tational Phys. 6, 253 (1966).

[4]Data discarded are differential cross sections at 0.95, 1.49, 1.70, 11.157, 18.98, and 19.50 MeV and polarizations at 0.300, 0.390, 0.500, 0.515, 4.04, 4.46, 4.665, 4.757, 4.78, 5.43, and 6.00 MeV.

[5]M. A. Preston, *Physics of the Nucleus* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1962), Appendix B.

[6]Data discarded are differential cross sections at 11.16, 17.45 (D10) and 19.5 MeV and polarizations at 4.46, 4.665, 5.43, 6.00, and 22.93 MeV.

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Discussion of a Particular Form of Cluster Wave Functions

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The meaning of a particular form of cluster wave functions, which have been used in a previous paper to describe the ground-state rotational band of ^{20}Ne, is discussed.

I. INTRODUCTION

Several forms of cluster wave functions have been proposed.[1] In a recent paper[2] we proposed a form of cluster wave functions which seems to be particularly suited for the nuclei ^{19}F and ^{20}Ne. With our cluster wave functions the excitation energies of the ground-state rotational band of ^{20}Ne were calculated, and reasonable agreement with the experimental values was obtained. However, what the wave functions actually represent is not evident. It is the purpose of this paper to discuss the meaning of these wave functions.

II. CLUSTER WAVE FUNCTIONS

For ^{20}Ne, cluster wave functions of the following form were used:

$$\Psi(JM)=N a[\Psi(^{16}\mathrm{O})\Psi_\alpha(JM)]=N'a[\Psi(^{16}\mathrm{O})\Psi'_\alpha(JM)]\,. \tag{1}$$

Here the function $\Psi_\alpha(JM)$ describes an α cluster with total angular momentum J and Z component M moving in a potential well generated by an inert ^{16}O core. The function $\Psi'_\alpha(JM)$ is obtained from $\Psi_\alpha(JM)$ by deleting terms containing single-particle states occupied by the core nucleons and normalizing. The function $\Psi(^{16}\mathrm{O})$ describes the inert ^{16}O core. a is an antisymmetrizer. N and N' are normalization constants. It is well known that continual existence of subunits in a nucleus is not allowed by the Pauli principle. In our model of ^{20}Ne the probability of finding an α cluster (in its internal ground state) outside the inert ^{16}O core is given by

$$P_\alpha=\left|\int\Psi^*_\alpha(JM)\Psi'_\alpha(JM)\right|^2. \tag{2}$$

Since the space part of Ψ_α is taken to be totally symmetric,[2] the total wave function Ψ_α is a product of a space function and a spin-isospin function:

$$\Psi_\alpha=\psi_\alpha\chi(S=0,\ T=0)\,, \tag{3}$$

where ψ_α denotes the space function, and $\chi(S=0, T=0)$ denotes a totally antisymmetric spin-isospin function with spin $S=0$ and isospin $T=0$. ψ_α can be expanded in terms of products of single-particle orbital wave functions:

$$\psi_\alpha=\sum_{i=1}^{n}C_i\,\psi_i\,, \tag{4}$$

where C_i are the expansion coefficients. Suppose all terms in (4) with $i>n'$ contain single-particle orbital states occupied by the core nucleons. Then the space part ψ'_α of Ψ'_α is the sum

$$\psi'_\alpha=B\sum_{i=1}^{n'}C_i\,\psi_i\,, \tag{5}$$

where $B=(\sum_{i=1}^{n'}|C_i|^2)^{-1/2}$ is a normalization constant. Note that $\Psi'_\alpha=\psi'_\alpha\chi(S=0,\ T=0)$. Thus we obtain

$$P_\alpha=\sum_{i=1}^{n'}|C_i|^2\,. \tag{6}$$

The value of P_α was calculated[3] to be 3.8% for all states of the ground-state rotational band of ^{20}Ne.

III. DISCUSSION

The smallness of the value of P_α casts doubt on the usefulness of the cluster wave functions (1). In Appendix A we shall show that, for the low-lying states of ^{20}Ne, the wave functions (1) represent states of two protons and two neutrons in the s-d shell with the largest probability of occurrence of an α cluster (in its internal ground state). As the breakup energy of an α cluster in ^{20}Ne is high, the states (1) may be realized there with a small amount of admixtures; these states should be energetically favored.

The result in Sec. II suggests that in general the probability of occurrence of an α cluster in the nuclear surface is small.

In α-decay theory it is a valid assumption[4, 5] that α-decay probability per unit time can be expressed as a product of two factors: (1) the probability of occurrence of an α cluster in the nuclear surface, and (2) the probability per unit time that the α cluster already formed will penetrate the Coulomb barrier. Our calculation of P_α for ^{20}Ne suggests that the first factor is much less than 1.

A complete basis for the states of ^{20}Ne can be defined in the way shown in Appendix B. Thus, using the notation of Appendix B (with $A = 20$ and $A' = 16$) we can expand $\Psi(JM)$ as

$$\Psi(JM) = \sum_{\mu\nu\kappa J_2} C_{\mu\nu\kappa J_2} \Phi(\mu\nu\kappa(J_2); JM). \tag{7}$$

The coefficients $C_{\mu\nu\kappa J_2}$ play the role of coefficients of fractional parentage in our coupling scheme. Suppose the basis is defined so that $\Phi(\mu_0\nu_0\kappa_0(J); JM) = \Psi(^{16}\mathrm{O})\Psi_\alpha(JM)$. Then the coefficient $C_{\mu_0\nu_0\kappa_0 J}$ is given by

$$\begin{aligned}\int [\Psi(^{16}\mathrm{O})\Psi_\alpha(JM)]^*\Psi(JM) &= \int [\Psi(^{16}\mathrm{O})\Psi_\alpha(JM)]^*\{N' a[\Psi(^{16}\mathrm{O})\Psi'_\alpha(JM)]\} \\ &= N' \int \Psi^*_\alpha(JM)\Psi'_\alpha(JM),\end{aligned} \tag{8}$$

where $N' = (16!4!/20!)^{1/2}$ is the normalization constant in (1). Thus it follows that

$$P_\alpha = (20!/16!4!)|C_{\mu_0\nu_0\kappa_0 J}|^2 .$$

This result is expected because $|C_{\mu_0\nu_0\kappa_0 J}|^2$ gives the probability of finding an α cluster (in its internal ground state) formed from the last four (17th to 20th) nucleons outside an ^{16}O core, while P_α gives the probability of finding an α cluster (in its internal ground state) formed from any four nucleons outside an ^{16}O core.

APPENDIX A

Let $\Psi'_\alpha(JM)$ be a wave function for two protons and two neutrons in the s-d shell, and $\Phi(JM)$ be any normalized wave function for two protons and two neutrons in the same oscillator shell and with the same total angular momentum J and Z component M. Note that $\Psi'_\alpha(JM)$, $\Psi_\alpha(JM)$ [from which $\Psi'_\alpha(JM)$ is obtained], and $\Phi(JM)$ are states having the same number of oscillator quanta. We now show that, among all states of two protons and two neutrons in the s-d shell and with total angular momentum J and M, $\Psi'_\alpha(JM)$ has the largest probability of occurrence of an α cluster (in its internal ground state). We assume, without loss of generality, that the space part of Φ has [4] symmetry; for the space part of Ψ_α has [4] symmetry, and wave functions (in the L-S coupling scheme) of different orbital symmetries are orthogonal. Thus Φ is a product of a space function ϕ and $\chi(S=0, T=0)$. The probability of finding the nucleons in an α-particle-like configuration in the state Φ is given by

$$\begin{aligned}\left|\int \Psi^*_\alpha \Phi\right|^2 &= \left|\int \psi^*_\alpha \phi\right|^2 = \left|\int \left(\sum_{i=1}^{n} C_i\psi_i\right)^* \phi\right|^2 \\ &= \left|\int \left(\sum_{i=1}^{n'} C_i\psi_i\right)^* \phi\right|^2 = \left|\int \frac{1}{B}\psi'^*_\alpha \phi\right|^2 \\ &= \left|\int \frac{1}{B}\Psi'^*_\alpha \Phi\right|^2 = P_\alpha \left|\int \Psi'^*_\alpha\Phi\right|^2 \le P_\alpha .\end{aligned} \tag{A1}$$

The last step follows because $|\int \Psi'^*_\alpha \Phi| \le 1$, where the equality holds only if $\Phi = \Psi'_\alpha$ with a possible difference in phase.

APPENDIX B

The shell-model Hamiltonian for A nucleons in an oscillator potential is

$$H = \sum_{i=1}^{A} (p_i^2/2m + \tfrac{1}{2} m\omega^2 r_i^2), \tag{B1}$$

where $\vec{p}_i$ and $\vec{r}_i$ are the momentum and coordinate of the ith nucleon, m is the nucleon mass, and ω is the classical oscillator frequency.

If we introduce the center-of-mass and relative coordinates for the subsystem consisting of the $(A'+1)$th, the $(A'+2)$th, ..., and the last nucleon, the Hamiltonian takes the form

$$H = H_{\mathrm{I}} + H_{\mathrm{II}} + H_{\mathrm{III}}$$

with

$$H_{\mathrm{I}} = \sum_{i=1}^{A'} (p_i^2/2m + \tfrac{1}{2} m\omega^2 r_i^2),$$

$$H_{\mathrm{II}} = P^2/2(A-A')m + \tfrac{1}{2}(A-A')m\omega^2 R^2, \tag{B2}$$

where $\vec{R}$ and $\vec{P}$ are the coordinate and conjugate momentum of the center of mass of the subsystem, and H_{III} is the Hamiltonian for the internal motion of the subsystem.

For the internal states of the two nucleon groups we can define complete sets of orthonormal wave functions in the L-S coupling.[6] The translational states of the second nucleon group [consisting of the $(A'+1)$th, ..., and the last nucleon] are labeled by three quantum numbers: a radial quantum number and the quantum numbers for the orbital angular momentum of the center of mass and its z component. Let the internal states of the two nucleon groups be described by wave functions denoted by $\Phi_\mu^{(i)}(1,\ldots,A')$ and $\Phi_\nu^{(i)}(A'+1,\ldots,A)$, respectively, and the translational state of the center of mass of the second nucleon group be described by a wave function denoted by $\Phi_\kappa^{(t)}(A'+1,\ldots,A)$. μ, ν, and κ denote (collectively) the respective quantum numbers. As $\Phi_\kappa^{(t)}$ is a function of the center-of-mass coordinate of the second nucleon group, it is symmetric in the last $(A-A')$ nucleons. To obtain complete wave functions of the A nucleons, $\Phi_\nu^{(i)}$ and $\Phi_\kappa^{(t)}$ are first coupled to a state $\Phi(\nu\kappa; J_2M_2)$ with a resultant angular momentum J_2 and z component M_2. $\Phi(\nu\kappa; J_2M_2)$ describes the motion of the second nucleon group in the oscillator potential. Next $\Phi_\mu^{(i)}$ and $\Phi(\nu\kappa; J_2M_2)$ are coupled to a total angular momentum J and z component M, resulting in a function $\Phi(\mu\nu\kappa(J_2); JM)$ which is antisymmetric in the first A' nucleons and in the last $(A-A')$ nucleons. Then by antisymmetrization of $\Phi(\mu\nu\kappa(J_2); JM)$ a complete wave function is obtained for the system of A nucleons. The functions $\Phi(\mu\nu\kappa(J_2); JM)$ constitute a complete basis for the (antisymmetrized) wave functions of the A nucleons.

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