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[1]L. Youn, G. Osentinshii, N. Sodnom, A. Govorov, I. Sizov, and V. Salatskii, Zh. Eksperim. i Teor. Fiz. 39, 225 (1960) [transl.: Soviet Phys.–JEPT 12, 163 (1961)].

[2]J. Barry, R. Batchelor, and B. Macefield, in *Proceedings of the Rutherford Jubilee Conference, Manchester, 1961,* edited by J. Birks (Heywood and Company, Ltd., London, England, 1962).

[3]B. Kühn and B. Schlenk, Nucl. Phys. 48, 353 (1963).

[4]D. Smith, N. Jarmie, and A. Lockett, Phys. Rev. 129, 495 (1963).

[5]T. Klopcic and S. E.Darden, in *Proceedings of the Third International Polarization Symposium,* edited by H. H. Barshall and W. Haeberli (University of Wisconsin Press, Madison, Wisconsin).

[6]G. Ohlsen, Nucl. Instr. Methods 37, 240 (1965).

[7]S. Bame and J. Perry, Phys. Rev. 107, 1616 (1957).

[8]A. F. Behof, J. M. Hevezi, and G. Spalek, Nucl. Phys. 84, 290 (1966).

[9]R. Shamu, Nucl. Instr. Methods 14, 297 (1961).

[10]C. Conde and A. Policarpo, Nucl. Instr. Methods 53, 7 (1967).

[11]G. Lietz, S. Trevino, A. Behof, and S. E. Darden, Nucl. Phys. 67, 193 (1965).

[12]T. R. Donoghue, private communication.

[13]W. Büsse, J. Christiansen, D. Hilscher, U. Morfeld, J. Scheer, and W. Schröder, Nucl. Phys. A100, 490 (1967).

[14]G. Morgan, Ph.D. thesis, Duke University, 1967 (unpublished).

[15]C. Hollandsworth, private communication.

[16]M. Ivanovich, P. Young, and G. Ohlsen, Nucl. Phys. A110, 441 (1968).

[17]A. Bacher, R. Spiger, and T. Tombrello, Nucl. Phys. A119, 481 (1968).

[18]Fortran-IV computer code and instructions written by P. D. Kunz, unpublished.

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Muon Capture in Li^6 in the Ditriton Channel*

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Within the context of the impulse approximation we analyze the reaction $\mu^- + Li^6 \to H^3 + H^3 + \nu$ using the current-current theory of weak interactions and an α-d model of Li^6 fitted to the binding energy and charge radius of Li^6. The model is tested by considering the channel $\mu^- + Li^6 \to He^6 + \nu$. Within the limitations of our stated approximations, total rates are estimated for these two modes and the combined triton distribution given for the ditriton channel.

I. INTRODUCTION

We hope to exhibit some of the pertinent details of the capture process

$$\mu^- + Li^6 \to H^3 + H^3 + \nu \tag{1.1}$$

which is a channel interesting in connection with a proposed experiment to measure the muon neutrino mass.[1] Discussion of the experimental aspects of muon capture in the above channel will be reserved for another communication. We present the analysis in a general form to facilitate its application to any of the remaining channels.

Theoretical studies of muon-capture processes in lithium are numerous,[2] though we are unaware of any consideration of the ditriton final state. Basically, an impulse approximation, in which the capture is effected on a single proton, has been coupled to weak-interaction theory to describe muon-capture processes. Recourse is made in this paper to these techniques using cluster models of the relevant nuclei for which pertinent momentum structures are exhibited. More particularly, an α-d model of Li^6 is assumed in which trial wave functions have been fitted to the lithium binding energy and charge radius using variational techniques.[3] The over-all validity of our model within the limitations of our approximations can be estimated roughly by comparison of predicted rates with experimental rates. We chose the He^6-ν channel for comparison, and found fair agreement as seen in Sec. V.

The analysis proceeds in the following fashion. In Sec. II we define the representation and mechanism of the capture process along with the appropriate wave functions for the two-triton final state, while Sec. III presents the parallel development for the He^6-ν channel. Numerical results and conclusions are given in Secs. IV and V.

II. ANALYSIS OF THE H^3-H^3-ν CHANNEL

A. Scattering Representation

In the process $\mu^- + Li^6 \to H^3 + H^3 + \nu$, the impulse scattering in the α-d model of Li^6 occurs as follows:

$$\begin{aligned}\mu^- + He^4 + d &\to (\mu^- + P_1) + H^3 + N_2 + P_2\\ &\to (\nu + N_1) + H^3 + N_2 + P_2\\ &\to \nu + H^3 + H^3\,, \end{aligned} \tag{2.1}$$

and also,

$$\begin{aligned}\mu^- + d + He^4 &\to (\mu^- + P_2) + N_2 + P_1 + H^3\\ &\to (\nu + N_1) + N_2 + P_1 + H^3\\ &\to \nu + H^3 + H^3\,. \end{aligned} \tag{2.2}$$

In Eq. (2.1), capture takes place on the α particle, while in Eq. (2.2) capture is effected by the deuteron. The particles not contained in the brackets are spectators. The five particles indicated in the intermediate state provide a convenient momentum basis for the amplitude, and the remaining two variables, corresponding to the internal degrees of freedom of the spectator H^3, labeled ξ_1, ξ_2, are chosen in configuration space. The weak current-current interaction operator H responsible for the process $\mu^- + P \to \nu + N$ is given by

$$H = \frac{G}{\sqrt{2}} \int d^3x \, [\bar{\psi}_\mu(x) \gamma^\alpha (1-\gamma_5) \psi_\nu(x)]^\dagger [\bar{\psi}_N(x) \gamma_\alpha (1-\lambda\gamma_5) \psi_P(x)]\,, \tag{2.3}$$

where G is the universal Fermi coupling constant; ψ_μ, ψ_ν, ψ_N, ψ_P are appropriate fields for μ^-, ν, N, and P; γ_α, γ^α, γ_5, are the usual Dirac matrices; and λ is the ratio of the axial-vector-to-vector coupling constant, i.e., $\lambda = C_A/C_V$ with $C_V = G$. The matrix element for the capture process takes the form in momentum space

$$\begin{aligned}\langle \nu H^3(p_3) H^3(q_3) | H | Li^6 \mu^- \rangle = \int d^{15}p' d^{15}q' d\xi_1 d\xi_2 \langle \nu H^3 H^3 | p_3' \xi_1 \xi_2 ; p_P' p_1' p_2' ; p_\nu' \rangle \\ \times \langle p_\nu' p_P' p_1' p_2' p_3' | H | q_3' q_2' q_1' q_N' q_\mu' \rangle \langle q_\mu' ; q_1' q_3' \xi_1 \xi_2 ; q_N' q_2' | Li^6 \mu^- \rangle\,. \end{aligned} \tag{2.4}$$

In the above equation $(p_\nu' p_P' p_1' p_2' p_3')$ refer to the Fourier three-momentum components of ν, P_1, N_1, N_2, and H^3 in the final state, while $(q_\mu' q_N' q_1' q_2' q_3')$ refer to the Fourier three-momentum components of μ^-, N_2, P_1, P_2, and H^3 in the initial state.[4] In the state vectors, sets of variables associated with different particles are separated by semicolons. The factored state vectors take the explicit form (where $m_N = m_p = m_1$; $m_j = jm_1$ and m_6 and q_6 refer to Li^6)

$$\langle \nu H^3(p_3) H^3(q_3) | p_3' \xi_1 \xi_2 ; p_P' p_1' p_2' ; p_\nu' \rangle = \delta^3(P_f - P_f') \delta^3(p_\nu - p_\nu') \delta^3(p_3 - p_3') \psi_3(\xi_1, \xi_2) \phi_{N-2}(\tfrac{2}{3}p_1' - \tfrac{1}{3}p_2' - \tfrac{1}{3}p_P') \phi_3(\tfrac{1}{2}p_P' - \tfrac{1}{2}p_2') \tag{2.5}$$

and

$$\begin{aligned}\langle q_\mu' ; q_1' q_3' \xi_1 \xi_2 ; q_N' q_2' | Li^6 \mu^- \rangle = \delta^3(P_i - P_i') \psi_{\mu-\mathrm{Li}} \left(q_\mu' - \frac{m_\mu}{m_\mu + m_6} P_i \right) \psi_{N-3}(\tfrac{3}{4}q_1' - \tfrac{1}{4}q_3') \\ \times \psi_{4-d}(\tfrac{2}{3}q_N' + \tfrac{2}{3}q_2' - \tfrac{1}{3}q_3' - \tfrac{1}{3}q_1') \psi_4(\xi_1, \xi_2) \psi_2(\tfrac{1}{2}q_N' - \tfrac{1}{2}q_2') \end{aligned} \tag{2.6}$$

for

$$\begin{aligned} P_i &= q_\mu + q_6\,, & P_i' &= q_\mu' + q_1' + q_2' + q_N' + q_3'\,; \\ P_f &= p_3 + q_3 + p_\nu\,, & P_f' &= p_\nu' + p_1' + p_2' + p_N' + p_3' \end{aligned} \tag{2.7}$$

and the spectator triton carrying momentum p_3 in the final state. The triton momenta p_3, q_3 are not individually identifiable in the laboratory and consequently the observed triton distribution will be the superposition of the two. Gaussian distributions are assumed for the internal states of the final tritons and the initial α particles,[5] so that

$$\psi_3(\xi_1, \xi_2) = N_3 \exp[-\beta^2(\tfrac{1}{3}\xi_2{}^2 + \tfrac{1}{4}\xi_1{}^2)]\,,$$
$$\psi_4(\xi_1, \xi_2) = N_4 \exp[-\alpha^2(\tfrac{1}{3}\xi_2{}^2 + \tfrac{1}{4}\xi_1{}^2)]\,, \tag{2.8}$$

while in the α-d model of Li^6,

$$\psi_{4-d}(q) = (2\pi)^{-3/2} F_4(c^{-3}e^{-q^2/4c^2} + y d^{-3} e^{-q^2/4d^2})\,,$$
$$\psi_2(q) = (2\pi)^{-3/2} N_2(a^{-3}e^{-q^2/4a^2} + z b^{-3}e^{-q^2/4b^2})\,, \tag{2.9}$$

where ψ_{4-d} represents the interaction of the center of mass of the α particle with the center of mass of the deuteron and a, b, c, d, y, z, N_3, N_4, F_4, and N_2 are constants.[6] The remaining wave functions representing the P_1-H^3, μ^--Li^6, N_1-(N_2+P_j), N_2-P_j $(j=1, 2)$ relative interactions are given by

$$\psi_{N-3}(q) = (2\pi)^{-3/2} F_3 e^{-2q^2/3\alpha^2}\,,$$
$$\psi_{\mu-\mathrm{Li}}(q) = (2\pi)^{-3/2} N_{10}\delta^3(q)\,,$$
$$\phi_{N-2}(q) = (2\pi)^{-3/2} F_2 e^{-3q^2/4\beta^2}\,, \tag{2.10}$$
$$\phi_3(q) = (2\pi)^{-3/2} Q_3 e^{-q^2/\beta^2}\,,$$

where α, β, F_3, N_{10}, F_2, and Q_3 are constants[6] and ψ_{N-3}, ϕ_{N-2}, ϕ_3 are obtained from the Gaussian triton wave functions and $\psi_{\mu-\mathrm{Li}}$ from the (1S) μ-atomic Coulomb wave functions by Fourier transformation. The normalization factors and constants appear in Ref. 6.

B. Weak Interactions and Impulse Scattering

In the impulse model[7] only one scattering center at a time effects transitions in the system and consequently the states of the spectators are changed. Applied to the matrix elements this means that for capture from the α particle

$$\langle p_\nu' p_P' p_1' p_2' p_3' | H | q_3' q_2' q_1' q_N' q_\mu' \rangle = \delta^3(q_N' - p_2')\delta^3(q_2' - p_P')\delta^3(q_3' - p_3')\langle p_1' p_\nu' | H | q_\mu' q_1' \rangle\,, \tag{2.11}$$

and, for capture from the deuteron,

$$\langle p_\nu' p_P' p_1' p_2' p_3' | H | q_3' q_2' q_1' q_N' q_\mu' \rangle = \delta^3(q_N' - p_2')\delta^3(q_1' - p_P')\delta^3(q_3' - p_3')\langle p_1' p_\nu' | H | q_\mu' q_2' \rangle\,. \tag{2.12}$$

Contracting the ν, N_1, μ^-, and P_j and using the commutation relationships for appropriate creation and annihilation operators, there results[8] $(j=1, 2)$,

$$\langle p_1' p_\nu' | H | q_\mu' q_j' \rangle = (2\pi)^{-3}\frac{G}{\sqrt{2}}\bar{u}_\nu(p_\nu')\gamma^\alpha(1-\gamma_5)u_\mu(q_\mu')\bar{u}_N(p_1')\gamma_\alpha(1-\lambda\gamma_5)u_P(q_j')\left(\frac{m_\nu m_N m_P m_\mu}{\epsilon_\nu \epsilon_N \epsilon_P \epsilon_\mu}\right)^{1/2}\delta^3(p_\nu' + p_1' - q_\mu' - q_j')\,, \tag{2.13}$$

where u_σ are Dirac spinors for particles σ with the spin indices suppressed. Performing the intermediate integrations in Eq. (2.4) by putting $\vec{q}\,' = \vec{p}_1' = 0$ in the spinors and neglecting angular dependence,[9] and after squaring, summing, and replacing spinor sums with their momentum projection operators,

$$\Gamma = (2\pi)^{-8}\int d^3p_\nu\, d^3p_3 d^3q_3 (2\epsilon_\nu)^{-1} K^2(p_3, q_3, p_\nu)\delta^4(P_i - p_3 - q_3 - p_\nu)\frac{G^2}{2} L^\dagger L\,, \tag{2.14}$$

where, for capture at rest ($\vec{P}_i = 0$) in the low-energy limit, $m_N m_P/\epsilon_N \epsilon_P \sim 1$,

$$L^\dagger L = 2(m_\mu m_N m_P)^{-1}\,\mathrm{Tr}\,\gamma^\beta(1-\gamma_5)\not{p}_\nu\gamma^\alpha(1-\gamma_5)(\not{k}_\mu + m_\mu)\,\mathrm{Tr}\,\gamma_\beta(1-\lambda\gamma_5)(\not{k}_N + m_N)\gamma_\alpha(1-\lambda\gamma_5)(\not{k}_P + m_P)\,, \tag{2.15}$$

we find for the coherent form factor

$$K(p_3, q_3, p_\nu) = 3\left(\frac{\pi}{\alpha^2 + \beta^2}\right)^3 N_{10} N_3{}^2 N_4 N_2 F_2 F_3 F_4 [2K_\alpha(p_3, q_3, p_\nu) + K_d(p_3, q_3, p_\nu)]\,, \tag{2.16}$$

where the α-particle and deuteron nuclear form factors are given by

$$K_\alpha(p_3, q_3, p_\nu) = \left[\left(\frac{4}{\alpha^2+\beta^2}\right)^{3/2} + z\left(\frac{2}{b^2+\beta^2}\right)^{3/2}\right] \exp\left(-\frac{q_3^2}{3\beta^2} - \frac{(3q_3+3p_\nu-p_3)^2}{24\alpha^2}\right)$$
$$\times\left[\left(\frac{4c^{-2}}{3\beta^{-2}+2\alpha^{-2}+c^{-2}}\right)^{3/2} + y\left(\frac{4d^{-2}}{3\beta^{-2}+2\alpha^{-2}+d^{-2}}\right)^{3/2}\right],$$

$$K_d(p_3, q_3, p_\nu) = \beta^{-3}\left[\left(\frac{4a^{-2}}{4\beta^{-2}+a^{-2}}\right)^{3/2} \exp\left(-\frac{p_\nu^2}{16a^2}\right) + z\left(\frac{4b^{-2}}{4\beta^{-2}+b^{-2}}\right)\exp\left(-\frac{p_\nu^2}{16b^2}\right)\right]$$
$$\times\left[\left(\frac{12c^{-2}}{8\alpha^{-2}+9\beta^{-2}+3c^{-2}}\right)^{3/2}\exp\left(-\frac{p_3^2}{4c^2}\right) + y\left(\frac{12d^{-2}}{8\alpha^{-2}+9\beta^{-2}+3d^{-2}}\right)^{3/2}\exp\left(-\frac{p_3^2}{4d^2}\right)\right]\exp\left(-\frac{q_3^2}{12\beta^2} - \frac{p_3^2}{24\alpha^2}\right). \tag{2.17}$$

In the laboratory frame,

$$P_i = (m_\mu + m_6, 0) = (\epsilon_i, 0),$$

$$h_\mu = (m_\mu, 0), \quad h_N = (q_3^0 - m_2, \vec{q}_3), \quad h_P = (q_3^0 + \epsilon_\nu - m_2, \vec{q}_3 + \vec{p}_\nu), \quad q_3 = (q_3^0, \vec{q}_3), \quad p_3 = (p_3^0, \vec{p}_3). \tag{2.18}$$

Evaluation of Eq. (2.15) yields

$$L^\dagger L = 8(m_N m_P m_\mu)^{-1}[(1-\lambda)^2 S_1 S_2 + (1+\lambda)^2 S_3 S_4 - (1+\lambda^2) S_5 m_N m_P] \tag{2.19}$$

with the definitions

$$S_1 = h_P \cdot p_\nu, \quad S_2 = h_\mu \cdot h_N, \quad S_3 = h_P \cdot h_\mu, \quad S_4 = h_N \cdot p_\nu, \quad S_5 = h_\mu \cdot p_\nu. \tag{2.20}$$

Evaluation of Eq. (2.14) follows in Sec. IV.

III. ANALYSIS OF THE He^6-ν CHANNEL

The process

$$\mu^- + Li^6 \to \nu + He^6 \tag{3.1}$$

can be envisioned in much the same was as the ditriton channel. Analogous to Eq. (2.4), we write

$$\langle \nu He^6 | H | Li^6 \mu^- \rangle = \int d^{15}q' d^{15}p' d\xi_1 d\xi_2 \langle \nu He^6 | p_3' p_P' p_1' p_2' \xi_1 \xi_2; p_\nu' \rangle \langle p_\nu' p_2' p_1' p_P' p_3' | H | q_3' q_N' q_1' q_2' q_\mu' \rangle \langle q_\mu'; q_1' q_3' \xi_1 \xi_2; q_N' q_2' | Li^6 \mu^- \rangle \tag{3.2}$$

with the initial state given in Eq. (2.6) and

$$\langle \nu He^6 | p_3' p_P' p_1' p_2' \xi_1 \xi_2; p_\nu' \rangle = \delta^3(P_f - P_f')\delta^3(p_\nu - p_\nu')\phi_6(\xi_1, \xi_2)\phi_{N-3}(\tfrac{3}{4}p_1' - \tfrac{1}{4}p_3')\phi_{4-d}(\tfrac{2}{3}p_P' + \tfrac{2}{3}p_2' - \tfrac{1}{3}p_3' - \tfrac{1}{3}p_1')\chi_2(\tfrac{1}{2}p_P' - \tfrac{1}{2}p_2'), \tag{3.3}$$

where the following wave functions describe the α-d, P_j-H^3, P_j-N_2 and He^6 intrinsic states ($j = 1, 2$):

$$\begin{aligned} \phi_{4-d}(q) &= (2\pi)^{-3/2} G_4 e^{-3q^2/8\gamma^2}, \\ \phi_{N-3}(q) &= (2\pi)^{-3/2} G_3 e^{-2q^2/3\gamma^2}, \\ \chi_2(q) &= (2\pi)^{-3/2} G_2 e^{-q^2/\gamma^2}, \\ \phi_6(\xi_1, \xi_2) &= N_6 \exp[-\gamma^2(\tfrac{1}{3}\xi_2^2 + \tfrac{1}{4}\xi_1^2)], \end{aligned} \tag{3.4}$$

with γ, G_4, G_3, G_2, and N_6 constants.[6] Performing the identical steps leading to Eq. (2.14) gives

$$\Gamma = (2\pi)^{-8}\int d^3p_\nu d^3p_6 (2\epsilon_\nu)^{-1} J^2(p_\nu)\delta^4(P_f - p_6 - p_\nu)\frac{G^2}{2} L^\dagger L, \tag{3.5}$$

where

$$L^\dagger L = 2(m_N m_P m_\mu)^{-1}[(1-\lambda)^2 t_1 t_2 + (1+\lambda)^2 t_3 t_4 - (1+\lambda^2) t_5 m_N m_P], \tag{3.6}$$

$$J(p_\nu) = 24\left(\frac{\pi}{\gamma^2+\alpha^2}\right)^3 N_4 N_6 N_2 G_4 G_3 N_{10} F_3 F_4 [2J_\alpha(p_\nu) + J_\alpha(p_\nu)], \tag{3.7}$$

and the α-particle and deuteron nuclear form factors are now given by

$$J_\alpha(p_\nu)=\left(\frac{3}{\alpha^2+\gamma^2}\right)^{3/2}\left[\left(\frac{8c^{-2}}{3\gamma^{-2}+2c^{-2}}\right)^{3/2}+y\left(\frac{8d^{-2}}{3\gamma^{-2}+2d^{-2}}\right)^{3/2}\right]\left[\left(\frac{1}{a^2+\gamma^2}\right)^{3/2}+z\left(\frac{1}{b^2+\gamma^2}\right)^{3/2}\right]\exp\left(-\frac{3p_\nu^2}{8\gamma^2}\right),$$

$$J_d(p_\nu)=\left(\frac{3\alpha^2}{8\gamma^2}\right)^{3/2}\left[\left(\frac{a^{-2}}{a^{-2}+4\gamma^{-2}}\right)^{3/2}+z\left(\frac{b^{-2}}{b^{-2}+4\gamma^{-2}}\right)^{3/2}\right]\left[\left(\frac{2c^{-2}}{\gamma^{-2}+2c^{-2}}\right)^{3/2}+y\left(\frac{2d^{-2}}{\gamma^{-2}+2d^{-2}}\right)^{3/2}\right]\exp\left(-\frac{11p_\nu^2}{36\gamma^2}\right). \quad (3.8)$$

Furthermore,

$$t_1=j_P\cdot p_\nu,\quad t_2=j_\mu\cdot j_N,\quad t_3=j_P\cdot j_\mu,\quad t_4=p_\nu\cdot j_N,\quad t_5=p_\nu\cdot j_\mu, \quad (3.9)$$

where

$$j_\mu=(m_\mu,0),\quad j_N=(\epsilon_6-m_6+m_\mu+m_N,\vec{p}_6),\quad j_P=(m_P,0),\quad p_\nu=(\epsilon_\nu,\vec{p}_\nu),\quad \epsilon_6^2=\vec{p}_6^{\,2}+\mu_6^2, \quad (3.10)$$

and μ_6, $\vec{p}_6$ designate the mass and momentum of He^6.

IV. RESULTS

A. $\mu^-+Li^6\to H^3+H^3+\nu$

Integrating Eq. (2.18), we obtain in the limit $m_\nu\to 0$,[10]

$$\Gamma=(2\pi)^{-6}\int d(\cos\theta)p_3^2dp_3q_3^2dq_3 \times\frac{G^2}{\epsilon_\nu m_N m_P m_\mu}K^2(p_3,q_3)\delta(\epsilon_\nu+q_3^0+p_3^0-\epsilon_i) \times[(1-\lambda)^2S_1S_2+(1+\lambda)^2S_3S_4-(1+\lambda^2)m_Nm_PS_5], \quad (4.1)$$

where $\epsilon_\nu=(\vec{q}_3^{\,2}+\vec{p}_3^{\,2}+2\vec{q}_3\cdot\vec{p}_3)^{1/2}$, $\hat{q}_3\cdot\hat{p}_3=\cos\theta$ and

$$S_1=\epsilon_\nu(q_3^0-m_2)+\vec{q}_3^{\,2}+\vec{q}_3\cdot\vec{p}_3,\quad S_2=m_\mu(q_3^0-m_2),\quad S_3=m_\mu(q_3^0+\epsilon_\nu-m_2),\quad S_4=\epsilon_\nu(q_3^0-m_2)+\vec{q}_3^{\,2}+\vec{q}_3\cdot\vec{p}_3,\quad S_5=\epsilon_\nu m_\mu. \quad (4.2)$$

From Eq. (4.1), we obtain for $\lambda=1.18$

$$\Gamma=6.401\times10^2\ \mathrm{sec}^{-1}. \quad (4.3)$$

Before we may take this to be a number relevant in the laboratory, it is necessary to point out that the effects of Pauli exclusion have not been included. We estimate that an over-all factor of $\frac{1}{4}$ is required to account for this effect, based on the analysis of Primakoff.[11] The estimate which we obtain, therefore, is

$$\Gamma(Li^6\to H^3+H^3)=1.60\times10^2\ \mathrm{sec}^{-1}, \quad (4.4)$$

neglecting final-state interactions of the tritons. We are pursuing this question and hope to include this effect through parametrization of the t-t interaction as a scattering length. This final-state interaction could further inhibit this channel. With these caveats, the sum of the "capture" and "spectator" triton kinetic energy (T) distributions is given in Fig. 1, and is obtained from Eq. (4.1):

$$\frac{d\Gamma}{dT}=\frac{m_3}{p_3}\frac{d\Gamma}{dp_3}+\frac{m_3}{q_3}\frac{d\Gamma}{dq_3}. \quad (4.5)$$

The curve has been arbitrarily normalized in the figure. As mentioned in Sec. I, the experimental significance of these quantities will be discussed elsewhere. Before the experiment is performed, however, the relative validity of the analysis can be gauged by comparing the calculated and experimental rates for the He^6-ν channel.

B. $\mu^-+Li^6\to He^6+\nu$

From Eq. (3.9) there results, in the same $m_\nu\to 0$ limit,

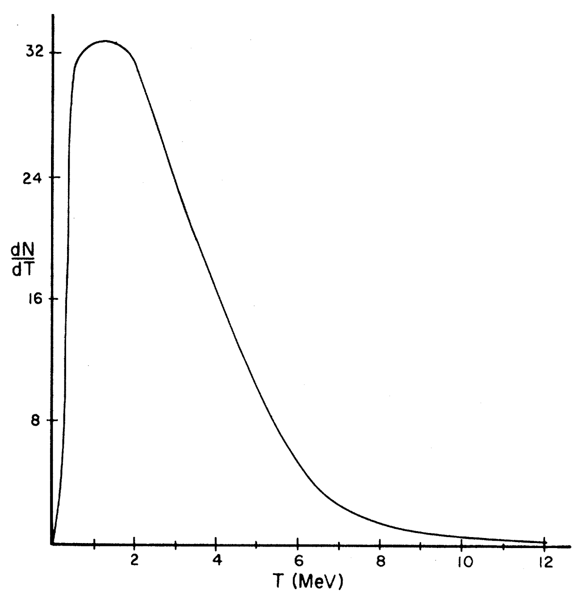

FIG. 1. Triton kinetic energy distribution.

$$\Gamma = (2\pi)^{-7}\left(\frac{\epsilon_0\mu_6}{\epsilon_0+\mu_6}\right)\frac{G^2}{m_N m_P m_\mu}J^2(\epsilon_0) \times[(1-\lambda)^2 t_1 t_2 + (1+\lambda)^2 t_3 t_4 - (1+\lambda)^2 m_N m_P t_5] \tag{4.6}$$

with the neutrino energy

$$\epsilon_0 = \mu_6\left\{\left[\frac{\mu_6 + 2(m_\mu + m_6 - \mu_6)}{\mu_6}\right]^{1/2} - 1\right\} \tag{4.7}$$

and,

$$\begin{aligned} t_1 &= m_N\epsilon_0\,,\\ t_2 &= m_\mu(\epsilon_6 - m_6 + m_\mu + m_N)\,,\\ t_3 &= m_\mu m_N\,,\\ t_4 &= \epsilon_0(\epsilon_6 - m_6 + m_\mu + m_N) + \epsilon_\nu^{\,2}\,,\\ t_5 &= m_\mu\,\epsilon_0\,. \end{aligned} \tag{4.8}$$

The impulse capture from the deuteron is quite significant for this channel, and we obtain from Eq. (4.6)

$$\Gamma = 5.92\times10^3\ \text{sec}^{-1}\,. \tag{4.9}$$

Estimating the suppression due to Pauli exclusion in Li^6 as before gives

$$\Gamma(Li^6 \to He^6) \cong 1.48\times10^3\ \text{sec}^{-1}\,, \tag{4.10}$$

which can be compared with the experimental value[12]

$$\Gamma_{exp}(Li^6 \to He^6) = 1.60^{+0.33}_{-0.13}\times 10^3\ \text{sec}^{-1}\,. \tag{4.11}$$

V. CONCLUSIONS

We have estimated within the context of the impulse approximation the total rate and differential triton energy distribution for the capture from rest to the ditriton channel

$$\mu^- + Li^6 \to \nu + H^3 + H^3 \tag{5.1}$$

using the α-d model of Li^6 with Gaussian wave functions. As a check on the model and our calculational procedure, we have investigated the channel.

$$\mu^- + Li^6 \to \nu + He^6\,. \tag{5.2}$$

The total rate for this reaction as calculated agrees roughly with the experimental number and we are encouraged to believe our estimate of the total rate for the ditriton capture channel *within the limitations of our approximations*.

Refinements in the calculation can be effected in various ways. We plan to include the exact calculation of the suppressive effect of Pauli exclusion by complete antisymmetrization of the initial- and final-state wave functions. We are also investigating the effect of using other wave functions, since the Gaussian wave function used may significantly underestimate the high-momentum components which affect the occurrence of high-energy triton events. The effects of final-state interactions between the tritons, as mentioned earlier, will be included by means of a t-t scattering length.

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[1]Proposal to the Atomic Energy Comission for support of a program in Medium Energy Physics at Northwestern University, Evanston, Illinois, April 6, 1970. Private correspondence from one of us (S.L.M.) to L. Rosen and P. Gram of Los Alamos. Preliminary Investigation of a Measurement of the Mass of the Muon Neutrino (Space Radiation Effects Laboratory proposal submitted March 25, 1970).

[2]Some useful references are given in the following list, which is not exhaustive: H. Primakoff, Phys. Rev. 31, 802 (1959); C. W. Kim and H. Primakoff, Phys. Rev. 140B, 566 (1965); A. Galindo and P. Pascual, Nucl. Phys. B14, 37 (1969); C. W. Kim and H. Primakoff, Phys. Rev. 139B, 1447 (1965); C. W. Kim and M. Ram, Phys. Rev. D. 1, 2651 (1970); A. Lodder and C. C. Jonker, Phys. Letters 15, 245 (1965).

[3]S. Fantoni and S. Rosati, to be published. The authors fit a trial wave function of the form

$$\psi(\rho,\zeta) = (e^{-a^2\zeta^2} + ze^{-b^2\zeta^2})(e^{-c^2\rho^2} + ye^{-d^2\rho^2})\,,$$

where ρ represents the separation of α, ζ the separation of N from P, and a, b, c, d, y, and z are constants, to the lithium binding energy of He^4, $E_b = E_{Li^6} - E_{He^4} = -3.69$ MeV, and the charge radius $r_c = 2.54$ F. Their fit gives $E_b = -3.03$ and $r_c = 2.25$ F.

[4]The states are normalized $\langle k|k'\rangle = \delta^3(k-k')$.

[5]See B. R. Wienke, Phys. Rev. D 1, 2541 (1970) for the manipulations leading to explicit representation of the cluster wave functions.

[6]All factors and constants appearing in Secs. II and III are listed here:

$$N_2=\pi^{-3/4}[(2a)^{-3}+2z(a^2+b^2)^{-3/2}+z^2(2b)^{-3}]^{-1/2}\pi^{3/2},$$

$$N_3=\left(\frac{\beta^2}{\sqrt{3}\,\pi}\right)^{3/2},\quad N_4=\left(\frac{\alpha^3}{2\pi^{3/2}}\right)^{3/2},\quad N_6=\left(\frac{\gamma^5}{\sqrt{6}\,\pi^{5/2}}\right)^{3/2},$$

$$N_{10}=\left(\frac{27}{\pi q_0{}^3}\right)^{1/2}(2\pi)^3,\quad Q_3=\left(\frac{4\pi}{\beta^2}\right)^{3/2}N_3,$$

$$F_2=\left(\frac{3\pi}{\beta^2}\right)^{3/2},\quad F_3=\left(\frac{8\pi}{3\alpha^2}\right)^{3/2},$$

$$F_4=\pi^{-3/4}[(2c)^{-3}+2y(c^2+d^2)^{-3/2}+y^2(2d)^{-3}]^{-1/2}\pi^{3/2},$$

$$G_4=\left(\frac{3\pi}{2\gamma^2}\right)^{3/2},\quad G_3=\left(\frac{8\pi}{3\gamma^2}\right)^{3/2};$$

$a=60$ MeV, $a_0=0.84\times10^{-13}$ m, $\alpha=146$ MeV,
$b=128$ MeV, $z=1.03$, $\beta=155$ MeV,
$c=56$ MeV, $y=-2.42$, $\gamma=128$ MeV.
$d=156$ MeV,

Furthermore, we use

$$\lambda=1.18,\quad Gm_P{}^2=1.02\times10^{-5}.$$

[7]For a discussion of the impulse model, see G. F. Chew and G. C. Wick, Phys. Rev. 85, 636 (1952); M. Gell-Mann and M. L. Goldberger, Phys. Rev. 91, 398 (1953); G. F. Chew and M. L. Goldberger, Phys. Rev. 87, 788 (1952); see also Ref. 5.

[8]The fields $\psi(x)$ are expanded in plane waves via

$$\psi(x)=(2\pi)^{-3/2}\int d^3k\left(\frac{m}{\epsilon}\right)^{1/2}\sum_{r=1,2}a^{(r)}(k)u^{(r)}(k)e^{ik\cdot x}+b^{(r)}(k)v^{(r)}(k)e^{ik\cdot x},$$

where r refers to the spin indices, $u(k)$ and $v(k)$ are Dirac spinors for particle and antiparticle, and the creation and annihilation operators $a^\dagger, b^\dagger$ and a,b satisfy the anticommunication relations

$$\{a^{(r)}(k),a^{(s)}(q)\}=\delta_{rs}\delta^3(k-q),$$
$$\{b^{(r)}(k),b^{(s)\dagger}(q)\}=\delta_{rs}\delta^3(k-q),$$
$$\{b^{(r)}(k),a^{(s)}(q)\}=\{b^{(r)\dagger}(k),a^{(s)}(q)\}=0,$$
$$\{b^{(r)}(k),a^{(s)}(q)\}=\{b^{(r)\dagger}(k),a^{(s)}(q)\}=0.$$

The operators $a,a^\dagger$ refer to particles, and $b,b^\dagger$ to antiparticles, and $\vec{k}^2+m^2=\epsilon^2$. The text supresses the spin indices r,s for simplicity.

[9]In the integration, we let

$$\int e^{-(k+p)^2}d^3k\to 4\pi e^{-p^2}\int e^{-k^2}k^2dk$$

as a good approximation. Putting $\vec{q}_1'=\vec{p}_1'=0$ in the spinors allows us to integrate over the momentum wave functions directly.

[10]The integration is easily done in the limit $m_\nu\to 0$. Since for capture at rest,

$$|\vec{p}_\nu|=|\vec{p}_3+\vec{q}_3|=(\vec{p}_3{}^2+\vec{q}_3{}^2+2|p_3||q_3|\cos\theta)^{1/2}=\epsilon_\nu.$$

The integration over the energy δ function is simply accomplished through the transformation.

[11]H. Primakoff, Phys. Rev. 31, 802 (1959).

[12]J. P. Deutsche, L. Grenacs, P. Igo-Kemenes, P. Lipnik, and P. C. Macq, Phys. Letters 26B, 315 (1968).

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Yield Curves for Li + Li and Li + Be Nuclear Reactions*

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Differential cross sections for $Li^{6,7}+Li^{6,7}$ and $Li^{6,7}+Be^9$ nuclear reactions leading to low-lying residual-nucleus states have been measured over the energy range from 4–14 MeV. In the $Li^7(Li^7, \alpha)Be^{10}(3.37)$ case, the energy range was extended to 2.1 and 21 MeV. Striking enhancement and weakening of cross sections for formation of the 0.72-MeV state in B^{10} can be attributed to cluster structure of lithium nuclei. Resonance-like peaks are observed in some yields. Their general character is consistent with an extended-structure interpretation of the reaction mechanism.

I. INTRODUCTION

Lithium-induced nuclear reactions could yield useful information about nuclear structure if the reaction mechanisms were well understood.[1] Unfortunately, this is not the case. At energies well below and well above the Coulomb barrier these reactions have been interpreted as primarily direct reactions.[2,3] Near the barrier the situation is unclear. An example of this is afforded by the results of Synder and Waggoner for the Li^7+Be^9 reaction near 6 MeV.[4] Integrated cross sections for nearly all reaction channels studied showed a close proportionality to $2J+1$, where J is the spin of a particular residual state; this is in agreement with a compound-nucleus mechanism.[5] However, cross sections were obtained from angular distributions which showed strong asym-