\langle r_2\rangle^2 . \qquad (4)$$

Taking $\langle r_B\rangle^2 = 3.168$ fm^2, $\langle r_{ch}(p)\rangle^2 = 0.722$ fm^2, $\langle r_{ch}(n)\rangle^2 = 0.126$ fm^2, and $\langle r_2\rangle^2 = 0.540$ fm^2, we obtain

$$\langle r_{ch}\rangle = 1.96 \text{ fm} \quad (\text{without } S'),$$

$$\langle r_{ch}\rangle = 2.00 \text{ fm} \quad (\text{with } S').$$

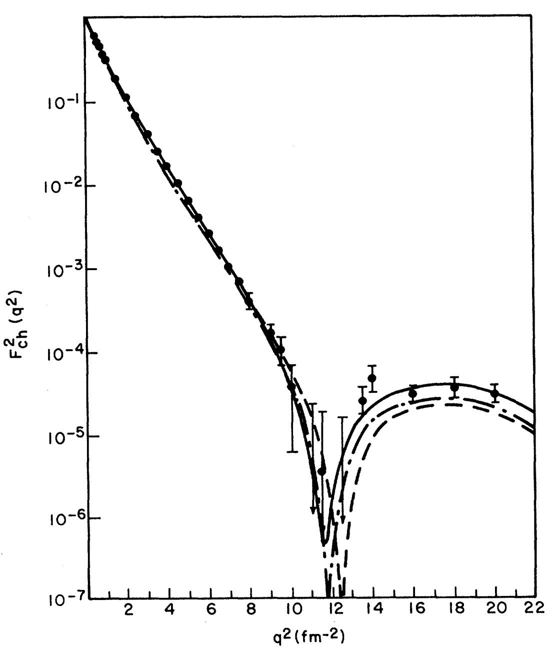

FIG. 1. He^3 charge form factor, the best-fit curve, and data of McCarthy *et al.* (Ref. 1). The dash and dash-dot lines are our predicted curves without and with S'-state admixture.

These values are larger than those extracted by McCarthy *et al.*[1] but must be considered satisfactory since $\langle r_B\rangle$, as noted in Ref. 3, is slightly too large.

Good fits to the magnetic form factor are clouded by the likely presence of magnetic effects. In this note, we are interested in evaluating the magnetic form factor with the same wave functions as were used for the charge form-factor calculations. It is hoped thereby that some knowledge of the exchange form factor F_{XV} may be gathered and that the difference between $\langle r_{mag}\rangle$ and $\langle r_{ch}\rangle$ may be explained.

From Eq. (2) using $F_{mag}(n)$ and $F_{mag}(p)$ from Ref. 11, we determine F_{mag}, which we show in Fig. 2. It appears that a simultaneous fit to the experimental data of F_{ch} and F_{mag} is impossible without magnetic-exchange contributions. In Fig. 3, we have plotted the F_{XV} required to yield the best-fit curve for F_{mag} from experiment. The F_{XV} so obtained agrees with Padgett's theoretical curve[15] at low q^2.

Now, a theoretical estimate of $\langle r_{mag}\rangle$ can be made if an expansion of the magnetic form factors similar to that in Eq. (3) is tried. Then

$$\mu\langle r_{mag}\rangle^2 = \mu(n)[\langle r_{mag}(n)\rangle^2 + \langle r_B\rangle^2] - \tfrac{2}{3}[\mu(p) + \mu(n)]\langle r_2\rangle^2 + [\mu - \mu(n)]\langle r_{XV}\rangle^2 . \qquad (5)$$

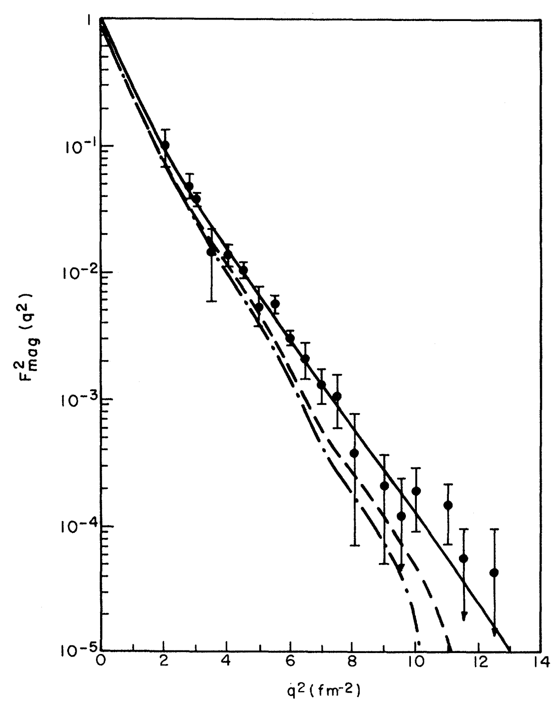

FIG. 2. He^3 magnetic form factor, the best-fit curve, and data of McCarthy *et al.* (Ref. 1). The dash and dash-dot lines are our predicted curves of F_{mag}, without and with S'-state admixture, normalized to $\mu(n)/\mu$ at $q^2 = 0.0$ fm^{-2}, i.e., F_{XV} is zero for all q^2.

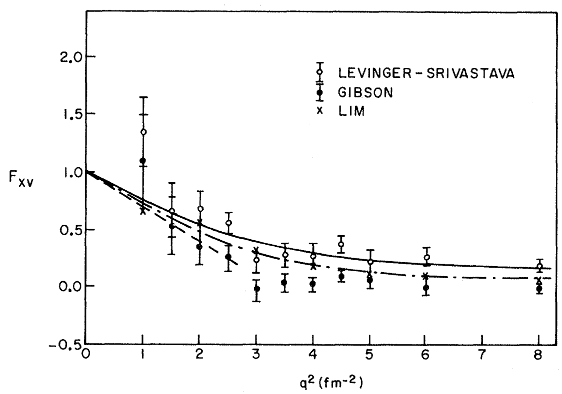

FIG. 3. The exchange magnetic form factor, F_{XV}. Data are those from the analyses of Levinger and Srivastava (Ref. 11) and Gibson (Ref. 12). Solid curve is the visual best-fit one of Ref. 11, dash curve is the F_{XV} for small q^2 calculated by Padgett (Ref. 15) and dash-dot curve is the F_{XV} required to fit F_{mag} of Ref. 1 without S' state.

Taking $\langle r_{mag}(n)\rangle^2 = 0.851$ fm^2, $\langle r_{mag}(p)\rangle^2 = 0.788$ fm^2, and $\langle r_{XV}\rangle^2 = 2.0$ fm^2 (extracted from the slope of Padgett's line), we find that

$$\langle r_{mag}\rangle = 1.95 \text{ fm} \quad (\text{exchange without } S'),$$

$$\langle r_{mag}\rangle = 1.99 \text{ fm} \quad (\text{exchange with } S').$$

Our calculations lead to

$$\Delta r = \langle r_{mag}\rangle - \langle r_{ch}\rangle ,$$

$$\Delta r = -0.01 \text{ fm} ,$$

in substantially better agreement with the latest Stanford result, where

$$\Delta r = 0.06 \text{ fm} ,$$

and in disagreement with the old value

$$\Delta r = -0.17 \text{ fm} .$$

It is clear from Eq. (5) that $\langle r_{XV}\rangle^2$ influences greatly the magnitude of $\langle r_{mag}\rangle$ and that we can achieve even better agreement with the new Δr if we assign a steeper slope to F_{XV}. It is also clear that the new experiment and our calculations point out that Schiff's intuitive explanation of Δr must be supplemented by consideration of magnetic-exchange effects. Definite conclusions about the size of the S' contributions from the Eikemeier-Hackenbroich potential must await a more detailed evaluation of F_2.

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