Multipole Admixtures of Transitions from Beta, Gamma, and Octupole Vibrational States in ^{154}Gd

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A NaI-Ge(Li) detector arrangement has been used to measure γ-γ directional correlations in ^{154}Gd from the decay of ^{154}Eu. From the NaI detector, gates were selected to span the 123-keV $2^+ \to 0^+$, and 248-keV $4^+ \to 2^+$ transition, and an equivalent region of the Compton distribution at 320 keV to obtain the background contribution to each gate. These gates opened 2048 channels of the memory of a 4096-channel analyzer for storage of data at 90, 135, and 180° from the Ge(Li) detector. The well-known 1274-123-keV and 873-123-keV correlations were used to obtain attenuation and Ge(Li) solid-angle corrections for cascades that involve the 123-keV 2^+ state. The following correlations (in keV) were measured, with the A_2 and A_4 coefficients, respectively, in parentheses: 692-123, $2_\beta^+ \to 2_g^+ \to 0^+(-0.145 \pm 0.022,\ 0.311 \pm 0.040)$; 1005-123, $3_\gamma^+ \to 2_g^+ \to 0^+(-0.239 \pm 0.018,\ -0.073 \pm 0.026)$; 1494-123, $3^- \to 2_g^+ \to 0^+(-0.075 \pm 0.029,\ -0.022 \pm 0.044)$; 1597-123, $2^- \to 2_g^+ \to 0^+(0.196 \pm 0.013,\ -0.017 \pm 0.021)$; 757-248, $3_\gamma^+ \to 4_g^+ \to 2_g^+ (0.161 \pm 0.004,\ -0.174 \pm 0.006)$, 677-248, $4_\beta^+ \to 4_g^+ \to 2_g^+(-0.168 \pm 0.020,\ 0.178 \pm 0.029)$, 444-248, $2_\beta^+ \to 4_g^+ \to 0^+(0.196 \pm 0.009,\ 0.095 \pm 0.011)$; 625-248, $2_\gamma^+ \to 4_g^+ \to 2^+(0.187 \pm 0.014,\ 0.108 \pm 0.019)$; 893-248, $4_\gamma^+ \to 4_g^+ \to 2^+(-0.037 \pm 0.010,\ 0.160 \pm 0.013)$; 881-248, $3^- \to 4_g^+ \to 2^+(-0.132 \pm 0.058,\ -0.059 \pm 0.086)$; 1188-248, $4^- \to 4_g^+ \to 2^+(0.224 \pm 0.028,\ -0.010 \pm 0.041)$; and 1246-248, $3^- \to 4_g^+ \to 2^+(-0.151 \pm 0.007,\ 0.010 \pm 0.008)$. The following $M1$ admixtures (in keV) were found in the transition from the β and γ bands: 692, $2_\beta^+ \to 2_g^+(99.2^{+0.3}_{-0.7}\ E2)$; 677, $4_\beta^+ \to 4_g^+$ (>80% $E2$ for 2σ limits); 1005, $3_\gamma^+ \to 2_g^+(99.78^{+0.20}_{-0.23}\ E2)$; 757, $3_\gamma^+ \to 4_g(97.0 \pm 0.3\%\ E2)$; and 893, $4_\gamma^+ \to 4_g^+(95.2 \pm 0.8\%\ E2)$. The transitions from the odd-parity states studied to the even-parity ground-state band are all $99.9 \pm 0.1\%\ E1$.

I. INTRODUCTION

There has been much interest in the last few years in the detailed properties of the β-vibrational bands in deformed rare-earth nuclei since the reports[1,2] of anomalous γ-ray branching ratios from these bands in ^{152}Sm and ^{154}Gd. It was noted that these discrepancies could be removed if large $M1$ components were present in the $2_\beta^+ \to 2_g^+$ and $4_\beta^+ \to 4_g^+$ transitions. Although such $M1$ components are allowed by the spin-parity selection rules, the rotational bands of the β and γ type were understood as being built on quadrupole vibrations of the deformed ground state.[3] Thus transitions from these vibrational levels to the ground-state level would be pure quadrupole radiations.[3] These predictions are well verified for the rotational levels built on the lowest γ-vibrational motion.[4] γ-γ directional correlation results have shown that the $M1$ components in the $2_\gamma^+ \to 2_g^+$ transitions are less than 5% (and in the most accurate measurements, less than 1%), as seen in the review of such measurements by Bodenstedt.[4] The branching ratios from the γ bands are also in near agreement[1,2] with theory which includes γ–ground-state band mixing.[5]

Although this long-standing prediction[3] for the β bands had never been tested, agreement with theory had been expected until the anomalous-branching-ratio problem occurred. The magnitudes of the $M1$ admixtures, 50%, needed to bring about agreement with the branching ratios in ^{152}Sm and ^{154}Gd were surprisingly high in view of the theoretical predictions[3] and the results from the γ bands. Mottelson[6] has pointed out, however, that the theory could be modified so as to include large $M1$ admixtures which could remove the difference between the theoretical and experimental branching ratios. Mottelson[6] emphasized the existence of a major crisis in the application of rotational relationships in these nuclei if the $M1$ components were not observed.

Information on β bands was sparse because of their weak population and then in only a few nuclei. The internal-conversion processes in the $2_\beta^+ \to 2_g^+$ and $4_\beta^+ \to 4_g^+$ transitions where $M1$ admixtures are allowed by spin- and parity-selection rules are dominated by $E0$ radiation[7] (≃90% $E0$), so little or no information on $M1$ components is obtained from the internal-conversion process. In the samarium and gadolinium nuclei, the decay schemes are so complex that γ-γ directional correlation studies involving these weak transitions from the β band were impossible with two NaI detectors. The development of high-resolution lithium-drifted germanium, Ge(Li), detectors with reasonably large volume changed this situation. Thus we have carried out extensive γ-γ directional correlation studies in ^{154}Eu with a NaI-Ge(Li) detector arrangement to determine the magnitude of the $M1$ admixtures in transitions from the 2_β^+ and 4_β^+ β-vibrational states and from the 3_γ^+ and 4_γ^+ γ-vibrational states for the first time. The spins of some of the odd-parity states have been firmly established and the $M2$ admixtures for transitions from these states determined. Some of the results of these studies have been reported previously.[8,9] Details of these measurements and additional ones are reported here. The results show that a new theoretical treatment of at least the β-type vibrational states is called for.

II. EXPERIMENTAL PROCEDURES

A. Correlation Apparatus

At the time our work was begun there were no published measurements of γ-γ directional correlations with Ge(Li) detectors. There were many possible choices of systems with one or two Ge(Li) detectors with digital gates or multichannel analysis.

These choices and their relative merits have been reviewed recently.[10] The low-energy γ spectrum from the decay of deformed rare-earth nuclei consists primarily of the $2^+ \rightarrow 0^+$ and $4^+ \rightarrow 2^+$ transitions. These transitions can be easily resolved in a NaI detector. The transitions from the β and γ bands are considerably higher in energy in the complex regions of the γ spectrum and require the high resolution of the Ge(Li) detectors for separation of their photopeaks from those of other transitions. Thus a NaI-Ge(Li) arragement is ideally suited for studies of the type of interest in this work (see Ref. 10 for additional discussion of this choice).

The next choice was whether to use gate selection on both transitions or multichannel analysis. For these studies in ^{154}Gd with the detectors then available, and perhaps even now, multichannel analysis was required on the Ge(Li) detector. This system is described in detail in the literature.[11] Basically it consists of a 2-in.$\times$2-in. NaI detector which can be rotated through 180° around a fixed Ge(Li) detector. Gate signals from the NaI detector in coincidence with signals from the Ge(Li) detector allow the data from the Ge(Li) detector to be analyzed and stored in a Nuclear Data 161F analyzer. Since this system had dual analog-to-digital convertors (ADC) and 2048 channels are usually sufficient to store a coincidence spectrum, a routing system allowed two or more separate energy gates to be selected from the NaI detector and the Ge(Li) spectrum in coincidence with each gate stored in separate sections of the memory. Thus the correlations of γ rays with two photopeaks or a photopeak and the Compton background under the peak could be measured simultaneously. The first measurements were taken with one gate before a two-gate routing system was installed.

In the first measurements, a 17-cm^3 true-coaxial Ge(Li) detector was used. The full width at half maximum (FWHM) of this detector at the count rates used in this work were from 4 to 5 keV at 700 keV. The detector was rigidly clamped to the correlation table. In later measurements a 36-cm^3 true-coaxial detector (3.8% efficiency relative to a 3-in.$\times$3-in. detector at 133 keV) with FWHM of 3.0 keV at 1333 keV was used. A TC135 preamplifier was integrally mounted to the detector and was connected to a TC200 linear amplifier with pole-zero cancellation adjustment. The TC200 was capacitor coupled to a TC250 biased amplifier and stretcher. The TC250 was used to bias off the spectrum below 500 keV so that the analyzer dead time could be reduced.

The source to Ge(Li) detector distance was 6 cm (the detector is 1.3 cm inside the detector cap). The NaI detector was 7 cm from the source and had a lead shield to reduce scattering of γ rays from one crystal to the other. After the first series of measurements, a graded shield of 0.014-in. lead, 0.032-in. cadmium, and 0.006-in. copper was placed in front of the Ge(Li) detector to absorb the 123- and 248-keV γ rays and x rays.

B. ^{154}Eu Source

The ^{154}Eu source was prepared by thermal-neutron capture on europium oxide enriched in ^{153}Eu and was from the same material used earlier.[2,7] The ^{152}Eu in the source was measured to be less than 0.6% from the singles γ-ray spectrum. The ^{154}Eu activity was picked up in distilled water and deposited in a cylindrical Lucite vial with i.d. of 3.5 mm. The depth of the liquid was about 4 mm. The vial is covered with a screw cap whose flanges were coated with Vaseline to seal the source for the prevention of evaporation. The source was centered with respect to the movable detector by a count-rate determination to better than 1%.

C. Data Acquisition and Reduction

Figure 1 gives the essential features of the decay scheme of ^{154}Eu as seen in the present work. A typical spectrum is shown in Figs. 2 and 3. Because of the low intensity of the transitions from the β band in ^{154}Gd from the decay of ^{154}Eu, long counting periods were required. Data were collected at 90, 135, 180, 225, and 270°. A coincidence run was taken at each angle for 23 h. Twenty to thirty cycles (each cycle consisted of 23 h at each of 90, 135, and 180°) were taken for each cascade. The NaI gate counts were continuously counted and a 10-min single spectra from the Ge(Li) detector was taken after each 23-h run.

In the initial measurements, only one NaI gate at a time was used. With the addition of a routing box, two NaI energy gates could be set and the spectra in coincidence with each recorded. In the first runs, the coincidences from the Compton background under the photopeaks of interest were measured in separate runs but only one background run was taken for every three photopeak runs. These data are group I for the 123-keV correlation (Table I). For group II and all the 248-keV correlations, the Compton background was measured simultaneously with the true correlation. The graded shield in the Ge(Li) detector also was not used in group I data collection.

Each coincidence run and each single spectrum was checked for consistency with each other and other runs. Periodic checks of the NaI gate spectra insured constant counts in the NaI gate. These gate counts were used to normalize out any small effects ($\lesssim 1\%$) from small differences in these counts. These gate counts and the Ge(Li) singles counts were used to correct for chance coincidences at each angle. The resolving time of the coincidence unit was $2\tau = 135$ nsec.

Because of the low-count rates in the principle peaks of interest data from at least five cycles were combined for the calculation of correlation coefficients. Five- and ten-cycle groups were compared with each other for internal consistency which was always within the limits of error.

Figure 4 shows the NaI spectrum and the arrows indicate the energy gates for the 123- and 248-keV correlations and the Compton background. If one sits on the apparently flat region between the 123- and 248-keV peaks, one finds that the small back-scattering contribution in this region strongly enhances the Compton edges at 180°. Such an edge occurs very near photopeaks of two transitions of interest in the correlation work. Thus the Compton background was set above the 248 keV at an

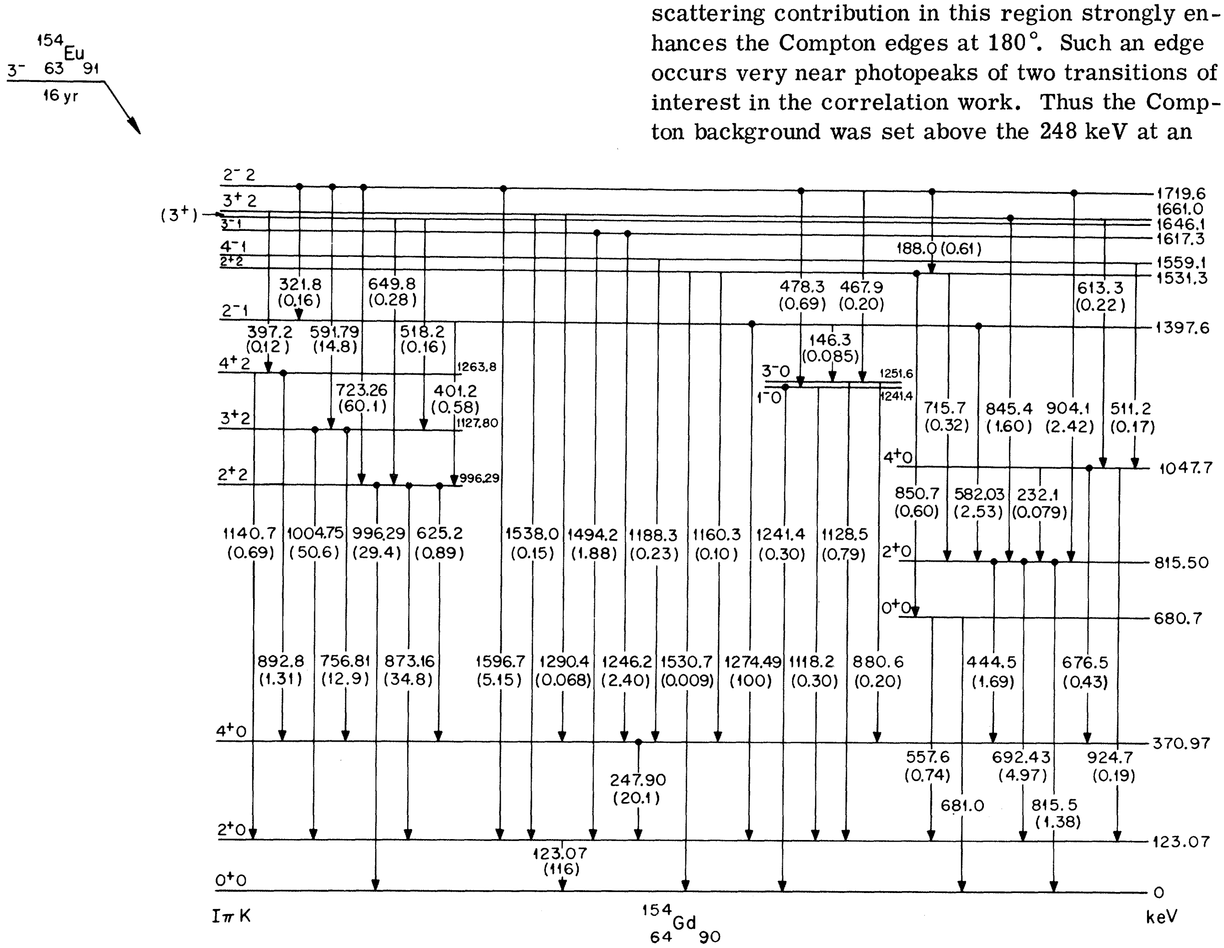

FIG. 1. Partial decay scheme of ^{154}Eu.

energy of 320 keV. The Compton distributions under the 123- and 248-keV peaks that are in coincidence with the transitions of interest arise from transitions of greater than 500 keV so the Compton distributions are essentially flat from 320 keV down. One sees in Fig. 4 that there is some ^{155}Eu in the sample. Since the decay energy[12] of ^{155}Eu is less than 250 keV, this isotope produced only a small contribution to the chance spectra.

In a sequence of 23-h runs at three angles, the statistics on the weak peaks, such as the 692-keV one, were not sufficient to yield accurate correlation coefficients. Thus one must combine several runs at one angle to obtain good statistics. Each day's run at a particular angle was added to the previous data channel by channel. From day to day a shift of the order of one channel in the location of a peak could occur. To prevent unnecessary resolution degradation, a strong peak near the weak one of interest was used as a standard, and the data were shifted to put this peak in the same channel.

A computer program was developed to sum the data and to subtract the chance and coincidence background corrections channel by channel. When sufficient data were accumulated, the data with

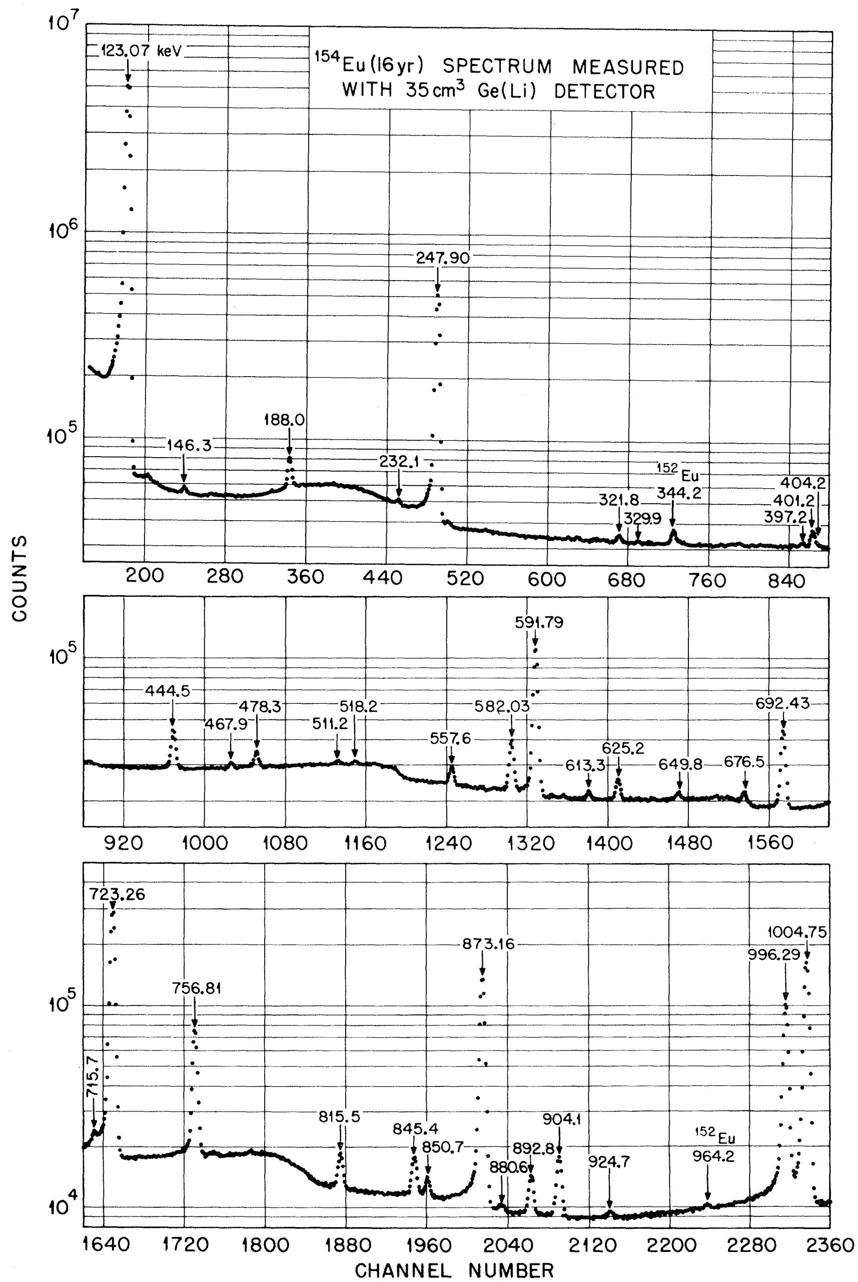

FIG. 2. The γ-ray spectrum of ^{154}Eu in the energy region 123–1005-keV region.

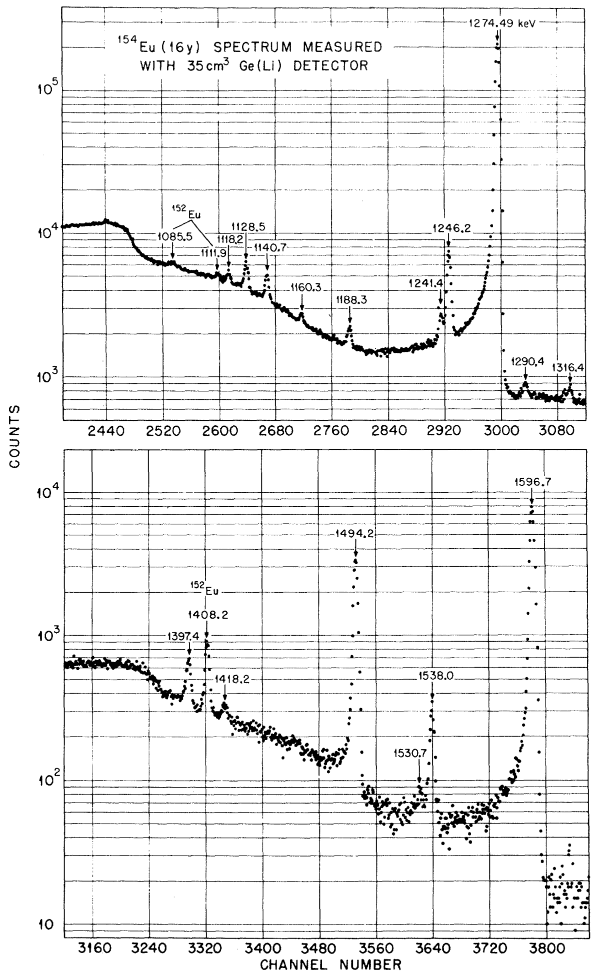

FIG. 3. The γ-ray spectrum of ^{154}Eu in the energy region 1010–1600-keV region.

chance and the background from Compton events in the NaI gate subtracted were plotted as shown in Fig. 5 and others that follow. The Compton backgrounds under each peak were drawn by hand as were curves that were considered to be upper and lower limits on these background curves for error analysis. The counts, less Compton background, in the 5, 7, 9, and 10 points similarly centered on each peak were summed and normalized to the NaI gate counts to form $W'(\theta_i)$, where $\theta_i = 90$, 135, and 180°.

The results of a γ-γ directional correlation are conveniently expressed in terms of the coefficients of a Legendre polynomial expansion of the directional correlation coefficient $W(\theta) = 1 + A_2P_2(\cos\theta) + A_4P_4(\cos\theta) + \cdots$.

Since only dipole and quadrupole radiations were studied, only terms through A_4 are necessary and a three-angle experiment suffices to determine the coefficients.[13] The A_2 and A_4 values are functions of the spins and multipole mixing parameters. Since the gate transitions have known multipolarities and go between levels of known spins, the correlation coefficients yield information on the upper transition multipolarities and the spins of the upper levels.

The experimentally measured relative coincidence intensities, $W'(\theta)$, yield A_2' and A_4' values which are related to the theoretical coefficients by $A_k' = Q_k\ G_k\ A_k$. The Q_k corrects for the finite solid angle subtended by the two detectors and G_k corrects for the attenuation of the correlation for reorientation of the nucleus when the intermediate state has a relatively long lifetime. The solid-angle corrections Q_k for the NaI detector were taken from tabulated values.[14] The solid-angle correlation for the Ge(Li) detectors was measured

TABLE I. Experimental results for 1274-, 873-, and 692-123-keV cascades in ^{154}Gd.

γ-γ (keV)	Spins and parities	Group	$Q_2G_2A_2$	$Q_4G_4A_4$	A_2	A_4
1274-123	$2^-, 2^+, 0^+$	I	0.202 ± 0.004	0.003 ± 0.004		
		II	0.205 ± 0.012	0.007 ± 0.012		
		Average	0.203 ± 0.004	0.005 ± 0.004	0.234 ± 0.006 [a]	
873-123	$2^+_\gamma, 2^+, 0^+$	I	-0.011 ± 0.013	0.204 ± 0.015		
		II	0.016 ± 0.018	0.174 ± 0.019		
		Average	0.002 ± 0.011	0.187 ± 0.013	0.002 ± 0.012 [b]	0.323 ± 0.001 [c]
692-123	$2^+_\beta, 2^+, 0^+$	I	-0.119 ± 0.023	0.148 ± 0.024		
		II	-0.134 ± 0.036	0.213 ± 0.038		
		Average	-0.126 ± 0.020	0.180 ± 0.020	-0.145 ± 0.022 [b]	0.311 ± 0.040 [d]

[a] See Steining and Deutsch (Ref. 15) – their value of 0.227 is corrected for the interfering cascade of 1246 keV (see text), 2(1)2(2)0 theory $A_2 = 0.250$, $M2$ admixture $= 0.1\%$.

[b] Corrected with $Q_2G_2 = 0.870 \pm 0.029$ obtained as ratio of $0.203 \pm 0.004/0.234 \pm 0.006$.

[c] A_4 (theory) compatible with A_2 (experimental).

[d] Corrected with $Q_4G_4 = 0.578 \pm 0.040$ obtained as ratio of $0.187 \pm 0.013/0.323 \pm 0.001$.

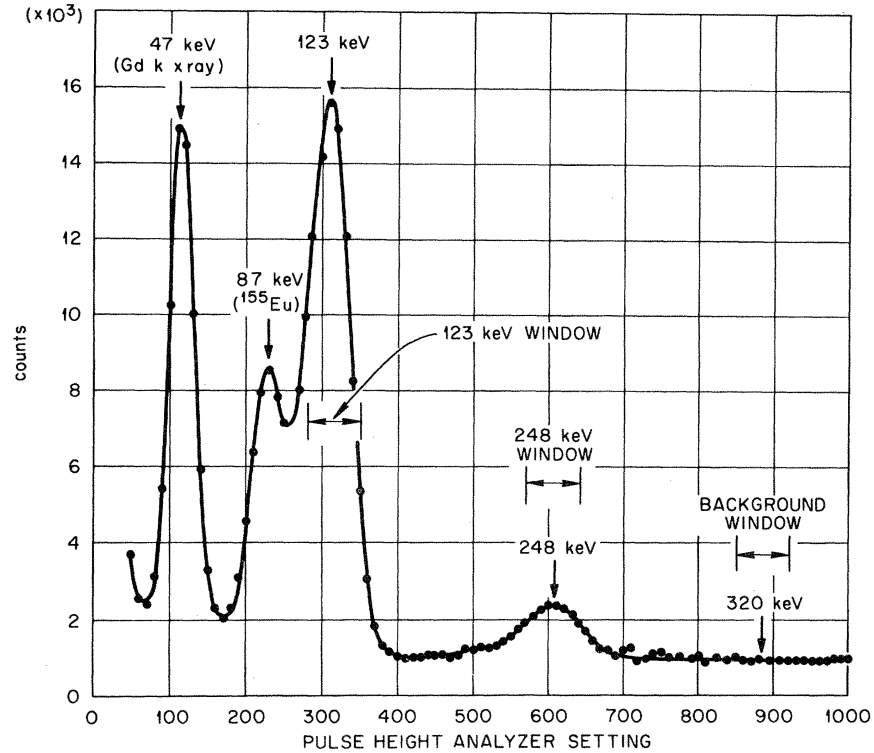

FIG. 4. NaI spectrum in the energy region 40–350-keV region for gate settings.

experimentally.

The lifetime of the 123-keV state is relatively long[12] (1.7 nsec) and attenuation effects are well-known for cascades through this level[15,16] particularly for solid sources. Recent work[17] indicates the attenuation is small, if present, in liquid sources such as we used. The possible G_2 and G_4 were included, however, together with the solid-angle corrections for the Ge(Li) detector. The product of these terms was measured by comparing our experimental results on two cascades in ^{154}Gd which had been carefully measured earlier.[15,18]

The experimental A_2 and A_4 coefficients were compared with the tables of Taylor and McPherson[19] to determine the spins and multipole mixtures. In our analysis an error was discovered in the 2(2, 3)4(2)2 table. A computer program was written for direct calculation of these coefficients from the F coefficients of Ferentz and Rosenzweg.[20] Our calculations agree with the other tables except for the $2 \to 4 \to 2$ cascades where an error was found.[21]

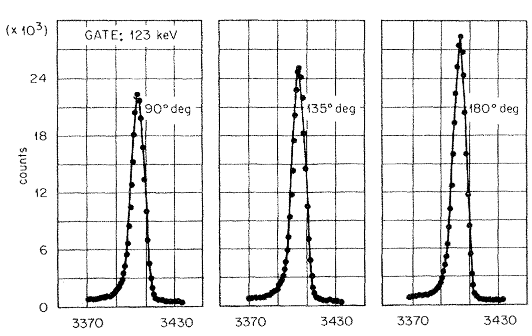

FIG. 5. The 1274.6-keV γ-ray peak in coincidence with 123-keV γ ray at three angles.

III. RESULTS

A. Cascades Through the 123-keV Level

In studies with the gate set on the 123-keV $2^+ \to 0^+$ transition (see Fig. 1) the 17-cm³ Ge(Li) detector was used. Data were taken at two different times. Group I consisted of 20 cycles and group II consisted of 10 cycles as described earlier. The second group had better resolution and true to chance ratio as well as simultaneous measurement of the Compton background. Although the statistics are better for group I, possible systematic effects are less for group II. Thus a straight average of the two sets of data was taken rather than a weight average.

Data in the strong 1274-123-keV cascade from the first 10 cycles of group I are shown in Fig. 5. Steining and Deutsch[15] have measured the correlation with NaI detectors with time-delay techniques to determine the unperturbed correlation function. Their result indicated an 0.1% $M2$ admixture in

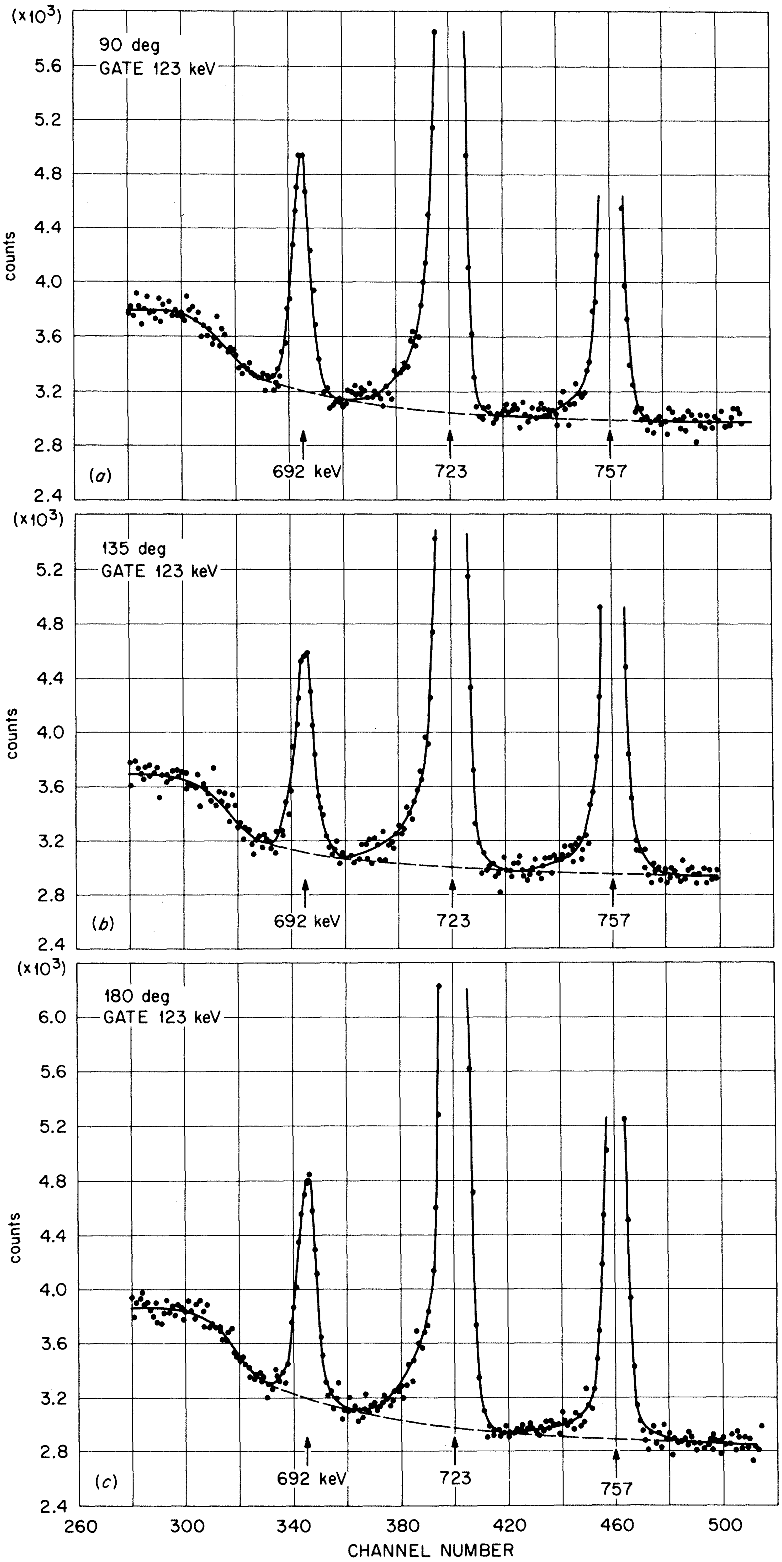

FIG. 6. Ge(Li) spectra at three angles in the energy region 600–780-keV region in coincidence with the 123-keV γ ray. The above spectra are corrected for chance coincidence.

the 1274-keV transition. The correlation has a large A_2 coefficient. From a comparison of our result with theirs, we obtained G_2 and Q_2 for the Ge(Li) detector. While Q_2 is a function of energy, the change in Q_2 for the energies of interest in our work 650-1600 keV, compared to 1274 keV, is small ($\lesssim 1\%$).

There is a weak (2.5%) 1246-keV transition that was present in the work of Steining and Deutsch[15] unknown to them. From our work on the 1246-(248)-123-keV (248 keV unobserved) and 1246-278-keV cascades, we find that the 1246-(248)-123-keV cascade has $A_2 = -0.14$. This is the theoretical value for a 3^- (D) 4^+ (Q) 2^+ (Q) 0^+ cascade. The multipolarities are measured in each case as 99.9% pure. Our experimental result agrees with this value but has a large error. Since the cascade involves essentially pure multipoles as determined from the 1246-248-keV correlation data, the theoretical value can be used with confidence. This leads to the following:

$$A_2(1274) = \left[1 + \frac{I_{1246}}{I_{1274}}\right] 0.227 \pm 0.007 \text{ (Ref. 15)} - \frac{I_{1246}}{I_{1274}}(-0.14)\,.$$

From this equation and γ-ray intensities,[22] we obtained $A_2(1274) = 0.234 \pm 0.006$. The theoretical value for a 2(1)2(2)0 sequence is 0.250. A comparison of this new result with theory leads to subsequent lowering of the $M2$ admixture to 0.04% and a raising of our G_2Q_2 results by 3.4%. This correction is small, however, and if neglected would not alter our later conclusions. The results are presented in Table I. One sees that there is good agreement between the data of the two groups.

The 873-123-keV, $2_\gamma^+ \to 2^+ \to 0^+$ cascade has also been well measured.[18] It has a large A_4 coefficient and this was used to obtain G_4 and Q_4 for the Ge(Li) detector. Here we have a double check. The A_2' for this correlation was corrected with the G_2Q_2 from the 1274-123-keV correlation and the A_4 consistent with this corrected A_2 was found. There is excellent agreement with our value of A_4 so obtained and recent accurate work.[18] This agreement serves as a good check on the operation of our system. Thus we can compare this A_4 value with our A_4' to get G_4Q_4 with confidence.

The 692-123-keV, $2_\beta^+ \to 2^+ \to 0^+$ cascade is the primary one of interest. One needs a $M1$ admixture of about 50% in the 2_β^+-2 transition in order to obtain consistent branching ratios from the 2_β^+ level. The data from group I are shown in Fig. 6. The group II data had somewhat poorer statistics but better separation between the 692-keV photopeak and the Compton edge on the left and the 723-keV peak on the right. The A_2 and A_4 coefficients are plotted against $Q = \delta^2/(1+\delta^2)$ in Fig. 7. Here $\delta = \langle f|L+1|i\rangle / \langle f|L|i\rangle$ [23] is for the upper transition

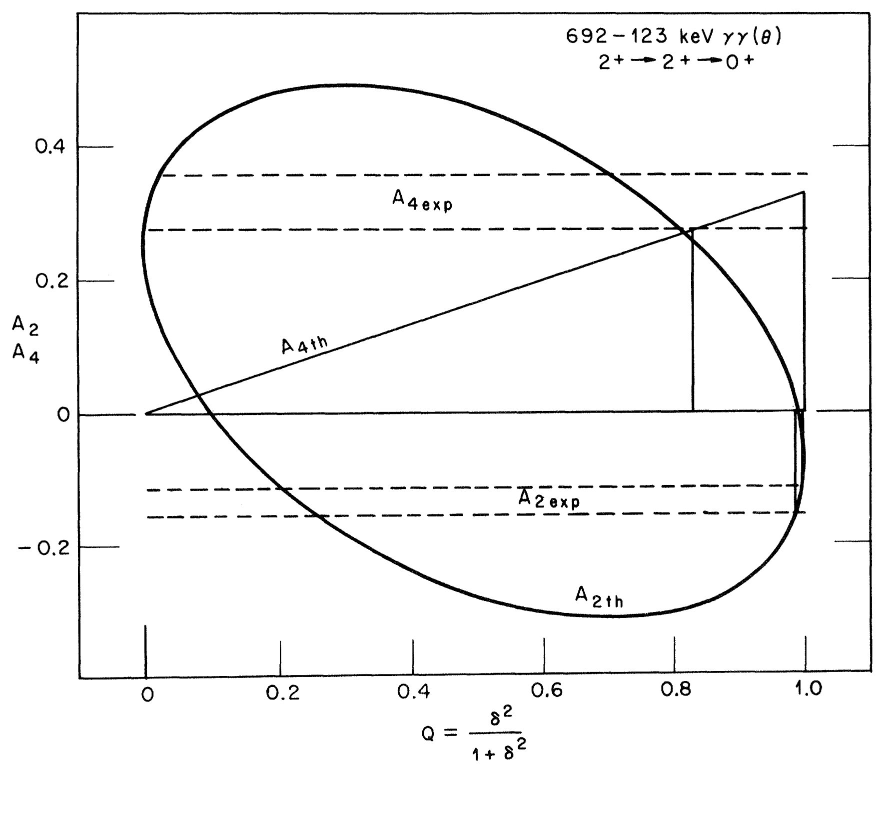

FIG. 7. Plot of A_2 and A_4 as a function of Q. The experimental A_2 and A_4 coefficients are shown as dashed lines.

TABLE II. Results of directional correlation measurements in ^{154}Gd. The δ's are given in the convention of Ref. 23.

γ-γ (keV)	Spin, parity	A_2	A_4	δ	% $E2$
692-123	2_β^+, $2^+, 0^\mp$	-0.145 ± 0.022	-0.311 ± 0.040	-11 ± 3	$99.18^{+0.31}_{-0.72}$
873-123	2_γ^+, $2^+, 0^+$			$10.1^{+1.4}_{-1.1}$	99.00^{+22}_{-25} [a]
1005-123	3_γ^+, $2^+, 0^+$	-0.239 ± 0.018	-0.073 ± 0.026	22^{+24}_{-7}	$99.78^{+0.20}_{-0.23}$
1494-123	3^-, $2^+, 0^+$	-0.075 ± 0.029	-0.022 ± 0.044	0	100 [b]
1597-123	2^-, $2^+, 0^+$	0.196 ± 0.013	-0.017 ± 0.021	0.07	>99.5 [b]
444-248	2_β^+, $4^+, 2^+$	0.196 ± 0.009	0.095 ± 0.011	-0.001 ± 0.001	99.9 ± 0.1
625-248	2_γ^+, $4^+, 2^+$	0.187 ± 0.014	0.108 ± 0.019	-0.023 ± 0.023	99.9 ± 0.1
677-248	4_β^+, $4^+, 2^+$	-0.168 ± 0.020	0.178 ± 0.029	-6 ± 2	97.0 ± 2 [c]
757-248	3_γ^+, $4^+, 2^+$	0.161 ± 0.004	-0.174 ± 0.006	5.7 ± 0.2	97.0 ± 0.3
881-248	3^-, $4^+, 2^+$	-0.132 ± 0.058	-0.059 ± 0.086	$0.01^{+0.06}_{-0.08}$	99.99 ± 0.48 [b]
893-248	4_γ^+, $4^+, 2^+$	-0.037 ± 0.010	0.160 ± 0.013	4.4 ± 0.5	95.2 ± 0.8
1188-248	4^-, $4^+, 2^+$	0.224 ± 0.028	-0.010 ± 0.041	0.15 ± 0.15	$97.8^{+2.2}_{-6.1}$ [b]
1246-248	3^-, $4^+, 2^+$	-0.151 ± 0.007	0.010 ± 0.008	-0.018 ± 0.015	99.9 ± 0.1 [b]

[a] From Ref. 10. The errors are in the numbers after the decimal.
[b] $E1$.
[c] For 2σ, $\delta = -6 \pm 4$ and $E2 > 80\%$.

in our case and i the initial state and f the intermediate state. The experimental values in Fig. 6 yield for the 692-keV transition $\delta = -(11 \pm 3)$. Thus the 692-keV transition is essentially pure $E2$. The 50% $M1$ needed for the branching ratios is definitely excluded.

Three other cascades were analyzed and the results are given in Table II. In Fig. 8 the data are compared with different spin sequences and for different choices of δ. The 1005-keV transition is known to be $E2$ from conversion-coefficient data.[7] From γ-$\gamma(\theta)$ work with NaI detectors, Debrunner and Kundig[24] limited the spin of the 1128-keV level to 1, 2, or 3. Our data uniquely determine for the first time the spin-3 assignment and precisely determine the $E2$ admixture of the 1005-keV transition. This admixture will be discussed in more detail later.

The 1596-123-keV cascade data yield a direct measurement of the spin of the 1719-keV level as 2 and a $\delta = -(0.07 \pm 0.02)$ for the 1596-keV transition. This spin and dipole character confirm the several measurements used previously to assign the spin and parity to this level.[25]

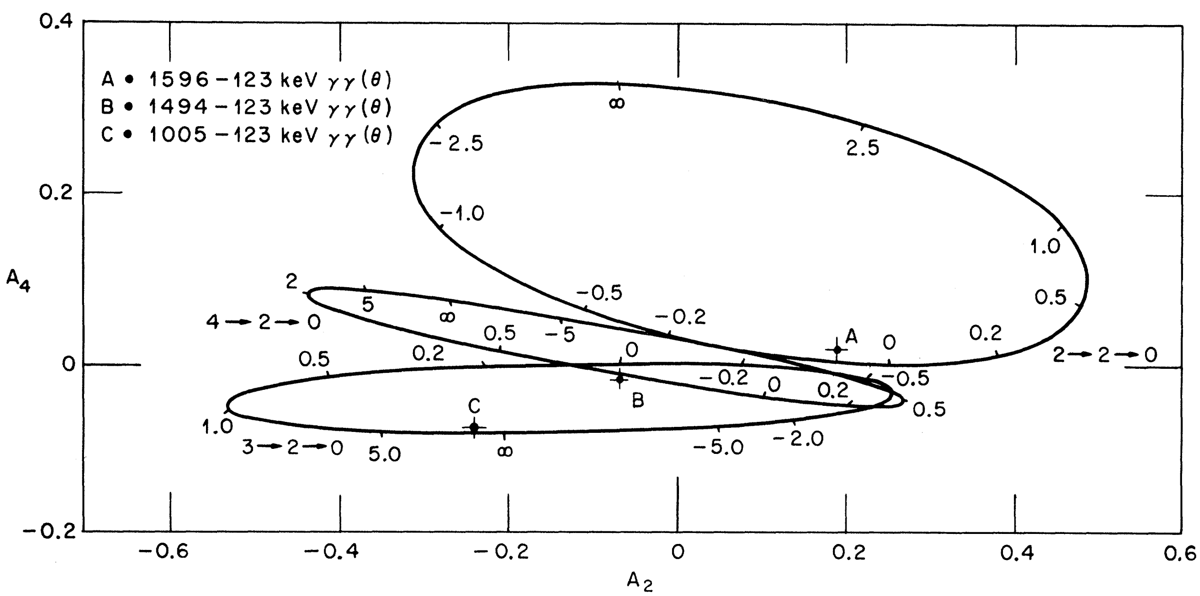

FIG. 8. The parametric plots of A_2 and A_4 as a function of δ for different spin sequences. The experimental values are shown as A, B, or C.

B. Correlations of Cascades Through the 371-keV Level

The lifetime of the 371-keV level is short enough[12] so that there are definitely no source perturbation effects. Since a new Ge(Li) detector was used, the solid angle of that detector has to be determined also. The NaI detector correction factor was taken from Yates[14] for 7-cm source to detector distance and 248-keV energy. The Ge(Li) detector solid-angle correction factors were calculated from the known geometry to by $Q_2 = 0.94 \pm 0.02$ and $Q_4 = 0.84 \pm 0.03$. These results were applied to the 757-248-keV $3^+_\gamma \to 2^+ \to 0^+$ cascade. These data yield $\delta = 5.7 \pm 0.2$ so that the 757-keV transition is essentially pure $E2$. Since the conversion-electron data also eliminate the choice of $\delta \approx 0.4$ allowed by A_2, the A_4 consistent with A_2 was used to obtain a new Q_4. A relatively large error had been assigned Q_4 for the Ge(Li) detector in our calculations since Q_4 is a large (≈16%) correction. This new Q_4 was about 2% lower. This 2% difference is much smaller, however, than the errors on any of the experimental Q_4A_4 values for these cascades. The lower value was used for the other cascades but the results are insensitive to errors of this order. The total Q_2 and Q_4 values for both detectors are 0.87 ± 0.02 and 0.63 ± 0.02, respectively.

The 248-keV, $4^+ \to 2^+$ transition from the 371-keV level was selected from the NaI detector as a gate and the contribution from the Compton background under the peak was simultaneously measured in the next experiments. Data were recorded for 30 cycles (3 angles per cycle, 23 h/angle) with the 3.8% Ge(Li) detector. Eight correlations were analyzed as previously described. The data were analyzed in three groups, 10 cycles each. The results of each group were consistent. These correlations serve to show the power of a NaI (gate)-Ge(Li) (spectrum) γ-$\gamma(\theta)$ system in a complex decay, as discussed elsewhere in more detail,[10] since the 248-keV transition is only 17% of the 123-keV one and the other members of these cascades vary from 11% to only 0.17% of the 123-keV transition.

The results of these measurements are presented in Fig. 9 and Table II. These are the first measurements of any of these correlations. The 757-248-keV cascade results are already given above.

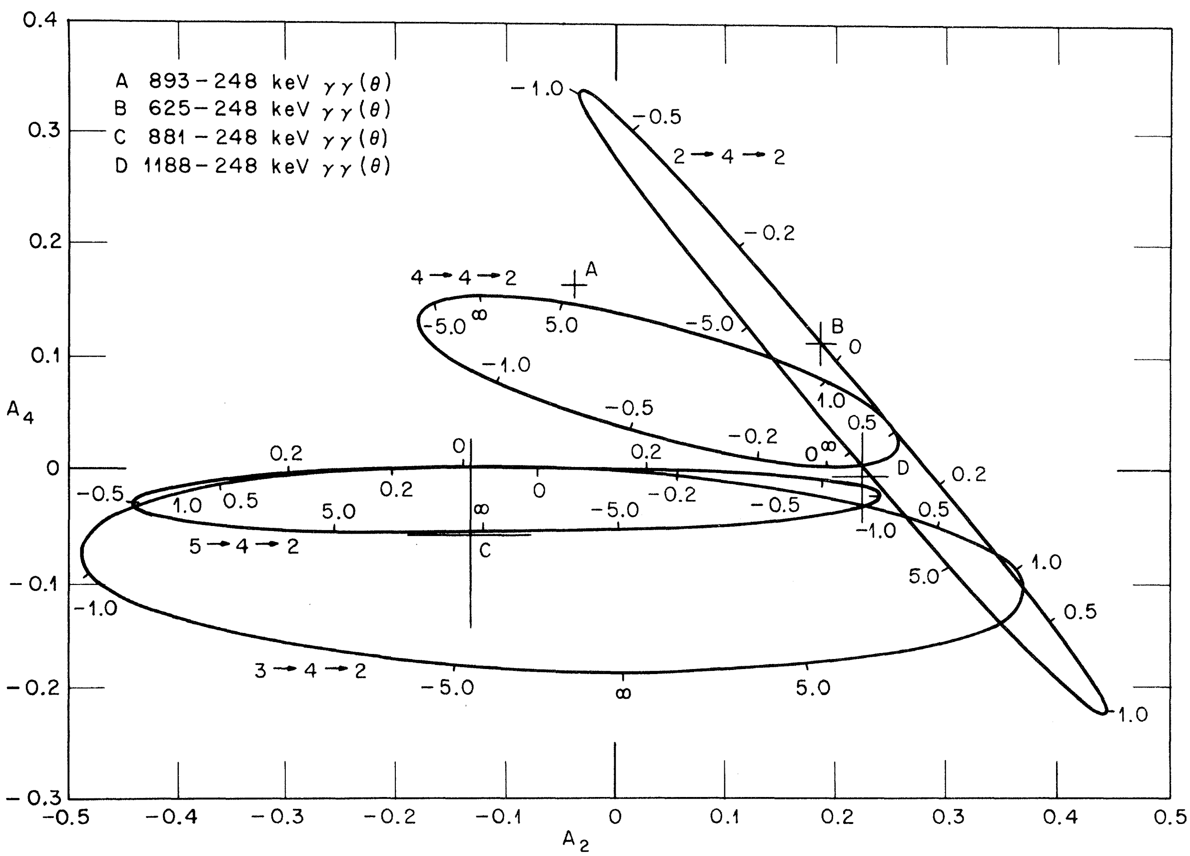

FIG. 9. The parametric plots of A_2 and A_4 as a function of δ for different spin sequences. The experimental values are shown as A, B, C, and D.

The $2 \to 4 \to 2$ cascades that originate from odd-parity levels, where one expects pure quadrupole and dipole character in the first transition, respectively, offer additional checks on the reliability of the system. The 444-248-keV, $2^+_\beta \to 4^+ \to 2^+$ correlation data yield similar results of less than 0.1% $M3$ in the first transition. The 3^-, 1251-keV state is known from (d, d') work.[26] The 881-248-keV correlation results confirm this spin (spin 5 is ruled out by decay to the 2^+, 123-keV state) and establish the quadrupole admixture as ($\lesssim$1%). as expected.

Although the 1264-keV level has long been considered the 4^+ member of the γ band on the basis of an energy fit,[25] the 893-248-keV correlation data uniquely determine the spin of the 1264-keV level as 4 for the first time. The $\delta = 4.4 \pm 0.5$ obtained from these measurements yield a small but definite $M1$ component in the $4^+_\gamma \to 4^+$ transition.

A state has been observed at 1560 keV[2,22,27] with a 4^- assignment suggested on the basis of lack of decay to the 123-keV state. The 1188-248-keV correlation data in fact agree with spin 2, 3, 4, or 5 with the following multipoles: ≃100% octupole, ≃15% quadrupole, ≃0% quadrupole, and ≃50% quadrupole, respectively. In light of the multipolarities of other transitions in this nucleus, the 4^- assignment is strongly favored by these data.

The 1617-keV level is known[2,22,27] and assigned 3^- from the decay to known levels and the lack of a ground-state transition. The 1246-248-keV correlation data determine the spin as 3 or 5. Spin 5 is excluded by decay to the 2^+, 123-keV state. Thus the spin of 3 is now established. From $\delta = -0.018 \pm 0.015$, the transition has $\lesssim$0.1% quadrupole radiation. This lack of quadrupole radiation supports the odd-parity assignment as no pure $M1$ transitions are observed in ^{154}Gd.

The 677-248-keV, $4^+_\beta \to 4^+ \to 2^+$ correlation data required careful analysis. The contribution from the Compton events in the gate gave rise to counts in the nearby 692-keV photopeak and the Compton edge just below 677-keV. The data with this Compton background contribution subtracted are shown in Fig. 10. There is still a small bump or low-energy tail at 135 and 180° on the 677 peak. This arises from a small contribution of the backscatter peak around 200 keV in the 248-keV gate so that the Compton edge just below the 677-keV peak is slightly enhanced at the large angles. The correlation results for the 5, 7, 9, and 10 counts around the peak gave consistent results so that this tail at 135 and 180° did not distort our final result. Without simultaneously measuring the background, these data would have been very difficult to analyze with any accuracy. The data yield $\delta = -(7^{+3}_{-2})$ for the $4^+_\beta \to 4^+$ transition which thus has only 1–4% $M1$ admixture.

Approximately 50% $M1$ admixture in the $4^+_\beta \to 4^+$ transition is needed to explain the branching ratios. There is a slow variation in A_2 with δ here.

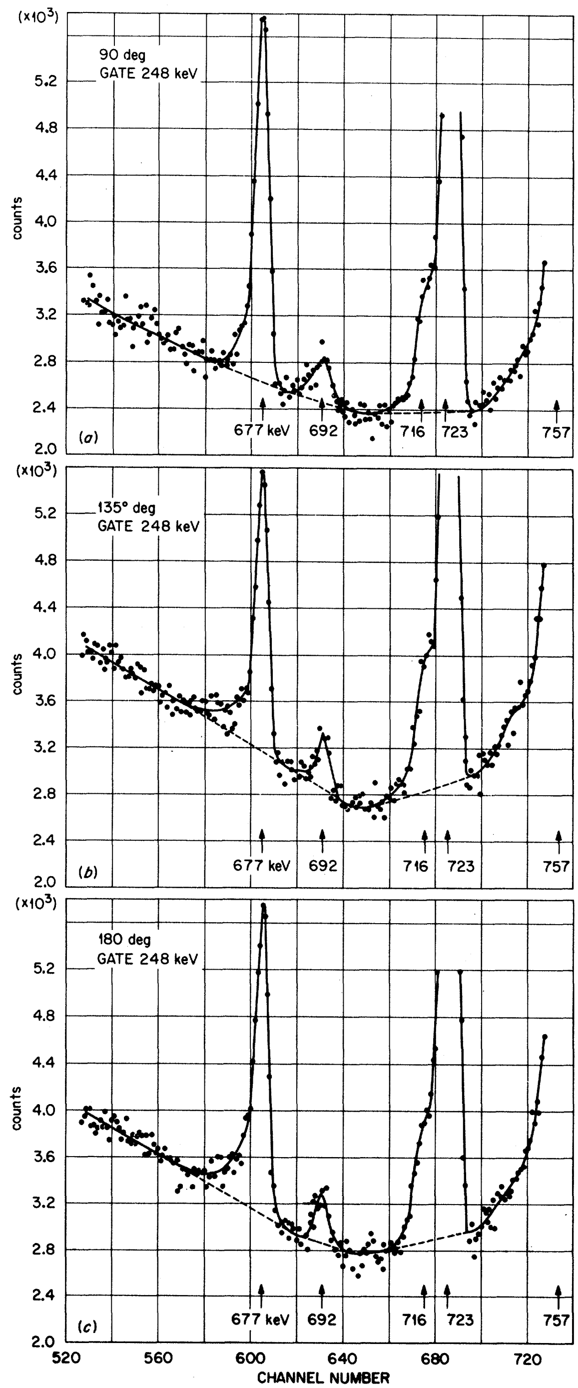

FIG. 10. Ge(Li) spectrum at three angles in the energy region 650–750-keV in coincidence with the 248-keV γ ray. The above spectrum is corrected for chance coincidence.

One can obtain an upper limit of 20% $M1$ from a 2σ variation of A_4. This rules out an $M1$ component as an explanation of the lack of agreement with theory with the experimental branching ratios here too.

IV. DISCUSSION AND CONCLUSIONS

These data on the $2_\beta^+ \to 2^+$ and $4_\beta^+ \to 4^+$ transitions have clearly ruled out the hoped for[6] large $M1$ admixtures which would explain the branching ratios from the β band in this nucleus. It has also been shown that β-γ band mixing does not explain the situation either.[2] It has now been shown[28] that it is the $2_\beta^+ \to 4^+$ transition that is responsible for the anomalous behavior of the branching ratios from the 2_β^+ state in ^{154}Gd.

In general the theoretical description of the $K=2$ γ-vibrational bands has been considered to be good.[6] However, in transitional nuclei like ^{152}Sm and ^{154}Gd, recent evidence[2,17] indicates that mixing of only the γ and ground-state bands is not sufficient to explain the branching ratios from the γ band. As can be seen in Table II the $M1$ component in the $2_\gamma^+ \to 2^+$ transition is very small ($1.00^{+0.25}_{-0.22}$%) as is also the case for the $3_\gamma^+ \to 2^+$ transition ($0.28^{+0.23}_{-0.13}$%). It is interesting to note that the $3_\gamma^+ \to 4^+$ has a small but definite $M1$ admixture (3.0 ± 0.3%). There is a still larger $M1$ component (4.8 ± 0.8%) in the $4_\gamma^+ \to 4^+$ transition. These differences in decays to the 4^+ member of the ground-state band could be additional clues as to the anomaly in the $2_\beta^+ \to 4^+$ transition. The $M1$ components in the transitions from the γ band are still too small to significantly alter the branching ratios. Our data also allow a larger $M1$ component in the $4_\beta^+ \to 4^+$ transition though small in both cases. The $2_\beta^+ \to 4^+$ and $2_\gamma^+ \to 4^+$ transitions are normal at least in regards to the absence of any $M3$ admixture ($\lesssim 0.1$%).

In ^{178}Hf a large $M1$ component ($\simeq 80$%) has been reported from γ-$\gamma(\theta)$ work[29] on a $2^+ \to 2^+$ transition from a supposedly β-vibrational state. This $M1$ component agreed with that needed to obtain a consistent band mixing parameter Z_β in the earlier branching-ratio data.[29] The presence of a large $M1$ component has been confirmed in this transition but not a consistent band mixing parameter.[30] Preliminary analysis of γ-$\gamma(\theta)$ work[31] on ^{156}Eu decay to ^{154}Gd, however, indicate that the $2_\beta^+ \to 2^+$ transition is pure $E2$ there also. What happens in these more strongly deformed nuclei will be considered in our subsequent work.[30,31]

Thus it appears that in transitional nuclei like ^{152}Sm and ^{154}Gd, we can definitely say there is at present a breakdown in the theoretical understandings of these states. Such breakdowns may be related to the softness of these nuclei or perhaps higher-order terms in the mixing of the β and ground-state band or admixtures of other K states. In any case, further theoretical work is called for to explain these discrepancies, which represent the first major crisis in the application of rotational relationships to nuclei with permanent deformations in their ground states.

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Blocking Effect in the Nuclear Pairing Theory: An Application to the Sn Isotopes

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The effect of blocking, for odd systems, is included in the gap equations of the nuclear pairing theory for a general two-body interaction. These equations are applied to calculate the one-quasiparticle low-energy spectra, the neutron separation energies, and the "odd-even mass differences" for the Sn isotopes. The apparent inflation of the effective pairing strengths in calculations in which the blocking effect is ignored is pointed out.

The Bardeen-Cooper-Schrieffer[1] (BCS) theory of superconductivity has been widely applied to treat the pairing correlation in deformed[2] as well as spherical[3] nuclear systems. As was pointed out by Soloviev,[2] the BCS trial wave function is suitable only for even systems. For an odd system, the "blocking effect" induced by the unpairing particle should be taken into account. The resulting gap equations, when the pairing interaction is assumed to be the constant pairing force, have previously been derived.[2,3] In this paper we give the gap equations for a general two-body interaction. Using a realistic nucleon-nucleon residual interaction we apply these equations to the Sn isotopes. The low-energy spectra of the odd-mass isotopes are reasonably well reproduced. The calculated neutron separation energies exhibit the experimentally observed odd-even staggering effect. This effect disappears when we neglect blocking. In calculating the odd-even mass differences, we show that the effective strength of the pairing force is inflated by 20 to 40% when the blocking effect is ignored.

The BCS trial wave function $|\Phi_0\rangle$, or the quasiparticle (q.p.) vacuum, can be obtained by performing the canonical Bogoljubov-Valatin (BV)[4] transformation e^S on the single-particle (s.p.) vacuum state $|0\rangle$:

$$|\Phi_0\rangle = e^S|0\rangle, \tag{1}$$

with e^S defined in terms of the occupational u and v factors,

$$a_\alpha{}^\dagger = e^S c_\alpha{}^\dagger e^{-S} = u_a c_\alpha{}^\dagger - (-)^{j_a - m_a} v_a c_{\bar{\alpha}}, \tag{2}$$

where $u_a{}^2 + v_a{}^2 = 1$, $c_\alpha{}^\dagger$ (c_α) and $a_\alpha{}^\dagger$ (a_α) are s.p. and q.p. creation (annihilation) operators, respec-