Physik 226, 364 (1969).

[5] K. A. Brueckner, J. R. Buchler, and M. M. Kelly, Phys. Rev. 173, 944 (1968).

[6] S. A. Afzal, A. A. Z. Ahmad, and S. Ali, Rev. Mod. Phys. 41, 247 (1969), and references therein.

[7] G. C. Morrison, H. T. Fortune, and R. H. Siemssen, in *Proceedings of the International Conference on Nuclear Reactions Induced by Heavy Ions, Heidelberg, Germany, 15-18 July 1969,* edited by R. Bock and W. R. Hering (North-Holland Publishing Company, Amsterdam, The Netherlands, 1970), p. 72.

[8] I. Dormehl, G. Gruber, K. Meier-Ewert, and K. Bethge, Jahresbericht 1969, Universität Heidelberg, p. 8.

[9] K. Nagatani, D. P. Boyd, P. F. Donovan, E. Beardsworth, and P. A. Assimakopoulos, Phys. Rev. Letters 24, 675 (1970).

[10] R. H. Siemssen, H. T. Fortune, J. W. Tippie, and J. L. Yntema, in *Proceedings of the International Conference on Nuclear Reactions Induced by Heavy Ions* (see Ref. 7), p. 174.

[11] J. L. Yntema, U. S. Atomic Energy Agency Report No. CONF-690301, 1969 (unpublished), pp. 321-325.

[12] S. Zawadski, unpublished.

[13] E. H. Auerbach, Brookhaven National Laboratory Report No. BNL-6562 (unpublished).

[14] J. V. Maher, R. H. Siemssen, M. W. Sachs, A. Weidinger, and D. A. Bromley, in *Proceedings of the International Conference on Nuclear Reactions Induced by Heavy Ions* (see Ref. 7), p. 60.

[15] R. H. Siemssen, H. T. Fortune, A. Richter, and J. W. Tippie, to be published.

[16] J. A. McIntyre, K. H. Wang, and L. C. Becker, Phys. Rev. 117, 1337 (1960).

[17] H. E. Conzett, A. Isoya, and E. Hadjimichael, in *Proceedings of the Third Conference on Reactions between Complex Nuclei,* edited by A. Ghiorso, R. M. Diamond, and H. E. Conzett (University of California Press, Los Angeles, California, 1963), p. 26.

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Study of p + He^3 and n + H^3 Systems with the Resonating-Group Method*

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The p + He^3 and n + H^3 systems are considered with the resonating-group method in the one-channel approximation. A purely central nucleon-nucleon potential which has different ranges in the singlet and triplet states is employed. The internal wave function of the three-nucleon cluster is a sum of two Gaussian functions, with its parameters adjusted to reproduce quite well the properties of the cluster. From the results obtained it is found that the agreement with experiment is quite satisfactory, although in the very low-energy region, detailed fit is not obtained, which is most probably due to the omission of the specific distortion effect in our calculation. An effective interaction between the clusters is also constructed. From this effective interaction it is concluded that the requirement of antisymmetry for the total wave function is very important. In particular, it creates an odd-even feature, wherein the effective potentials in the odd-l and even-l states are quite different.

I. INTRODUCTION

In two previous investigations[1, 2] single-channel resonating-group calculations have been performed using a nucleon-nucleon potential which has the feature of having different ranges in the singlet and triplet states and which yields a very good fit to the low-energy two-nucleon scattering data. These calculations were made on the $\alpha+\alpha$[1] and $\alpha+N$[2] scattering systems, where the low compressibility of the α particle and the high reaction thresholds provided us with an excellent opportunity to employ the one-channel approximation over a wide energy range. Excellent agreement with experiment was obtained in these calculations. In this study we will use the same central potential to examine the p+He^3 and n+H^3 systems,[3] as another step in our continual effort to study the few-nucleon problems with the resonating-group method. Here, however, it is expected that the results will be somewhat worse than those obtained in the $\alpha+\alpha$ and $\alpha+N$ cases; this is so, since the He^3 or the H^3 cluster is not as incompressible as the α cluster, and hence the use of a one-channel approximation, with the subsequent omission of the specific distortion effect, will result in some lack of detailed agreement with the experimental data especially in the very low-energy region.

In previous resonating-group calculations, the wave function used to describe the He^3 or the H^3 cluster has invariably been assumed to consist of a single-Gaussian function.[4-6] With such a wave function, even though it is possible to get a correct value for the rms radius of the nucleon distribution, the calculated binding energy is too low and the body-form-factor data are not described too well. In this calculation we shall therefore use an improved wave function for the three-nucleon cluster, which consists of a sum of two Gaussian functions with three adjustable parameters. These latter parameters are then chosen to yield satisfactory values not only for the rms radius but also for the binding energy and the body form factors.

As in the $\alpha+N$ case,[2] we shall also construct an effective local interaction between the proton and the He^3 cluster by using the wave function obtained from the resonating-group calculation. Here again, the main purpose is to assess the importance of the antisymmetrization procedure and to study the features of this effective interaction in the hope that we can then use the information gained to construct effective potentials in heavier systems where a straightforward resonating-group formulation is impractical.

In the next section, a brief formulation of the problem will be given, together with a discussion of the internal wave function for the three-nucleon cluster, and the various approximations made in the calculation. In Sec. III we present the results for the phase shifts and differential cross sections as well as a description of the effective local interaction between the clusters. Finally, in Sec. IV, concluding remarks are made.

II. FORMULATION

In the one-channel approximation, the wave function for the $p+\text{He}^3$ or the $n+\text{H}^3$ system is given by

$$\Psi_s = \mathcal{A}\,[\phi_{\text{CL}}\,\xi_s(\sigma,\tau)\,F_s(\vec{R}_N-\vec{R}_{\text{CL}})]\,, \tag{1}$$

where $\mathcal{A}$ is an antisymmetrization operator and $\xi_s(\sigma,\tau)$ is an appropriate charge-spin function with the subscript s denoting the total spin of the system, which can be either 0 or 1. The function ϕ_{CL} describes the spatial behavior of the three-nucleon cluster (either He^3 or H^3); it is assumed to be of the form

$$\phi_{\text{CL}} = \exp[-\tfrac{1}{2}\alpha_1 \sum_{i=1}^{3} (\vec{r}_i-\vec{R}_{\text{CL}})^2] + c\exp[-\tfrac{1}{2}\alpha_2 \sum_{i=1}^{3} (\vec{r}_i-\vec{R}_{\text{CL}})^2]\,, \tag{2}$$

with $\vec{R}_{\text{CL}}$ being the position vector of the center of mass of the three-nucleon cluster. The parameters α_1, α_2, and c are chosen to yield a good fit to the experimental binding-energy and form-factor data; they are chosen as

$$\begin{aligned}\alpha_1 &= 0.25\ \text{F}^{-2}\,,\\ \alpha_2 &= 0.71\ \text{F}^{-2}\,,\\ c &= 3.17\,.\end{aligned} \tag{3}$$

In previous resonating-group calculations involving a He^3 or a H^3 cluster,[5, 6] a single-Gaussian function

$$\phi'_{\text{CL}} = \exp[-\tfrac{1}{2}\alpha \sum_{i=1}^{3} (\vec{r}_i-\vec{R}_{\text{CL}})^2] \tag{4}$$

has been used for the spatial part of the three-nucleon wave function, with α chosen as $0.36\ \text{F}^{-2}$. In this study, we shall compute the differential cross sections with both the function ϕ_{CL} and the function ϕ'_{CL}. The purpose is, of course, to see whether an improvement in the cluster wave function can lead to a significant change in the scattering results. If this should turn out to be the case, then all these previous calculations involving three-nucleon clusters would have to be reexamined with the two-Gaussian wave function ϕ_{CL} given in Eqs. (2) and (3).

The scattering function $F_s(\vec{R}_N-\vec{R}_{\text{CL}})$, with $\vec{R}_N$ denoting the position vector of the incident nucleon, describes the relative motion of the two clusters and is determined from the variational equation

$$\langle \delta\Psi_s | H - E' | \Psi_s \rangle = 0\,, \tag{5}$$

where E' is the total energy composed of the internal energy of the three-nucleon cluster and the relative energy E in the c.m. system, and H is the Hamiltonian given by

$$H = -\frac{\hbar^2}{2M}\sum_{i=1}^{4}\nabla_i^2 + \sum_{i>j=1}^{4} V_{ij}\,. \tag{6}$$

The nucleon-nucleon potential V_{ij} is chosen as

$$V_{ij} = [\tfrac{1}{2}(1+P_{ij}^{\sigma})V_t + \tfrac{1}{2}(1-P_{ij}^{\sigma})V_s] \times [\tfrac{1}{2}u + \tfrac{1}{2}(2-u)P_{ij}^{r}] + \frac{e^2}{4r_{ij}}(1+\tau_{iz})(1+\tau_{jz})\,, \tag{7}$$

where P_{ij}^{σ} and P_{ij}^{r} are the spin- and space-exchange operators, respectively, and τ_{iz} and τ_{jz} are the z components of the isospin operators for the ith and jth particles, respectively. The quantities V_t and V_s are the S-state triplet and singlet potentials given by

$$\begin{aligned}V_t &= -V_{0t}\,e^{-\kappa_t r^2}\,,\\ V_s &= -V_{0s}\,e^{-\kappa_s r^2}\,.\end{aligned} \tag{8}$$

The constants V_{0t}, κ_t, V_{0s}, and κ_s are adjusted to yield the correct values for the two-nucleon effective-range parameters; they are found to be[1]

$$\begin{aligned}V_{0t} &= 66.92\ \text{MeV}, \quad \kappa_t = 0.415\ \text{F}^{-2}\,,\\ V_{0s} &= 29.05\ \text{MeV}, \quad \kappa_s = 0.292\ \text{F}^{-2}\,.\end{aligned} \tag{9}$$

The parameter u, which cannot be determined from the low-energy two-nucleon s-wave scattering data, will be treated as an adjustable parameter which will be fixed by the requirement that an over-all good fit to the low-energy experimental $p+\mathrm{He}^3$ scattering data be obtained.

It should be mentioned that the introduction of an adjustable parameter u is a necessary and desirable procedure. To make our calculation feasible, we have made some simplifying assumptions which will be discussed in the following paragraphs. These assumptions will necessarily introduce some defects into our calculation and it is our hope that with the freedom available in the choice of u, these defects can be partially corrected. It is important, of course, that the value of u determined by the above-mentioned best-fit criterion should be close to 1. This is so, since a value of $u=1$ corresponds to a pure Serber potential and it is known that the experimental two-nucleon scattering data favor a near-Serber exchange mixture for the nucleon-nucleon potential. If the resultant value of u should turn out to be quite different from 1, then we should take it as a clear indication that this crude procedure of varying u is not accurate enough and a more elaborate calculation, with less severe assumptions, must be performed.

To facilitate computations, we have employed a rather simple central nucleon-nucleon potential which contains no repulsive core but does yield correct values for the effective-range parameters. This is quite likely sufficient for the description of the mutual interaction between the clusters,[7] since we are dealing mainly with relatively low-energy scattering phenomena where the relative energies between the nucleons in different clusters are rather small. On the other hand, the lack of saturating character in this potential certainly means that the internal properties of the cluster cannot be properly explained. In resonating-group calculations,[8] this is crudely compensated for by an appropriate choice of parameters in the cluster wave functions using experimental information. It is realized, of course, that this procedure of fixing the cluster parameters and using a simple nucleon-nucleon potential will introduce uncertainties into the results; however, the excellent agreement with experiment obtained in the $\alpha+\alpha$[1] and $\alpha+N$[2] cases where a similar procedure was used does indicate that at least in the low- and medium-energy regions this procedure does not introduce serious errors.

It is also noted that our nucleon-nucleon potential does not have a noncentral component. This is again not too serious, since there are no sharp resonance levels in the $p+\mathrm{He}^3$ and $n+\mathrm{H}^3$ systems. In the $N+\alpha$ case[2] where a two-nucleon spin-orbit component was included in the nucleon-nucleon potential, it was found that in the energy region where no sharp resonance level exists, the main difference between the differential cross sections calculated with and without a spin-orbit component occurs only around deep diffraction minima where, in any case, our calculated result is not expected to be in detailed agreement with experiment owing to the omission of reaction channels.

As in previous calculations,[8] a one-channel approximation is again adopted here. This means that the specific distortion effect, i.e., the distortion effect over and above that already implicitly given by the antisymmetrization procedure, is not properly accounted for. In the very low-energy region where this effect has its largest influence, it is expected that the adoption of this approximation will cause some discrepancy between the calculated and the experimental results. Thus, in this particular energy region, a better calculation will eventually be needed. In this respect, it is noted that a procedure recently suggested by Wildermuth and his collaborators[9] seems interesting and will be a subject of our investigation in the future.

Using Eq. (5), an integrodifferential equation of the form

$$\left\{\frac{\hbar^2}{2\mu}\left[\frac{d^2}{dr^2}-\frac{l(l+1)}{r^2}\right]+E-V_{Ns}(r)-V_C(r)\right\}f_{ls}(r)=\int_0^\infty k_{ls}(r,r')f_{ls}(r')dr' , \tag{10}$$

can be derived,[10] with $f_{ls}(r)$ defined by the equation

$$F_s(\vec{r})=\sum_l \frac{1}{r}f_{ls}(r)P_l(\cos\theta) . \tag{11}$$

In Eq. (10), μ represents the reduced mass, while E represents the relative energy of the two clusters in the c.m. system. The explicit forms for the direct nuclear potential $V_{Ns}(r)$, the direct Coulomb potential $V_C(r)$, and the kernel function $k_{ls}(r,r')$ are given in the Appendix. By solving Eq. (10) with the proper boundary conditions, scattering phase shifts can be determined. In addition, using the resultant functions $f_{ls}(r)$, we can construct an effective local, E-, l-, and s-dependent potential

$$V_{ls}^*(r)=V_{Ns}(r)+\frac{1}{f_{ls}(r)}\int_0^\infty k_{ls}(r,r')f_{ls}(r')dr' , \tag{12}$$

between the clusters, which, because of the particular way of construction, will obviously yield the same phase shifts as those calculated with the resonating-group method.

Finally, we discuss the choice of parameters used in the internal wave functions $\phi_{\rm CL}$ and $\phi'_{\rm CL}$ defined in Eqs. (2) and (4), respectively. As has been mentioned, the lack of saturation character in the nucleon-nucleon potential is crudely compensated for by an appropriate choice of parameters in the cluster wave functions. In the case of a single-Gaussian function, the procedure is to fix the width parameter α so that the experimentally determined value for the rms radius of the nucleon distribution is correctly given. Thus, for the He^3 and H^3 clusters, a choice of $\alpha = 0.36$ F^{-2} in the function $\phi'_{\rm CL}$ gives the correct rms radius of 1.67 F. It is found, however, that the use of such a single-Gaussian function, while giving a correct rms radius, yields not only a value of 4.57 MeV for the H^3 binding energy which does not agree too well with the experimental value of 8.48 MeV, but also values for the three-nucleon body form factors which are too low for $q^2 \gtrsim 1$ F^{-2}. This is shown in Fig. 1, where the dashed line represents the values of the body form factor for the three-nucleon system calculated using $\phi'_{\rm CL}$ with $\alpha = 0.36$ F^{-2}, and the crosses represent the empirical values[11] deduced from experimental data.

The parameters of the two-Gaussian wave function $\phi_{\rm CL}$ are determined by a three-parameter search to obtain a best fit to the body-form-factor data, subject to the constraint that the rms radius of the nucleon distribution be fixed at the correct value of 1.67 F. With the set of parameters given in Eq. (3), we obtain a binding energy of 6.91 MeV for H^3 and a very good fit to the body-form-factor data as illustrated by the solid line in Fig. 1. It should be mentioned that there are several other sets of parameters which give about the same quality of fit as does the set chosen here, but it is found that the $p+\mathrm{He}^3$ and $n+\mathrm{H}^3$ scattering results obtained using these sets are all quite similar.

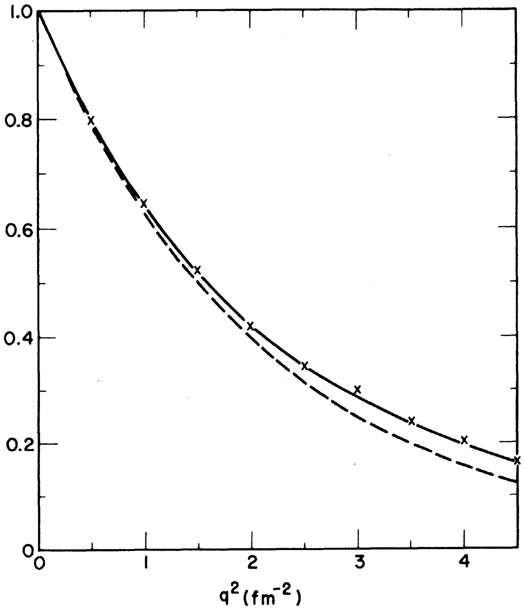

FIG. 1. Body form factors for He^3 or H^3 as a function of q^2 calculated with the two-Gaussian wave function (solid line) and the one-Gaussian wave function (dashed line). The experimental points are taken from Ref. 11.

III. RESULTS

A. Phase Shifts and Differential Cross Sections

As there exists a large amount of $p+\mathrm{He}^3$ data from a variety of sources and as the $p+\mathrm{He}^3$ data are generally expected to be somewhat more accurate than the $n+\mathrm{H}^3$ data, we have adopted the procedure of fixing the parameter u from the $p+\mathrm{He}^3$ data. In this way it is found that, while the optimum value of u cannot be pinpointed using the low-energy data, the value of $u = 1$ does yield an over-all good fit; therefore, in the following discussion, we shall use the results obtained with $u = 1$ for both the $p+\mathrm{He}^3$ and the $n+\mathrm{H}^3$ systems.

First, we compare the results obtained using one-Gaussian and two-Gaussian internal wave

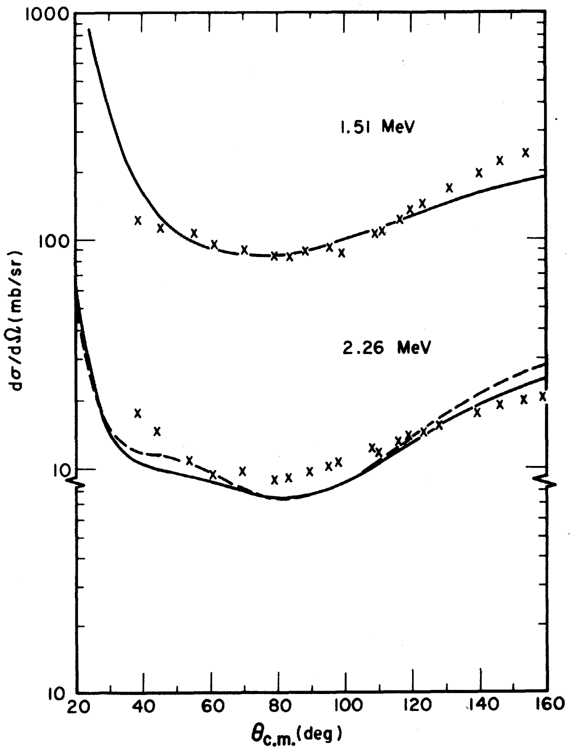

FIG. 2. Comparison of $p+\mathrm{He}^3$ differential cross sections calculated using $u = 1$ with the two-Gaussian wave function (solid lines) and the one-Gaussian wave function (dashed line) with experimental data at 1.51 and 2.26 MeV. The experimental data are those of Ref. 12.

functions. In Figs. 2 and 3 are shown the $p+\mathrm{He}^3$ differential cross sections calculated at 2.26 and 6.38 MeV with the wave function ϕ_{CL} (solid lines) and the wave function ϕ'_{CL} (dashed lines). From these figures, we see that at these energies, the difference between the cross sections amounts to about 20% in the forward direction and about 10% in the backward direction. At higher energies, e.g., at 23.25 MeV, the difference becomes even smaller. This shows that if a very accurate calculation is desired, it is certainly important to employ a good internal wave function. On the other hand, since these small percentage differences are probably within the uncertainties caused by the various approximations adopted in our calculation, it is our viewpoint that the use of a two-Gaussian wave function for the He^3 or the H^3 cluster, which does complicate somewhat the numerical computations, is not entirely called for at this moment.

Next, the results obtained using the two-Gaussian wave function ϕ_{CL}, represented by solid lines in Figs. 2–6, will be compared with experimental measurements in both the $p+\mathrm{He}^3$ and the $n+\mathrm{H}^3$ cases. In Fig. 2, comparisons are made for the differential cross sections at c.m. energies of 1.51 and 2.26 MeV with the experimental $p+\mathrm{He}^3$ data of Tombrello *et al.*[12] From this figure it is seen that a fair but not detailed agreement is obtained. As has been mentioned in Sec. II, it is our belief that the lack of detailed agreement is due to the omission of the specific distortion effect in our calculation. In Fig. 3, we compare our results at 3.41, 6.38, and 8.61 MeV with the experimental $p+\mathrm{He}^3$ data of Clegg *et al.*[13] Here we see that the calculated results are quite good, thus supporting our contention that the specific distortion effect is important mainly in the very low-energy region. In Fig. 4, the $p+\mathrm{He}^3$ result at 23.25 MeV is shown together with the experimental data of Kim *et al.*[14] From this figure we find that even at an energy well above the reaction threshold, the agreement with experiment is still satisfactory. The discrepancy between theory and experiment around the Coulomb-interference minimum at about 15° and the deep diffraction minimum at about 120° is a consequence of the fact that we have omitted reaction channels and used a purely central nucleon-nucleon potential in our calculation.

In Figs. 5 and 6 we compare the results for $n+\mathrm{H}^3$ scattering with the data of Seagrave, Cranberg, and Simmons[15] at 0.75, 1.5, and 4.5 MeV, and the

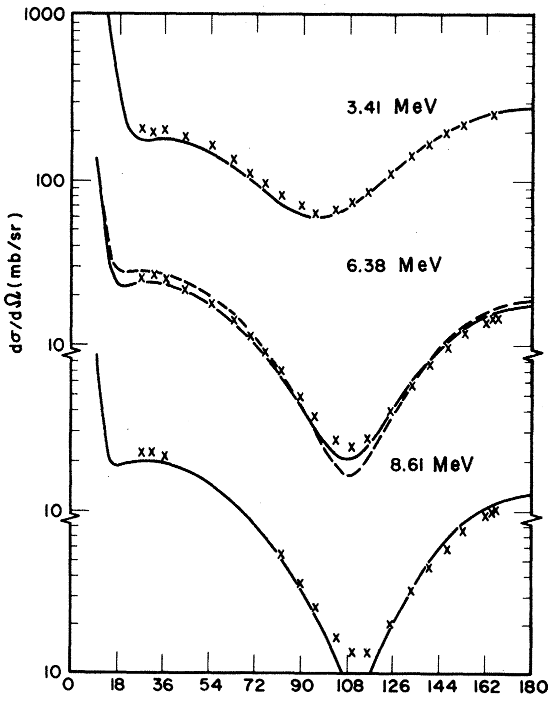

FIG. 3. Comparison of $p+\mathrm{He}^3$ differential cross sections calculated using $u=1$ with the two-Gaussian wave function (solid lines) and one-Gaussian wave function (dashed line) with experimental data at 3.41, 6.38, and 8.61 MeV. The experimental data are those of Ref. 13.

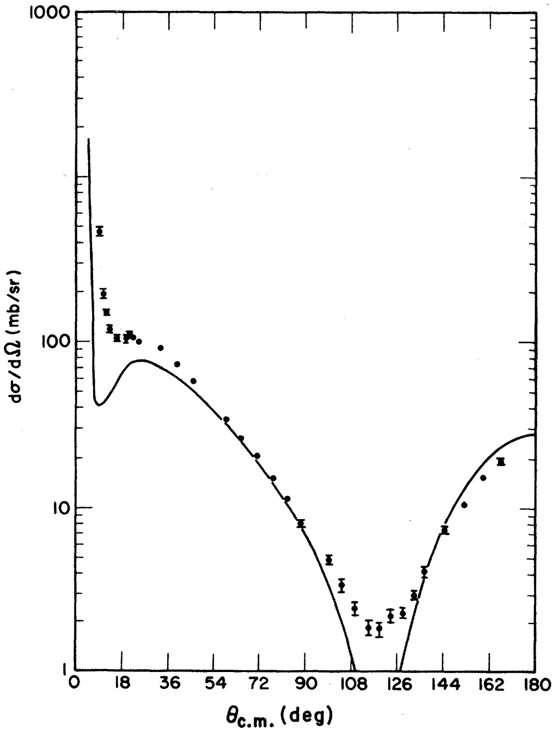

FIG. 4. Comparison of $p+\mathrm{He}^3$ differential cross sections calculated using $u=1$ and the two-Gaussian wave function with experimental data at 23.25 MeV. The experimental data are those of Ref. 14.

data of Kootsey[16] at 10.6 MeV. Here also, it is noted that in this low-energy region, there is a trend toward improved fits at higher energies, just as in the $p+\mathrm{He}^3$ case.

As has been mentioned,[8] the resonating-group phase shifts are useful as starting values in a detailed phase-shift analysis. In fact, such a procedure has been adopted in the phase-shift analyses of Bacher, Spiger, and Tombrello[17] for $\mathrm{He}^3+\mathrm{He}^3$ scattering, and Schwandt *et al.*[18] for $\mathrm{He}^3+\mathrm{He}^4$ scattering. In both of these cases, the final values for the phase shifts turned out to be rather similar to the starting resonating-group values. Thus, it seems to us that the $p+\mathrm{He}^3$ phase shifts calculated here might also be useful for such a purpose. Therefore, we have listed in Table I the values of the $p+\mathrm{He}^3$ phase shifts δ_{ls} for $l=0-6$ obtained with the wave function ϕ_{CL} in the energy region of 0.5-40 MeV.

It is interesting to note that in his phase-shift analysis of the $p+\mathrm{He}^3$ scattering data, Tombrello[19] has found that the $l=0$ phase shifts in the singlet state are larger than those in the triplet state, while our results given in Table I indicate the opposite. We should mention, however, that the $l=0$ phase shifts in the singlet and triplet states do not really differ greatly in magnitude. Hence, it is quite likely that a further phase-shift analysis of the experimental data might in fact favor the phase-shift behavior found in this calculation.

Recently, Harbison *et al.*[20] have made a $p+\mathrm{He}^3$ phase-shift analysis at 22.88 MeV and found five phase-shift sets which fit the experimental data about equally well. Out of these five sets, our resonating-group calculation favors their set 1(D) which has the same behavior for the $l=0$ phase shifts as is found in this study. In Table II, we compare the phase shifts of set 1(D) with the corresponding unsplit phase shifts obtained in this calculation. From this table we see that the agreement is fairly good, thus providing further evidence that the resonating-group results are useful even at relatively high energies.

B. Effective Interaction

We now discuss the effective local $p+\mathrm{He}^3$ interaction defined by Eq. (12). In Fig. 7 are plotted the effective potentials in the triplet state[21] at 4.5 MeV for $l=0-3$, calculated using both the wave function ϕ_{CL} (solid lines) and the wave function ϕ'_{CL} (dashed lines). From this figure, it is clear that the main features of the effective potentials calculated with these two functions are essentially

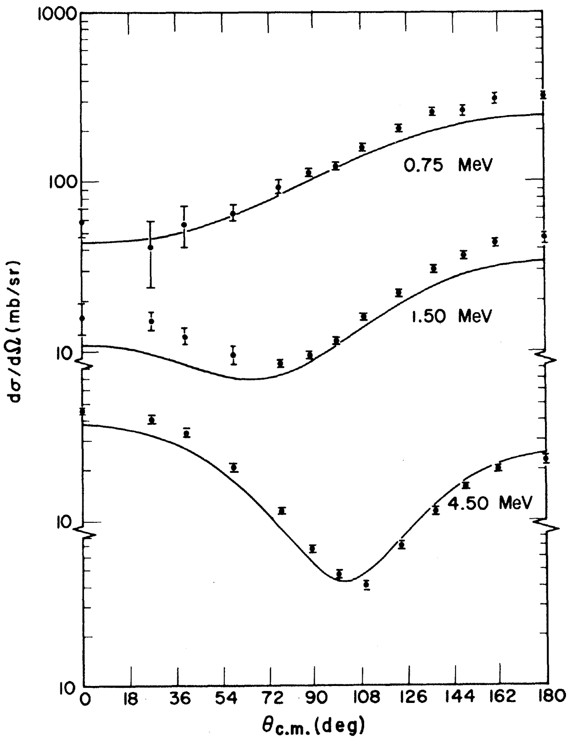

FIG. 5. Comparison of $n+\mathrm{H}^3$ differential cross sections calculated using $u=1$ and the two-Gaussian wave function with experimental data at 0.75, 1.50, and 4.50 MeV. The experimental data are those of Ref. 15.

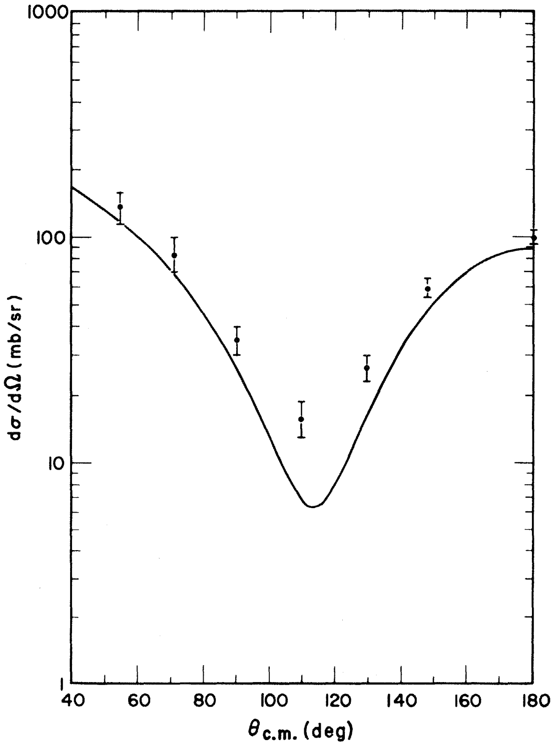

FIG. 6. Comparison of $n+\mathrm{H}^3$ differential cross sections calculated using $u=1$ and the two-Gaussian wave function with experimental data at 10.58 MeV. The experimental data are those of Ref. 16.

TABLE I. $p+\mathrm{He}^3$ phase shifts δ_{ls}, in degrees, calculated with the wave function ϕ_{CL} and $u=1$.

E (MeV)	$l=0$ δ_{00}	δ_{01}	$l=1$ δ_{10}	δ_{11}	$l=2$ δ_{20}	δ_{21}	$l=3$ δ_{30}	δ_{31}	$l=4$ δ_{40}	δ_{41}	$l=5$ δ_{50}	δ_{51}	$l=6$ δ_{60}	δ_{61}
0.5	−11.8	−11.0	1.3	1.3										
1.0	−23.6	−22.2	5.1	5.3	−0.1	−0.1								
1.5	−32.9	−30.9	10.6	11.2	−0.2	−0.2								
2.0	−40.4	−38.0	16.6	18.0	−0.4	−0.3								
2.5	−46.9	−44.0	22.3	24.9	−0.6	−0.5								
3.0	−52.5	−49.3	27.2	31.2	−0.9	−0.8	0.1	0.1						
4.0	−62.0	−58.1	34.3	40.6	−1.5	−1.3	0.2	0.2						
5.0	−69.7	−65.3	38.2	46.3	−2.1	−1.8	0.3	0.3						
6.0	−76.3	−71.5	40.2	49.5	−2.7	−2.2	0.5	0.4						
8.0	−87.0	−81.3	41.3	52.2	−3.4	−2.8	1.0	0.8	−0.1	−0.1				
10.0	−95.4	−89.1	40.7	52.6	−3.5	−2.9	1.4	1.2	−0.1	−0.1				
12.0	−102.2	−95.5	39.7	52.0	−3.3	−2.6	1.9	1.6	−0.2	−0.2				
14.0	−108.0	−100.9	38.5	50.9	−2.6	−2.0	2.3	2.1	−0.2	−0.2	0.1	0.1		
16.0	−112.9	−105.5	37.3	49.7	−1.8	−1.3	2.7	2.5	−0.2	−0.2	0.1	0.1		
18.0	−117.2	−109.6	36.1	48.4	−0.8	−0.4	3.1	2.9	−0.2	−0.1	0.1	0.1		
20.0	−121.0	−113.2	35.0	47.0	0.2	0.5	3.4	3.3	−0.1	−0.1	0.2	0.1		
22.0	−124.3	−116.4	33.9	45.7	1.3	1.5	3.8	3.8	0.0	0.1	0.2	0.2		
25.0	−128.7	−120.6	32.4	43.8	2.7	2.9	4.3	4.5	0.2	0.2	0.3	0.2		
30.0	−134.7	−126.5	30.2	40.9	4.9	5.3	5.2	5.6	0.6	0.6	0.4	0.4		
35.0	−139.4	−131.2	28.3	38.3	6.7	7.4	5.9	6.7	1.0	1.0	0.6	0.5	0.1	0.1
40.0	−143.1	−135.0	26.7	36.1	8.1	9.2	6.7	7.6	1.5	1.4	0.8	0.7	0.2	0.1

TABLE II. A comparison between the phase shifts δ_{ls} obtained in this calculation and the phase shifts δ_{ls}^{J} of set 1(D) of Harbison *et al*. for the $p+\mathrm{He}^3$ system at 22.88 MeV.

	Set 1(D) of Harbison *et al.*	This calculation
δ_{00}^{0}	−139.0	−125.4
δ_{01}^{1}	−114.0	−117.5
δ_{10}^{1}	36.0	33.6
δ_{11}^{0}	33.0	
δ_{11}^{1}	31.5	45.3
δ_{11}^{2}	55.0	
δ_{20}^{2}	11.5	1.7
δ_{21}^{1}	0.0	
δ_{21}^{2}	4.5	1.9
δ_{21}^{3}	6.0	
δ_{30}^{3}	5.0	3.9
δ_{31}^{2}	1.0	
δ_{31}^{3}	2.0	4.0
δ_{31}^{4}	7.5	

the same. This indicates that, in future investigations, if a study of the properties of the effective interaction is the main purpose, then the use of a relatively simple cluster wave function, such as ϕ'_{CL}, will be quite sufficient.

From Fig. 7, it is also seen that the effective potentials in the odd-l states are quite different from those in the even-l states. Thus, the odd-even effect, known to exist in the $\alpha+N^2$ and He^3

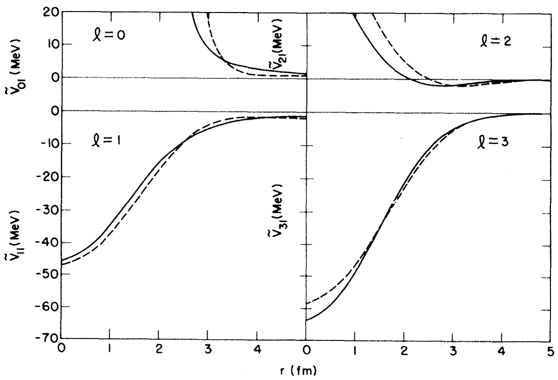

FIG. 7. Effective triplet-state $p+\mathrm{He}^3$ potentials calculated at 4.5 MeV with the two-Gaussian wave function (solid lines) and the one-Gaussian wave function (dashed lines) for $l=0$–3.

$+\alpha^6$ systems, is again very much apparent here. Also, it should be mentioned that the effective potentials in all l states are very different from the direct nuclear potential $V_{Ns}(r)$, which is strong evidence that the antisymmetrization procedure plays an important role in calculations on light nuclear systems.

IV. CONCLUSION

In this study we have considered the $p+\mathrm{He}^3$ and the $n+\mathrm{H}^3$ scattering problems using the resonating-group method in the one-channel approximation. The nucleon-nucleon potential used is a purely central potential which has different ranges in the singlet and triplet states and which gives a very good fit to the low-energy two-nucleon scattering data. The internal wave functions for the He^3 and H^3 clusters are composed of a sum of two Gaussian functions, with the parameters adjusted to reproduce quite well the essential properties of these clusters.

Previous resonating-group calculations[8] have always used a one-Gaussian wave function for the He^3 or the H^3 cluster. To have some idea about how reliable these calculations were, we have compared the $p+\mathrm{He}^3$ differential cross sections calculated with two-Gaussian and one-Gaussian cluster wave functions. The result shows that the differences between the cross sections are 10-20% at energies below about 10 MeV and become even smaller at higher energies. The fact that these differences are rather small indicates that these previous resonating-group calculations involving three-nucleon clusters are good enough and further considerations using an improved cluster wave function do not seem to be worthwhile at the present moment.

Since the He^3 and H^3 clusters are not tightly bound, it is expected that the omission of the specific distortion effect may lead to some disagreement with experiment in the very low-energy region.[22] Indeed, a comparison of the calculated differential cross sections with experimental data shows that the agreement is only fair at very low energies, but does improve significantly as the energy is increased.

Even at relatively high energies where the one-channel approximation is expected to be less valid, it is found that the agreement between theory and experiment is quite satisfactory. In fact, we have been able to use our results to pick one out of the five phase-shift sets obtained recently by Harbison *et al.*[20] from a phase-shift analysis of their $p+\mathrm{He}^3$ experimental data at 22.88 MeV.

The property of the effective local $p+\mathrm{He}^3$ interaction is also studied. From this study we find that, as in many other cases,[8] the antisymmetrization procedure is quite important and the odd-even effect, wherein the effective potentials in the odd-l and even-l states are appreciably different, is very much apparent.

APPENDIX. EXPRESSIONS FOR THE DIRECT POTENTIALS AND THE KERNEL FUNCTION CALCULATED WITH THE WAVE FUNCTION ϕ_{CL}

The expressions for the direct nuclear potential V_{Ns}, the direct Coulomb potential V_C, and the kernel function k_{ls} are as follows:
(i) $V_{Ns}(r)$

$$V_{Ns}(r) = -\frac{1}{N}\sum_{i=1}^{2} V_{0i}\gamma_{si}\sum_{j=1}^{4} c_j\left(\frac{\pi^2}{3\alpha_j^2}\right)^{3/2} \times\left(\frac{3\alpha_j}{3\alpha_j+2\kappa_i}\right)^{3/2}\exp\left(-\frac{3\alpha_j\kappa_i}{3\alpha_j+2\kappa_i}r^2\right), \tag{A1}$$

with

$$N=\sum_{j=1}^{4} c_j\left(\frac{\pi^2}{3\alpha_j^2}\right)^{3/2}, \tag{A2}$$

and

$$\gamma_{0i} = 3w_i - m_i - 2h_i\,,$$
$$\gamma_{1i} = 3w_i - m_i + 2b_i - 2h_i\,. \tag{A3}$$

In Eqs. (A1)-(A3), the quantities involving the subscript i ($i=1,2$) are defined as

$$V_{01}=V_{0t}\,,\quad V_{02}=V_{0s}\,,$$
$$\kappa_1=\kappa_t\,,\quad \kappa_2=\kappa_s\,,$$
$$w_1=b_1=w_2=-b_2=\tfrac{1}{4}u\,,$$
$$m_1=h_1=m_2=-h_2=\tfrac{1}{4}(2-u)\,,$$

while the quantities involving the subscript j ($j=1$, 2, 3, 4) are defined as

$$\alpha_1=\alpha_1\,,\quad \alpha_2=\alpha_2\,,\quad \alpha_3=\alpha_4=\tfrac{1}{2}(\alpha_1+\alpha_2)\,,$$
$$c_1=1\,,\quad c_2=c^2\,,\quad c_3=c_4=c\,.$$

(ii) $V_C(r)$

$$V_C(r)=\frac{1}{N}\frac{ZZ'e^2}{r}\sum_{j=1}^{4} c_j\left(\frac{\pi^2}{3\alpha_j^2}\right)^{3/2}\Phi\left[\left(\frac{3\alpha_j}{2}\right)^{1/2}r\right], \tag{A4}$$

with Z and Z' being the atomic numbers of the three-nucleon cluster and the nucleon, respectively, and

$$\Phi(\nu)=\frac{2}{\sqrt{\pi}}\int_0^{\nu} e^{-t^2}\,dt\,.$$

(iii) $k_{ls}(r, r')$

$$k_{ls}=-\frac{\hbar^2}{2M}\mathcal{T}-\sum_{i=1}^{2} V_{0i}\mathcal{V}_{si}+E'\mathcal{E}\,, \tag{A5}$$

where M is the nucleon mass and E' is given by

$$E' = E + E_{\mathrm{CL}}\,,$$

with

$$E_{\mathrm{CL}} = \frac{1}{N}\left\{ \frac{\hbar^2}{2M}\left[3\alpha_1\left(\frac{\pi^2}{3\alpha_1^2}\right)^{3/2} + 3c^2\alpha_2\left(\frac{\pi^2}{3\alpha_2^2}\right)^{3/2} + 6c\left(\frac{2\alpha_1\alpha_2}{\alpha_1+\alpha_2}\right)\left(\frac{4\pi^2}{3(\alpha_1+\alpha_2)^2}\right)^{3/2}\right]\right.$$

$$\left. - \sum_{i=1}^{2} V_{0i}(3w_i + 3m_i)\sum_{j=1}^{4} c_j\left(\frac{\pi^2}{3\alpha_j^2}\right)^{3/2}\left(\frac{\alpha_j}{\alpha_j+2\kappa_i}\right)^{3/2} + \frac{Z(Z-1)e^2}{2}\sum_{j=1}^{4} c_j\left(\frac{\pi^2}{3\alpha_j^2}\right)^{3/2}\left(\frac{2\alpha_j}{\pi}\right)^{1/2}\right\}.$$

In Eq. (A5), the quantities $\mathcal{T}$, $\mathcal{V}_{si}$, and $\mathcal{E}$ are defined in the following way:

$$\mathcal{T} = \frac{1}{N}\sum_{j=1}^{4}\left(\tfrac{3}{4}\right)^3 c_j\left(\frac{\pi^2}{3\alpha_j^2}\right)^{3/2}\left(\frac{3\alpha_j}{2\pi}\right)^{3/2}\exp\left[\tfrac{3}{8}H_j(r^2-r'^2)\right]\exp\left[-\tfrac{15}{32}\,\alpha_j(r^2+r'^2)\right]$$

$$\times\left[(D_j - F_{1j}r^2 - F_{2j}r'^2)\,S_l\left(\tfrac{9}{16}\,\alpha_j\right) - G_j r r'\,T_l\left(\tfrac{9}{16}\,\alpha_j\right)\right],$$

where

$$S_l(\nu) = \frac{4\pi}{\nu}\,\mathcal{J}_{l+1/2}(\nu r r')\,,$$

$$T_l(\nu) = \frac{4\pi}{\nu}\left[\mathcal{J}_{l+3/2}(\nu r r') - \frac{l}{\nu r r'}\,\mathcal{J}_{l+1/2}(\nu r r')\right],$$

with $\mathcal{J}(\nu r r')$ being a hyperbolic spherical Bessel function, and

$$F_{11} = \tfrac{81}{64}\,\alpha_1^2\,,\quad F_{12} = \tfrac{81}{64}\,\alpha_2^2\,,\quad F_{13} = \tfrac{27}{256}(\alpha_1^2 + 2\alpha_1\alpha_2 + 9\alpha_2^2)\,,\quad F_{14} = \tfrac{27}{256}(9\alpha_1^2 + 2\alpha_1\alpha_2 + \alpha_2^2)\,,$$

$$F_{21} = F_{11}\,,\quad F_{22} = F_{12}\,,\quad F_{23} = F_{14}\,,\quad F_{24} = F_{13}\,,$$

$$H_1 = 0\,,\quad H_2 = 0\,,\quad H_3 = \tfrac{1}{2}(\alpha_1 - \alpha_2)\,,\quad H_4 = \tfrac{1}{2}(\alpha_2 - \alpha_1)\,,$$

$$D_1 = \tfrac{33}{4}\,\alpha_1\,,\quad D_2 = \tfrac{33}{4}\,\alpha_2\,,\quad D_3 = D_4 = \frac{27\alpha_1^2 + 78\alpha_1\alpha_2 + 27\alpha_2^2}{8(\alpha_1+\alpha_2)}\,,$$

$$G_1 = \tfrac{63}{32}\,\alpha_1^2\,,\quad G_2 = \tfrac{63}{32}\,\alpha_2^2\,,\quad G_3 = G_4 = \tfrac{9}{128}(9\alpha_1^2 + 10\alpha_1\alpha_2 + 9\alpha_2^2)\,.$$

$$\mathcal{V}_{si} = \frac{1}{N}\sum_{j=1}^{4}\left(\tfrac{3}{4}\right)^3 C_j\left(\frac{\pi^2}{3\alpha_j^2}\right)^{3/2}\left(\frac{3\alpha_j}{2\pi}\right)^{3/2}\exp\left[\tfrac{3}{8}H_j(r^2-r'^2)\right]$$

$$\times\left\{\beta_{si}S_l\left[\tfrac{9}{16}\,(\alpha_j - 2\kappa_i)\right]\exp\left[-\frac{15\alpha_j + 18\kappa_i}{32}\,(r^2+r'^2)\right] + \lambda_{si}\left(\frac{\alpha_j}{\alpha_j+2\kappa_i}\right)^{3/2} S_l\left(\tfrac{9}{16}\,\alpha_j\right)\exp\left[-\frac{15\alpha_j}{32}(r^2+r'^2)\right]\right.$$

$$+\nu_{si}\left(\frac{2\alpha_j}{2\alpha_j+\kappa_i}\right)^{3/2} S_l\left(\frac{9}{8}\,\frac{\alpha_j^2 + 2\alpha_j\kappa_i}{2\alpha_j+\kappa_i}\right)\left[\exp\left(-\frac{3}{16}\,\frac{5\alpha_j^2 + 16\alpha_j\kappa_i}{2\alpha_j+\kappa_i}\,r^2 - \frac{3}{16}\,\frac{5\alpha_j^2 + 4\alpha_j\kappa_i}{2\alpha_j+\kappa_i}\,r'^2\right)\right.$$

$$\left.\left. + \exp\left(-\frac{3}{16}\,\frac{5\alpha_j^2 + 4\alpha_j\kappa_i}{2\alpha_j+\kappa_i}\,r^2 - \frac{3}{16}\,\frac{5\alpha_j^2 + 16\alpha_j\kappa_i}{2\alpha_j+\kappa_i}\,r'^2\right)\right]\right\},$$

where

$$\beta_{0i} = -w_i + 3m_i - 2b_i\,,\qquad \beta_{1i} = -w_i + 3m_i - 2b_i + 2h_i\,,$$

$$\lambda_{0i} = -w_i - m_i - 2b_i - 2h_i\,,\qquad \lambda_{1i} = -w_i - m_i\,,$$

$$\nu_{0i} = -2w_i - 2m_i + 2b_i + 2h_i\,,\qquad \nu_{1i} = -2w_i - 2m_i\,.$$

$$\mathcal{E} = \frac{1}{N}\sum_{j=1}^{4}\left(\tfrac{3}{4}\right)^3 c_j\left(\frac{\pi^2}{3\alpha_j^2}\right)^{3/2}\left(\frac{3\alpha_j}{2\pi}\right)^{3/2}\exp\left[\tfrac{3}{8}H_j(r^2-r'^2)\right] S_l\left(\tfrac{9}{16}\,\alpha_j\right)\exp\left[-\frac{15\alpha_j}{32}(r^2+r'^2)\right].$$

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[1] I. Reichstein and Y. C. Tang, Nucl. Phys. A139, 144 (1969).

[2] I. Reichstein and Y. C. Tang, Nucl. Phys. A158, 529 (1970).

[3] Previous studies on the $p+\mathrm{He}^3$ and $n+\mathrm{H}^3$ systems have been performed using a simpler potential and a single-Gaussian wave function for the He^3 or the H^3 cluster by groups named in Ref. 4.

[4] B. H. Bransden and H. H. Robertson, Proc. Phys. Soc. (London) A72, 770 (1958); B. H. Bransden, H. H. Robertson, and P. Swan, Proc. Phys. Soc. (London) A69, 877 (1956).

[5] D. R. Thompson and Y. C. Tang, Nucl. Phys. A106, 591 (1968); W. Sünkel and K. Wildermuth, Phys. Letters 26B, 655 (1968).

[6] R. E. Brown and Y. C. Tang, Phys. Rev. 176, 1235 (1968); Y. C. Tang, E. Schmid, and K. Wildermuth, Phys. Rev. 131, 2631 (1963).

[7] L. D. Pearlstein, Y. C. Tang, and K. Wildermuth, Nucl. Phys. 18, 23 (1960).

[8] Y. C. Tang, in *Proceedings of the International Conference on Clustering Phenomena in Nuclei, Bochum, Germany, 1969* (International Atomic Energy Agency, Vienna, Austria, 1969), p. 109.

[9] H. Federsel, E. J. Kanellopoulos, W. Sünkel, and K. Wildermuth, Phys. Letters 33B, 140 (1970).

[10] It should be pointed out that the exchange-Coulomb term is not included in the $p+\mathrm{He}^3$ calculation. It was found in Ref. 2 that this is a rather unimportant term which can be compensated for by a small adjustment in u, so long as the $l=0$ phase shift is insensitive to the parameter u, which is found to be the case in this calculation.

[11] Y. C. Tang and R. C. Herndon, Phys. Letters 18, 42 (1965).

[12] T. A. Tombrello, C. M. Jones, G. C. Phillips, and J. L. Weil, Nucl. Phys. 39, 541 (1962).

[13] T. B. Clegg, A. C. L. Bernard, J. B. Swint, and J. L. Weil, Nucl. Phys. 50, 621 (1964).

[14] C. C. Kim, S. M. Bunch, D. W. Devins, and H. H. Forster, Nucl. Phys. 58, 32 (1964).

[15] J. D. Seagrave, L. Cranberg, and J. E. Simmons, Phys. Rev. 119, 1981 (1960).

[16] J. M. Kootsey, Nucl. Phys. A113, 65 (1968).

[17] A. D. Bacher, R. J. Spiger, and T. A. Tombrello, Nucl. Phys. A119, 481 (1968).

[18] P. Schwandt, B. W. Ridley, S. Hayakawa, L. Put, and J. J. Kraushaar, Phys. Letters 30B, 30 (1969).

[19] T. A. Tombrello, Phys. Rev. 138, B40 (1965). See also L. W. Morrow and W. Haeberli, Nucl. Phys. A126, 225 (1969).

[20] S. A. Harbison, R. J. Griffiths, N. M. Stewart, A. R. Johnston, and G. T. A. Squier, Nucl. Phys. A150, 570 (1970).

[21] The effective potentials in the singlet state have similar features and hence are not shown. In addition, it should be mentioned that the curve for $l=0$ in Fig. 7 is actually for the effective potential $\tilde{V}_{01}$, which is simply related to the effective potential V^*_{01}, as was explained in a paper by D. R. Thompson, I. Reichstein, W. McClure, and Y. C. Tang, Phys. Rev. 185, 1351 (1969).

[22] This was found to be the case in $d+\alpha$ scattering by D. R. Thompson and Y. C. Tang, Phys. Rev. 179, 971 (1969).