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## $\beta$ Decay and Nuclear Structure in $A = 99$ Nuclei

T. Nagarajan,\* M. Ravindranath, and K. Venkata Reddy

Laboratories for Nuclear Research, Andhra University, Waltair, India

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An analysis of the  $\beta$  spectrum of  $\text{Mo}^{99}$  taking into account the correct shape of the outer  $\beta$  group revealed  $\beta$  groups with end points  $1214 \pm 1$  (84%),  $840 \pm 5$  (2%),  $450 \pm 10$  (14%) keV. The shape of the outer  $\beta$  group ( $\frac{1}{2}^+ \rightarrow \frac{1}{2}^-$ ) feeding the 142-keV level in  $\text{Tc}^{99}$  is expected to be statistical, obeying the  $\xi$  approximation. The measured shape-factor coefficient  $a$  is  $-0.01 \pm 0.007$ . On the basis of shell-model matrix elements, the conserved-vector-current prediction for  $\Lambda$  gave a positive value for  $a$ , whatever the value of  $\Lambda_0$ . Treating both  $\Lambda$  and  $\Lambda_0$  as free parameters, a fit to the experimental shape gave  $3.4 \leq \Lambda \leq 5$  and  $2.3 \leq \Lambda_0 \leq 3.7$ . The shape of the inner group (840 keV) was found to be consistent with first-forbidden nonunique character, and within experimental accuracy it was consistent with the shell-model prediction, even though the 514-keV ( $\frac{3}{2}^-$ ) level fed by this  $\beta$  group is considered as arising from the weak coupling of a  $p_{1/2}$  proton to the  $2^+$  first excited state of the core. But the large  $\log ft$  value of this transition may perhaps be explained by a detailed "microscopic" picture of the core.

### I. INTRODUCTION

The decay of  $\text{Mo}^{99}$  has been investigated by many,<sup>1-7</sup> and there is a general agreement on the existence of two  $\beta$  groups with maximum energies of 1230 and 445 keV. Vlasov *et al.*<sup>6</sup> observed a  $\beta$  transition to the ground state of  $\text{Tc}^{99}$  and two further inner groups which led them to postulate levels at 780 and 1062 keV for  $\text{Tc}^{99}$ . Cretzu *et al.*<sup>8</sup> could not confirm these additional  $\beta$  groups, but they detected a weak 245-keV  $\beta$  group feeding a level at 1130 keV in  $\text{Tc}^{99}$ . Further, the spins of

levels at 514, 922, and 1131 keV are not uniquely established.<sup>9,10</sup> The absence of  $\beta$  transitions to the ground state and the levels at 140 and 181 keV in  $\text{Tc}^{99}$  can be understood on the shell-model prediction that these states arise out of ( $g_{9/2}$ )<sup>3</sup> proton configurations. Recently the energy levels of  $\text{Tc}^{99}$  have been reproduced<sup>11</sup> with configuration interaction of the form  $(\nu d_{5/2})^6 (\pi g_{9/2})^3$  taking into account admixtures from  $p_{1/2}$  proton orbitals. In view of these considerations, a detailed study of the shapes of  $\beta$  groups can contribute some information to the level structure of  $\text{Tc}^{99}$ . The intense

outer  $\beta$  group is a  $\frac{1}{2}^+ \rightarrow \frac{1}{2}^-$  transition with  $\log ft = 7.1$ . Since  $\frac{1}{2}^+ \rightarrow \frac{1}{2}^-$  transitions do not involve the  $B_{ij}$  matrix element, and a cancellation is unlikely in both  $V$  and  $Y$ , they should normally be consistent with the  $\xi$  approximation. In the absence of any configuration admixture from neighboring orbitals, it should be possible to explain all  $\frac{1}{2}^+ \rightarrow \frac{1}{2}^-$  transitions on simple shell-model considerations. The spin of 514-keV level can be  $(\frac{3}{2}^-)$  or  $(\frac{5}{2}^-)$  from the  $\log ft$  value of 8.5 for the  $\beta$  group to this level. A determination of the shape of the 840-keV  $\beta$  transition can give further information on whether the spin change involved is 1(yes) or 2(yes), and also the structure of the 514-keV level can be inferred.

## II. EXPERIMENTAL PROCEDURE

In this investigation, the  $\beta$  spectra were studied by means of a thoroughly tested Siegbahn-Slatis spectrometer equipped with a cylindrical-well-type NE102 detector.<sup>12-14</sup> The counting efficiency of the detector is unity down to 50 keV. The backscattering correction is 0.2% at 80 keV for discrimination biases employed in the present work. The numerous control experiments and the mode of analysis were described earlier.<sup>12-17</sup> Several samples of  $\text{Mo}^{99}$  were obtained from the Isotope Division, Bhabha Atomic Research Centre, Bombay.  $\text{Mo}^{99}$  is a uranium fission product. The  $\text{Mo}^{99}$  fraction is separated from the fission fragments by alumina-gel chromatography. Carrier-free  $\text{Mo}^{99}$  is finally obtained as ammonium molybdate in dilute hydrox-

ide solution. The sources were prepared by evaporating a drop of source material on thin aluminized Mylar foils. Insulin was used to define the source area and help uniform spreading of the source. All sources used were thinner than  $100 \mu\text{g}/\text{cm}^2$ .

The Fermi-Kurie (FK) plot obtained by Cretzu *et al.*<sup>8</sup> exhibited a slow decrease of counting rate with increasing current beyond the end point of 1234-keV  $\beta$  group. This is indicative of the occurrence of appreciable scattering in the spectrometer. In such spectrometers it is difficult to establish the presence or absence of very weak high-energy components when intense inner groups are present. In view of this, the end-point energy of the outer  $\beta$  group determined by Cretzu and Hohmuth<sup>8</sup> cannot be regarded as accurate. In the present spectrometer the narrow gap between the annular slit and the chamber is closed, and it was found that the apparent high-energy component arising from scattering is absent. The existence or absence of the ground-state transition can therefore be ascertained with confidence. In addition the application of proper resolution corrections<sup>13</sup> and the study of shape factor as a function of end point,<sup>17</sup> enable one to determine both the shape and end-point energy very accurately.

During the first run, the  $\beta$  continuum was scanned from 100 to 1300 keV roughly in steps of 25 keV. The background at every point was taken by closing the central baffle. A FORTRAN program FERMKURI corrects<sup>12-14, 17</sup> the count rates for the

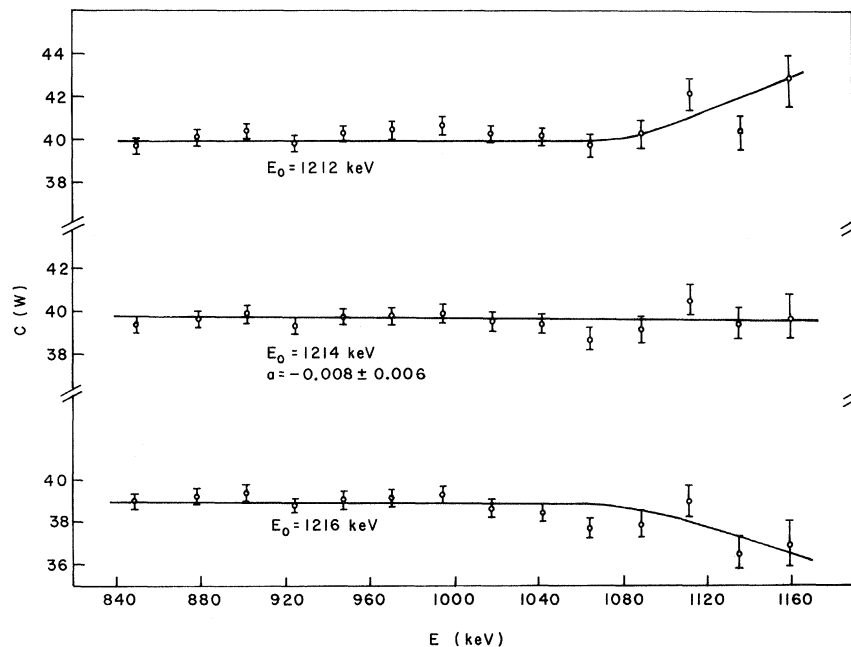


FIG. 1. The shape factor of 1214-keV  $\beta$  component of  $\text{Mo}^{99}$ . The figure shows the behavior of the shape-factor curve near the end-point energy when the end-point energy is varied around its true value.

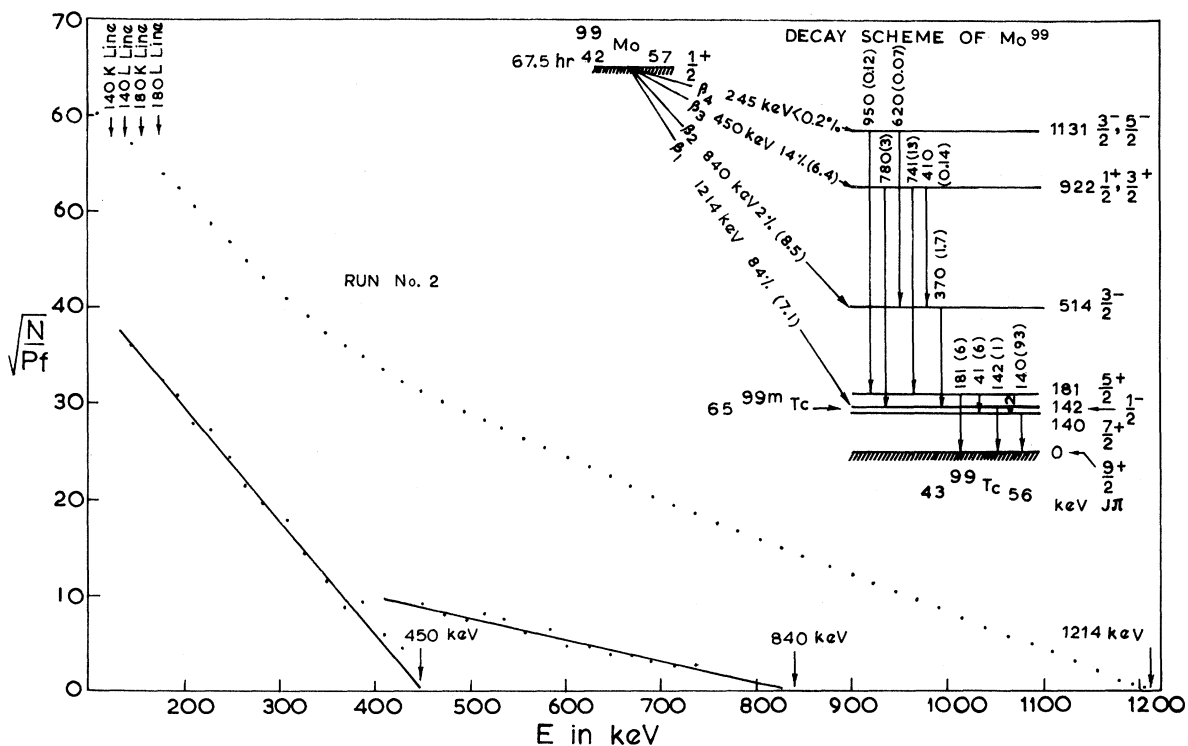


FIG. 2. Fermi-Kurie analysis of  $\beta$  spectrum of  $\text{Mo}^{99}$ . The decay scheme is from Ref. 21. The end-point energies and intensities of  $\beta$  groups are from the present work. As the uncertainties increase with the number of subtractions, the very weak ( $<0.2\%$ ) 245-keV group cannot be revealed.

short half-life (67.5 h), for finite resolution and backscattering effects. The program draws the FK plot after finding the exact Fermi function from the tables of Bhalla and Rose.<sup>18</sup> There is no evidence from the FK plot for the existence of a  $\beta$  group with an end point higher than 1214 keV.

The second run was used for shape-factor analysis in the energy region of 840 to 1160 keV and the

end-point energy is varied by the program "BETASHAP"<sup>12, 14, 17</sup> (Fig. 1). The shape-factor curve did not blow up or down for  $E_0 = 1214$  keV. A weighted least-square fit corresponding to this end-point energy yielded  $C(W) = k[1 - (0.008 \pm 0.006)W]$ . The high-energy portion of the  $\beta$  spectrum corrected with the above shape is subtracted from the gross spectrum. A weak inner

TABLE I. Data on the  $\beta$  groups of  $\text{Mo}^{99}$ .

Bunker and Canada (Ref. 1)	Medicus, Maeder, and Schneider (Ref. 2)	Varma and Mandeville (Ref. 4)	Levi and Papineau (Ref. 5)	Vlasov <i>et al.</i> (Ref. 6)	Cretzy and Hohmuth (Ref. 8)	Dorikens-Vanpraet <i>et al.</i> (Ref. 10)	This work
					245 (0.2)	240 $\pm$ 10 (0.25 $\pm$ 0.1)	245 (0.2)
445 $\pm$ 10 (20)	540 $\pm$ 5 (15)	450 (14)	410	370 $\pm$ 30 (16)	450 (14)	440 $\pm$ 10 (20 $\pm$ 5)	$\log ft = 8.3$ 450 $\pm$ 10 (14)
		870 (1)	800 (3)	640 $\pm$ 40 (7)	880 (1.7)	880 $\pm$ 25 (5.3)	840 $\pm$ 5 (2)
							$\log ft = 8.5$
1230 $\pm$ 1 (80)	1225 $\pm$ 15 (85)	1230 (85)	1180	1235 $\pm$ 5 (75) 1410 $\pm$ 15 (2)	1230 (84)	1230 $\pm$ 10 (75 $\pm$ 10)	1214 $\pm$ 1 (84)
							$\log ft = 7.1$

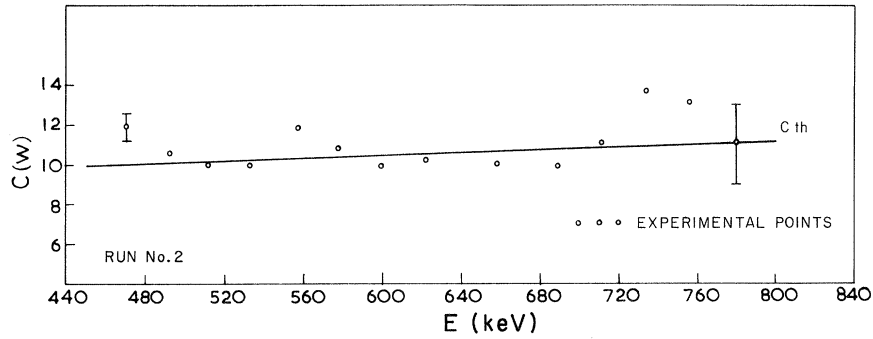


FIG. 3. The experimental shape of the weak 840-keV  $\beta$  component of  $\text{Mo}^{99}$  is compared with the theoretical shape factor  $C_{th}(W) = 1 + 0.049W - 0.0047/W + 0.034W^2$  computed from single-particle matrix elements. The 840-keV component results after the subtraction of the shape-corrected 1214-keV component. The error bars include the subtraction error.

group with end point at 840 keV is revealed (Fig. 2). Upon subtraction of this group, an inner group with an end point of 450 keV is obtained. The very weak 245-keV group reported by Cretzu *et al.*<sup>8</sup> could not be revealed, but its intensity should be less than 0.2%. The  $\beta$  end points, intensities, and  $\log ft$  values of the present measurements are compared in Table I with those of other workers.

After subtracting the 1214-keV  $\beta$  component corrected for its shape, the resulting spectrum was subjected to shape analysis in the energy region 440–840 keV (Fig. 3). The error bars in Fig. 3 also include a factor of 5 for the subtraction of the outer group. The statistical fluctuation of the points is large, but the energy dependence of the shape is certainly not unique. On account of the large fluctuations of about 20%, no attempt was made to evaluate the shape-factor coefficients. The single-particle estimate of the shape factor of the 840-keV  $\beta$  group normalized at 600 keV is

also shown in Fig. 3. The theoretical prediction is not inconsistent with the experiment.

In order to extend the region of shape-factor investigation of the outer  $\beta$  group to lower energies, the spectrum of the 840-keV  $\beta$  group was constructed according to the prescription

$$N_2(p)dp = kfp C(W)(W_{02} - W)^2 dp, \quad (1)$$

where the intensity factor  $k = 0.02A/A'$ .  $A$  is the area of the gross spectrum of  $\text{Mo}^{99}$  and  $A'$  is the area of (1) constructed with  $k = 1$ . The shape of the outer  $\beta$  group was analyzed after subtracting the spectrum (1) with statistical shape and also with the theoretical shape of the single-particle estimate (7). The results were not sensitive to the shape of the 840-keV transition on account of its very low-intensity (see Fig. 4). However the shape of the outer group is a little affected by the subtraction of this weak low-energy  $\beta$  group. The above results are summarized in Tables II and III.

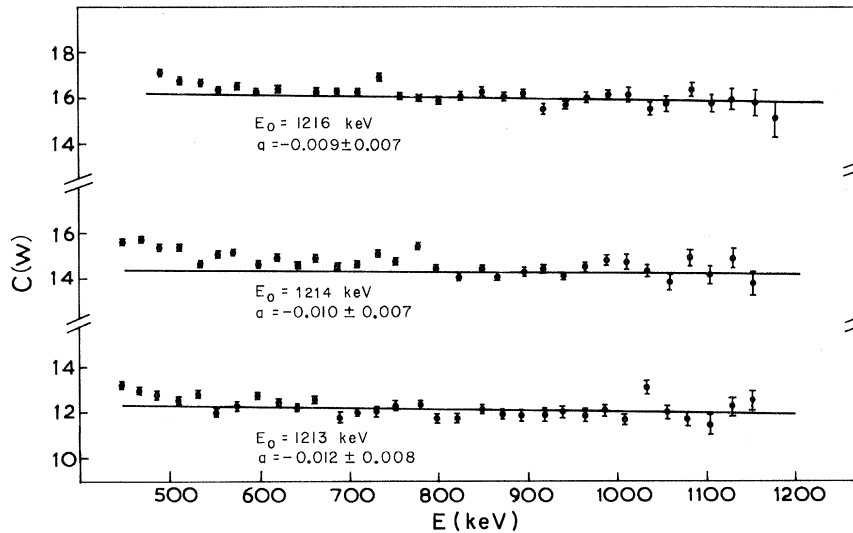


FIG. 4. Shape factors of the 1214-keV  $\beta$  component of  $\text{Mo}^{99}$  for different runs. The analysis is extended down to 500 keV after subtracting the 2% 840-keV inner group with statistical shape.

TABLE II. Different analysis of 1214-keV  $\beta$  group for run No. 2.

Energy range (keV)	$E_0$ (keV)	$a(m_0c^2)^{-1}$
840–1160	1214 $\pm$ 1	-0.008 $\pm$ 0.006
450–1150 (Without correcting for the inner group)	1214 $\pm$ 2	-0.011 $\pm$ 0.007
450–1150 (840-keV group subtracted with statistical shape)	1214 $\pm$ 2	-0.010 $\pm$ 0.007
450–1150 (840-keV group subtracted with theoretical shape)	1214 $\pm$ 2	-0.010 $\pm$ 0.007

## III. DISCUSSION

With neutron number  $N = 56$ , the  $2d$  shell is completely occupied. The spin  $\frac{1}{2}$  for the ground states of  $\text{Mo}^{99}$  and  $\text{Zr}^{97}$  shows that the 57th neutron is in the  $3s_{1/2}$  state. On the other hand, the nucleus  $\text{Ru}^{101}$  ( $N = 57$ ) has a spin ( $\frac{5}{2}^+$ ). Spins and parities for the nuclei  $\text{Sr}^{95}$ ,  $\text{Pd}^{103}$ , and  $\text{Cd}^{105}$  ( $N = 57$ ) are not known. With the same neutron number, the spin of the ground state depends on the number of proton pairs. However a spin  $\frac{1}{2}$  for the ground state of  $\text{Mo}^{99}$  is established by the measurements of Cohen and Chubinsky<sup>19</sup> and Hjarth and Cohen.<sup>20</sup> Experimentally, the ground level of  $\text{Tc}^{99}$  is determined to be  $\frac{3}{2}$  from atomic spectra and  $u$  appears to belong to the  $g_{9/2}$  Schmidt group.<sup>21</sup> The ground level and the levels at 140 keV ( $\frac{7}{2}^+$ ) and 181 keV ( $\frac{5}{2}^+$ ) for  $\text{Tc}^{99}$  arise from the proton configuration ( $\pi g_{9/2}$ )<sup>3</sup>. However the level at 142 keV ( $\frac{1}{2}^-$ ) appears to be a pure single-particle excitation<sup>10</sup> with the levels at 520 keV ( $\frac{3}{2}^-$ ) and 1131 keV arising from a coupling of a  $p_{1/2}$  proton to the  $2^+$  excited state of the core  $\text{Mo}^{98}$ .<sup>22</sup>

IV. INTERPRETATION OF THE SHAPE OF THE 1214-keV  $\beta$  GROUP

Assuming the initial and final states to be pure single-particle states, namely  $s_{1/2}$  and  $p_{1/2}$ , the single-particle matrix elements in the formalism of Rose and Osborn<sup>23</sup> are

$$x/u = 0.42; \quad w/u = 0.8662. \quad (2)$$

As to the two relativistic matrix elements, theoretical predictions are available for the ratios  $\Lambda = (\int i\vec{\alpha})/\xi \int \vec{r}$  and  $\Lambda_0 = (-\int i\gamma_5)/\xi \int \vec{\sigma} \cdot \vec{r}$ . The conserved-vector-current (CVC) prediction<sup>24</sup> for  $\Lambda$  is

$$\Lambda_{\text{CVC}} = (W_0 \mp 2.5)/\xi \pm \lambda_{\text{CVC}}, \quad (3)$$

where  $\lambda_{\text{CVC}} = 2.4$ <sup>24a</sup> and  $2.33$ .<sup>24b</sup> Using Fujita's value of  $\lambda_{\text{CVC}}$ ,  $\Lambda_{\text{CVC}} = 2.48$  for the 1214-keV transition of  $\text{Mo}^{99}$ ,  $\Lambda_0 = 2$ <sup>25</sup> and  $1$ .<sup>26</sup> Since the parameter  $\Lambda_0$  fluctuates from nucleus to nucleus, the parameter " $a$ " in the shape-factor expression was calculated using Kotani relations<sup>27</sup> and treating  $\Lambda_0$  as variable. The slopes of the calculated shape factor were always found to be positive; hence  $a > 0$ . This result contradicts the experimental shape factor ( $a = -0.01$ ). The same difficulty was encountered by Beekhuis<sup>28</sup> in the interpretation of the  $\text{Ce}^{141}$  ( $\frac{7}{2}^- \rightarrow \frac{7}{2}^+$ ) shape factor on the basis of single-particle estimates. The experience of the present work in the nuclei  $\text{Ag}^{111}$  and  $\text{La}^{140}$ , as well as those of others in connection with the experimental determination of the ratio by using several experimental observables, is that the experimental values of  $\Lambda$  do not strictly correspond to the CVC predictions. According to Binstoyle,<sup>29</sup> the CVC relations are only an approximation and errors of 10% are to be expected. In view of these considerations both  $\Lambda$  and  $\Lambda_0$  are treated as variables (Fig. 5).

The Kotani parameter  $C$  is zero for  $\frac{1}{2}^+ \rightarrow \frac{1}{2}^-$  transitions. The coefficients  $a$  and  $b$  were computed as functions of  $\Lambda$  and  $\Lambda_0$ . Imposing the condition  $b = 0$ , yields:  $\Lambda_0 = 3.55 - 0.7938\Lambda$ . Further impos-

TABLE III. Shape factor results of 1214-keV  $\beta$  group of  $\text{Mo}^{99}$  for different runs.

Run No.	Energy range (keV)	$E_0$ (keV)	$a(m_0c^2)^{-1}$
1	450–1150 (840-keV group subtracted with statistical shape)	1216 $\pm$ 2	-0.009 $\pm$ 0.007
2	...	1214 $\pm$ 2	-0.010 $\pm$ 0.007
3	...	1214 $\pm$ 2	-0.010 $\pm$ 0.007
4	...	1214 $\pm$ 2	-0.012 $\pm$ 0.008
	Mean	1214 $\pm$ 2	-0.01 $\pm$ 0.007

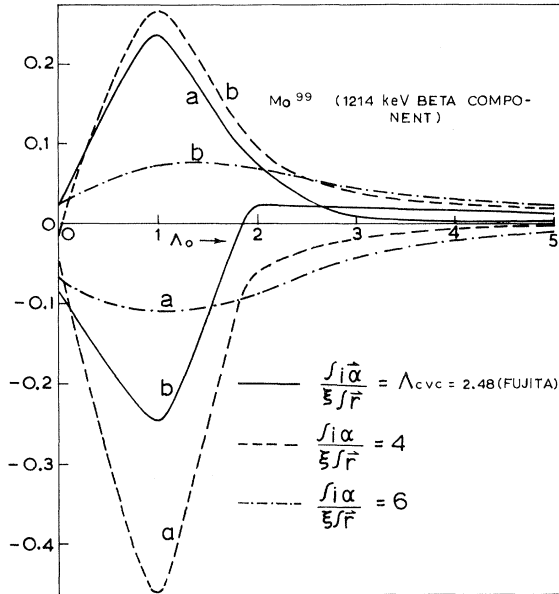


FIG. 5. Shape-factor coefficients  $a$  and  $b$  as functions of  $\Lambda$  and  $\Lambda_0$ .

ing the experimental condition  $a = -0.017$ , no solution exists for  $\Lambda$ . This is due to the presence of a small hyperbolic term in the shape factor which cannot be set to zero. The experimental shape factor (Fig. 6) was once again fitted with  $C(W) = k(1 + aW + b/W)$ , yielding  $a = -0.01$  and  $b = 0.005$ . The theoretical shape factor was calculated using Kotani's expressions<sup>27</sup> and the single-particle matrix elements [Eq. (1)], and the parameters  $\Lambda$  and  $\Lambda_0$  were varied in a grid with a computer program by requiring that

$$\chi^2 = \sum_i \frac{[C_{th}(W_i) - C_{exp}(W_i)]^2}{\sigma_{exp}^2(W_i)}$$

be a minimum. Both the theoretical and experi-

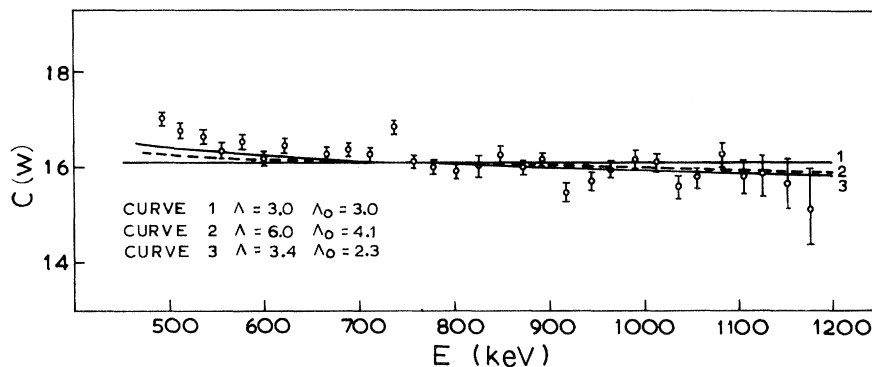


FIG. 6. Comparison of the theoretical shape factor of the single-particle model with experiment for the 1214-keV  $\beta$  component of Mo<sup>99</sup> for different choices of  $\Lambda$ . For each value of  $\Lambda$  the value of  $\Lambda_0$  is so chosen as to give the best fit to experimental points. The experimental points are best fitted by theoretical curve only in the region of parameter values  $3.4 \leq \Lambda \leq 5$  and  $2.3 \leq \Lambda_0 \leq 3.7$ .

mental shape factors were normalized at 720 keV.  $\chi^2/m - n$  ranged from 1.7 to 3.8 for the region of parameter values  $3.4 \leq \Lambda \leq 5$  and  $2.3 \leq \Lambda_0 \leq 3.7$ . The curves corresponding to the three sets of parameter values are shown in Fig. 6. The above parameter values are not far from their corresponding theoretical predictions. The differences are essentially without any significance as the uncertainties in predicting these ratios are very large.<sup>30</sup> So it can be concluded that the 1214-keV ( $\frac{1}{2}^+ - \frac{1}{2}^-$ ) transitions of Mo<sup>99</sup> exhibits a small negative slope for the energy dependence of the shape factor in accordance with large Coulomb energy approximations; and this effect can be adequately explained on the basis of the simple  $j-j$  coupling shell-model assignments of spins and parities to the connecting states.

#### V. INTERPRETATION OF THE 840-keV TRANSITION OF Mo<sup>99</sup>

This transition proceeds from a  $\frac{1}{2}^+$  state to the  $\frac{3}{2}^-$  state of Tc<sup>99</sup>. The neutron and proton configurations of Mo<sup>99</sup> are  $50 + (2d_{5/2})^6(3s_{1/2})^1$  and  $28 + (1f_{5/2})^6(2p_{3/2})^4(2p_{1/2})^2(1g_{9/2})^2$ , respectively. The 510 keV ( $\frac{3}{2}^-$ ) and 1131 keV ( $\frac{5}{2}^-$ ) may be a doublet arising from the weak coupling of the  $p_{1/2}$  proton to the  $2^+$  excited state of the core. In the above model, the  $\frac{3}{2}^-$  final-state wave function is

$$|J, M\rangle = \alpha |p_{3/2}; J = \frac{3}{2}, M\rangle + \beta |p_{1/2}; J_c = 2; J, M'\rangle. \quad (4)$$

$\alpha$  and  $\beta$  are the amplitudes of the pure particle excitation and pure de-Shalit-type excitations. For the Mo<sup>99</sup> ground state, in the lowest-seniority approximation ( $\nu = 1$ ), one assumes

$$|J', M'\rangle = |s_{1/2}; J' = \frac{1}{2}, M'\rangle. \quad (5)$$

Even though the detailed description of the  $\nu = 3$

member of the wave function (4) may be rather complicated, the reduced matrix elements of the 840-keV transition are composed of only two single-particle matrix elements. Following Delabaye, Deutsch, and Lipnik,<sup>31</sup> one writes

$$\langle J = \frac{3}{2} \| T_{KL\gamma} \| J' = \frac{1}{2} \rangle = \alpha a_K \langle p_{3/2} \| T_{KL\gamma} \| s_{1/2} \rangle + \beta' b_K \langle p_{1/2} \| T_{KL\gamma} \| s_{1/2} \rangle,$$

where the operators  $T_{KL\gamma}$  are defined in Chap. II;  $a_K$  and  $b_K$  are decoupling coefficients.<sup>32</sup>  $\beta'$  is the amplitude of the  $|p_{1/2}; [(g_{9/2})^2]_{J_c=2}; J, M\rangle$  proton-excitation component. This "microscopic" description of the core as arising out of a  $(g_{9/2})^2$  configuration is consistent with the work of Vervier,<sup>11</sup> who could successfully explain the levels of Tc<sup>99</sup> on the basis of  $(g_{9/2})^3(p_{1/2})^2$  configuration interaction by taking the  $(g_{9/2})^2$  interaction from the experimental spectrum of Mo<sup>98</sup>.

The ratio of matrix elements of the  $\frac{1}{2}^+ - \frac{3}{2}^-$  transition are given by

$$\frac{x}{u} = -\frac{C_Y}{C_A} \frac{1}{\sqrt{2}} \frac{\langle J = \frac{3}{2} \| T_{110} \| J' = \frac{1}{2} \rangle}{\langle J = \frac{3}{2} \| T_{111} \| J' = \frac{1}{2} \rangle},$$

$$\frac{z}{u} = \sqrt{2} \frac{\langle J = \frac{3}{2} \| T_{211} \| J' = \frac{1}{2} \rangle}{\langle J = \frac{3}{2} \| T_{111} \| J' = \frac{1}{2} \rangle}.$$

Taking the single-particle matrix elements from Rose and Osborn,<sup>23, 33</sup> one obtains

$$x/z = 0.266, \quad u/z = -0.316, \quad (6)$$

independently of  $\alpha$  and  $\beta'$ . Thus the matrix elements obtained in this simple treatment are the same as single-particle matrix elements and hence do not permit a distinction between single-particle and core-excitation mechanisms. The same conclusion was arrived at by Delabaye, Deutsch, and Lipnik<sup>31</sup> for Ag<sup>111</sup> and for stripping reactions.<sup>34</sup> Assuming the CVC relation  $(\int i\vec{\alpha})/\xi \int \vec{r} = 2.42$ , one obtains the theoretical shape factor

$$C_{th} = 1 + 0.049W - 0.0047/W + 0.03W^2. \quad (7)$$

The single-particle matrix elements predict a shape-factor increase of about 10% in the energy

range 450–800 keV and a 840- $\beta$ -370- $\gamma$  correlation anisotropy of the same order. On account of the very weak intensity of this  $\beta$  group the uncertainty in the shape factor is 20%, and an energy dependence of shape factor of 10% cannot be checked quantitatively. At any rate, the experimental results (Fig. 3) do not seem to exclude the above prediction. Very recently, Appalacharyulu, Sastry, and Jnanananda<sup>35</sup> have measured the angular correlation of 840- $\beta$ -370- $\gamma$  cascade, and they find an anisotropy of 2 to 3% with large uncertainty (25%). Both these results are consistent with the  $\xi$  approximation, as well as with the single-particle estimates. Even though the single-particle description of the states can explain the spectrum shape factor and angular-correlation anisotropy, the  $\log ft$  value of 8.5 for this transition is an order of magnitude higher than those given by matrix elements (6). This may be accounted for by performing explicit calculations with a "microscopic" description of the  $2^+$  core-excitation state in terms of single-particle configuration mixing. This implies a seniority-3 description to the initial state also. In this case, the number of adjustable parameters will be large enough to obtain a good fit to all the experimental observables. Thus a pure single-particle description of the 514-keV level ( $\frac{3}{2}^-$ ) cannot account for the high  $\log ft$  value. Our conclusion is consistent with the very detailed work of Vervier,<sup>11</sup> who reproduced the levels of Tc<sup>99</sup> by assuming a configuration interaction of the type  $(\nu d_{5/2})^n (\pi g_{9/2})^m (\pi p_{1/2})^l$  with  $0 \leq l \leq 2$ ,  $0 \leq m \leq 5$ , and  $n = 6$ .

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\*Present address: Department of Nuclear Physics, University of Madras, Madras-25, India.

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## Beta Decay of $\text{Ag}^{111}$

T. Nagarajan,\* M. Ravindranath, and K. Venkata Reddy

*Laboratories for Nuclear Research, Andhra University, Waltair, India*

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A new very weak  $\beta$  group  $414 \pm 10$  (0.9%) keV is revealed in addition to the three  $\beta$  groups with end points  $1035 \pm 2$  (92%), 789 (1.1%), and  $693 \pm 3$  (6.0%). The 414-keV  $\beta$  group feeds the 621-keV level of  $\text{Cd}^{111}$  introduced by Sastry, Lakshminarayana, and Jnanananda. The large shape deviation (17%) reported by Langer *et al.* for the 693-keV  $\beta$  group indicated a breakdown of the  $\xi$  approximation for this group, whereas correlation experiments and theoretical predictions indicated statistical shape. The present measurement  $C(W) = k[1 - (0.01 \pm 0.06)W]$  confirms the latter conclusions and is in quantitative agreement with the shell-model predictions of Delabaye, Deutsch, and Lipnik. The characteristics of  $\frac{1}{2}^- \rightarrow \frac{1}{2}^+$  and  $\frac{1}{2}^- \rightarrow \frac{3}{2}^+$  transitions are just similar to those of  $\text{Mo}^{99}$  and a theoretical fit of the outer  $\beta$  group of  $\text{Ag}^{111}$  yields  $3.2 \leq \Lambda \leq 3.3$  and  $1.2 \leq \Lambda_0 \leq 1.4$ .

### I. INTRODUCTION

Three  $\beta$  groups of the decay of  $\text{Ag}^{111}$  are fairly well established<sup>1,2</sup>; namely, those with end-point energies 1044, 793, and 690 keV. Sastry, Lakshminarayana, and Jnanananda<sup>3</sup> introduced a level at 610 keV fed by an additional  $\beta$  group of very weak intensity. The shape of the 693-keV  $\beta$  branch leading to the second excited state of  $\text{Cd}^{111}$  was first measured by Robinson and Langer<sup>2</sup> with a  $4\pi$  anthracene spectrometer adapted for coincidence. They found a large deviation from linearity of 17% which, along with the large  $\log ft$  value of 7.8, suggests cancellation among the matrix elements contributing to this decay. This indicates a clear

breakdown of the  $\xi$  approximation. The criterion for the applicability of the  $\xi$  approximation,  $\xi \gg W_0$ , is in this case well fulfilled ( $\xi \sim 10$ ,  $W_0 = 2.3$ ), so it would be expected that the  $\xi$  approximation should describe the data to within  $1/\xi \sim 10\%$ . In fact, Hamilton, Pettersson, and Hollander<sup>4</sup> and Seshagiri Rao<sup>5</sup> reported nearly isotropic angular correlation between the 693-keV  $\beta$  ray and the 342-keV  $\gamma$  ray. After this work was completed, Lehman<sup>6</sup> reported statistical shape for 693-keV  $\beta$  branch, in complete contradiction to Robinson and Langer.<sup>2</sup> Recently Delabaye, Deutsch, and Lipnik<sup>7</sup> calculated the matrix elements of this transition, on the basis of the extreme single-particle model, and they found that the results are compatible with the  $\xi$  ap-