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$^6Li(\alpha, 2\alpha)$ Reaction as a Test of the Existence of α Particles in ^6Li

G. Deconninck
Centre de Physique Nucléaire Université de Louvain, Heverlé, Belgique

and

A. Giorni, J. P. Longequeue, J. P. Maillard, and Tsan Ung Chan*
Institut des Sciences Nucléaires, Cédex 257 38, Grenoble-Gare, France
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The presence of a strong discontinuity in the free α-α scattering amplitude at 40 MeV is used as a probe for the identification of α clusters in nuclei. The $^6Li(\alpha, 2\alpha)$ reaction is investigated at 37.5 and 43.5 MeV with the deuteron as spectator in order to produce quasifree scattering conditions. The distributions measured at these two energies are found to be very different in magnitude as well as in structure, and this phenomenon is attributed to the presence of α-α scattering in the α-^6Li collision. Calculations based on the plane-wave impulse approximation are unable to reproduce the distributions even qualitatively. A calculation based on the graph formalism which takes into account the exact phase-shift amplitude of the free α-α collision fits the two distributions with the same set of parameters. One may conclude that these observations are in favor of the existence of α particles inside the ^6Li nucleus.

I. INTRODUCTION

The observation of α-particle emission in the outgoing channel of many nuclear reactions is usually attributed to the presence of α particles in the nuclear matter. In the case of $(\alpha, 2\alpha)$ reactions, this observation is generally interpreted in terms of knockout of α particles from the target nucleus.[1-3] Balashov[4] has discussed the off-energy-shell effects in the $(\alpha, 2\alpha)$ reaction, and has shown that from the two possible relative energies, before and after the interaction, only the last one gives good results. In this work we shall take advantage of the existence of resonances between clusters in order to check the existence of such clusters in ^6Li.

The analysis of $(\alpha, 2\alpha)$ experiments is generally based on the following hypotheses:
(i) the existence of α particles inside the nucleus and a special model for the ground state, generally an α-particle model;
(ii) a reaction mechanism, the one most used being the plane-wave impulse approximation (PWIA);
(iii) the utilization of an "on-the-energy-shell" matrix element for the calculation of "off-the-energy-shell" collisions.

The cross section predicted by the PWIA is given by

$$\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_1} = K\left(\frac{d\sigma}{d\Omega}\right)_{\alpha-\alpha} |\phi(\vec{k})|^2,$$

where K is a kinematic factor and $\phi(\vec{k})$ the Fourier transform of the α-particle wave function in the target ground state. Usually this formula is used as follows: $\phi(\vec{k})$ is extracted from the measured cross section and then used to find the wave function.

The aim of this work is to estimate the validity of such an α-particle model and also to determine the accuracy of the PWIA. In a preceding paper[5] we suggested the use of the resonances between particles and clusters as a "probe" to investigate phenomenologically the nature of nuclear-reaction mechanisms in collisions between light nuclei. The well-known cross-section variation occurring in the α-α scattering in the vicinity of 40 MeV (Fig. 1) will prove to be a very sensitive tool for this purpose. The free α-α elastic scattering cross section suffers strong variations in the neighborhood of 40 MeV in the (α, d) and (α, p) reactions on ^{6}Li, and we suggested this phenomenon may be associated with the presence of the $(\alpha$-$\alpha)$ resonances. The ^{6}Li$(\alpha, 2\alpha)$D reaction has been selected because of the high probability of an α cluster in the ground state of this nucleus and also for the low value of the binding energy of this α particle (E_b = 1.47 MeV). Moreover, since the experimental arrangement is such that "quasifree" scattering events will be selected, the deuteron being a "spectator", this will considerably simplify the calculations and will give an immediate physical meaning to the result. In the PWIA hypothesis, the bound cross section is proportional to the free cross section except for the kinematic factor. In this case every modification in the free cross section must also be present in the bound cross section, and this is a direct test of the PWIA. In this paper we shall perform more-realistic calculations, which are made feasible by selecting the events corresponding to $k = 0$, thereby introducing large simplifications into the integrations.

II. EXPERIMENTAL METHOD

The analyzed α-particle beam of the Grenoble variable-energy cyclotron has been used at 37.5 and 43.5 MeV. Li targets were bombarded in a 40-in. scattering chamber with a beam of 100 nA. These targets were prepared by evaporation of ^{6}Li on a thin nickel backing (50 μg/cm^2). The estimated thickness is 500 μg/cm^2. All the manipulations inside the scattering chamber were made under a dry atmosphere in order to avoid destruction of the target. A block diagram of the experimental arrangement is given in Fig. 2. The scattering chamber is equipped with two particle detectors (labeled 1 and 2) which select two particles emerging from the ^{6}Li$(\alpha, 2\alpha)$D reaction. The detector 1 is a 350-μ surface-barrier detector which serves to detect α particles of 6 to 20 MeV at angles θ_1 ranging from 44 to 70°. Detector 2 is a telescope made of two surface-barrier detectors 100 and 700 μ thick which serves to select α particles of 20 to 40 MeV at angles θ_2 ranging from 22 to 46°. The particle selection is obtained by a Goulding identifier system. The coincidence between the two α particles is measured by a time-to-pulse-height converter with an over-all time

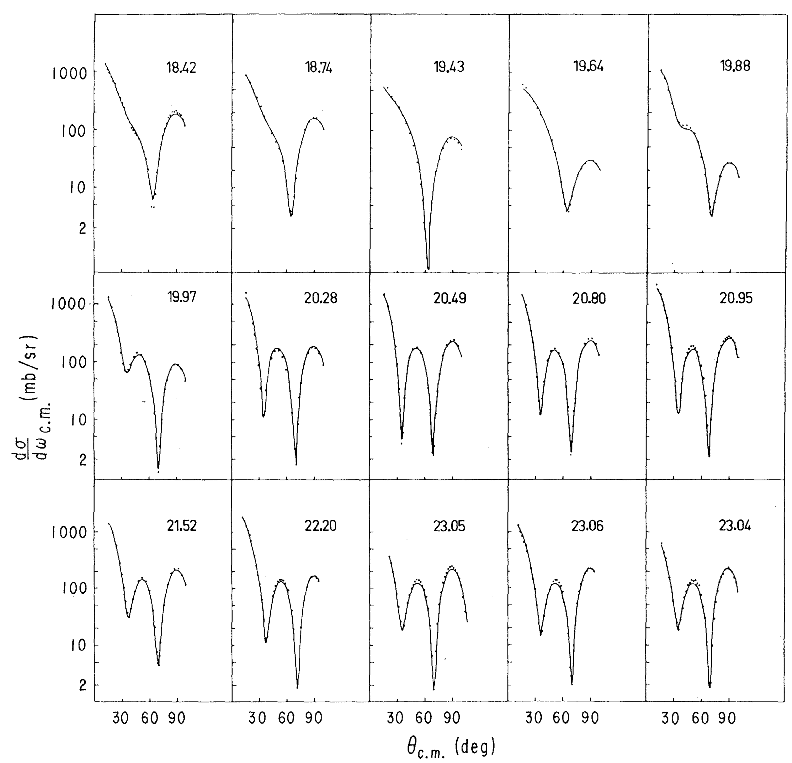

FIG. 1. Angular distribution of α-α scattering at different energies showing a large change near 40 MeV (lab) (from Ref. 9).

resolution of 5 nsec. The effective time resolution was due to the cyclotron pulses which are separated by 50 nsec. The coincidence signal is then used for triggering the linear chains. Since in free scattering collisions the sum E_1+E_2 must be equal to the total available energy, we have simplified the identification procedure by directly measuring this sum E_1+E_2 by means of a sum amplifier. A simplified block diagram of the electronics is represented in Fig. 2. After selection of the coincidence, the two signals E_1 and E_1+E_2 were sent to a biparameter analyzer where kinematic lines as well as sequential peaks can be observed.

In a collision where the residual deuteron is at rest in the laboratory frame, the incident and the two outgoing-particle trajectories lie in the same plane. Moreover, since the variations in E_1 and E_2 with θ_1 and θ_2 are of the order of 500 keV/deg, it is very important to work with small solid angles and to line up the collimators carefully. Counter 2 was collimated by a slit (2 mm$\times$5 mm)

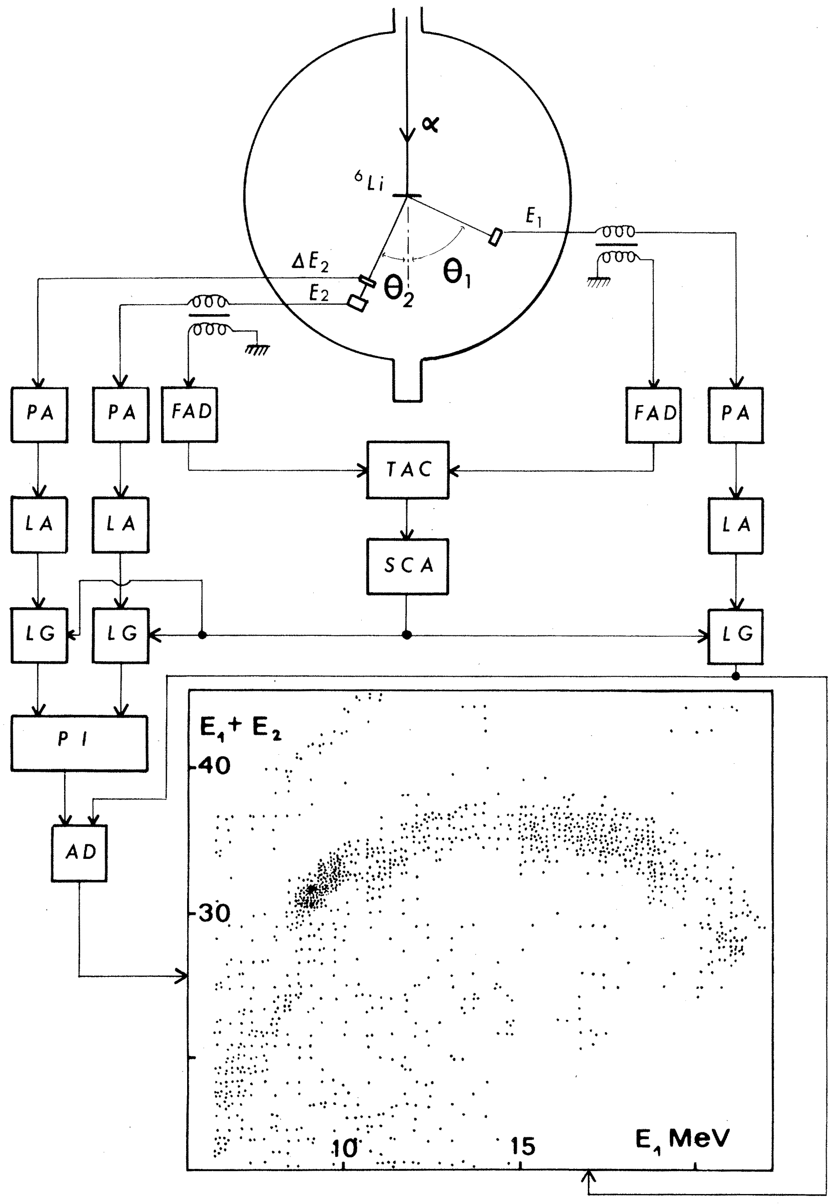

FIG. 2. Block diagram of the experiment ^{6}Li(α, 2α)D. The biparametric spectrum represents a measurement at the angles $\theta_{\alpha 1}=42.7°$ and $\theta_{\alpha 2}=45°$ with $T_{\alpha 0}=37.5$ MeV. We observe the sequential decay near the point $E_1=9$ MeV and $E_1+E_2\simeq 32$ MeV, and the quasifree α-α scattering near the point $E_1\simeq 15$ MeV and $E_1+E_2=36$ MeV. Symbols: PA=preamplifier, LA=linear amplifier, LG=linear gate, PI=particle identifier, AD=sum amplifier, FAD=fast amplifier discriminator, TAC=time-to-amplitude converter, SCA=single-channel analyzer.

located at a distance of 300 mm from the beam spot, and counter 1 was collimated by a slit (3 mm $\times$ 10 mm) located 70 mm from the beam spot. With this arrangement, we are sure to detect events with the deuteron at rest if the relative position of counter 1 and 2 is such that the difference $(\theta_1 - \theta_2)$ is in the neighborhood of 90°. Those events corresponding to $E_d < 80$ keV were considered to be good events.

The energy calibration of the cyclotron was done with counter 1 by comparison of the elastically scattered α particles from a thin C^{12} target at 140° with the α particles from a Th*C* source. Counter 2 was then calibrated with elastic and inelastic α particles on C^{12} at forward angles.

Measurements on the ^{6}Li$(\alpha, 2\alpha)$ reaction were taken at different positions of counters 1 and 2. They were monitored by integrating the current in a Faraday cup.

An example of a recorded spectrum is presented in Fig. 2. One observes a sharp peak due to sequential decay of the 2.18-MeV level in ^{6}Li and a kinematic line due to the ^{6}Li$(\alpha, 2\alpha)$ reaction. Several measurements were taken at different times at the same angle in order to avoid systematic errors. The results are summarized in Fig. 3 where

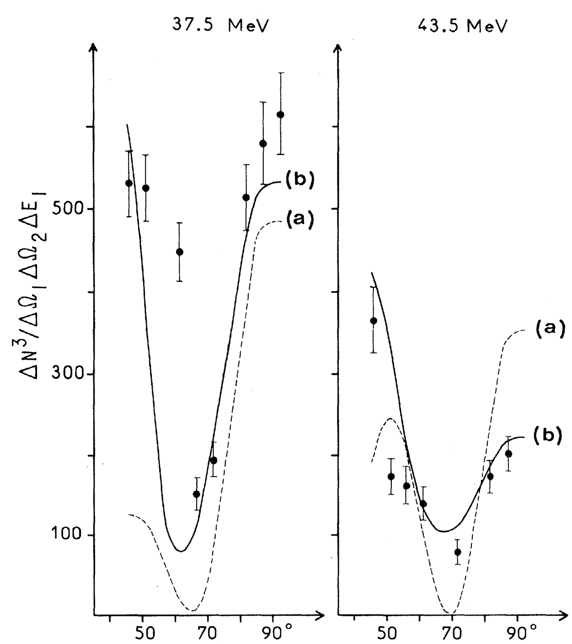

FIG. 3. Angular distribution of the ^{6}Li$(\alpha, 2\alpha)$D reaction with the deuteron as spectator. The experimental points are plotted versus the angle $\theta_{C.M.}(\alpha\alpha)$ of the quasifree α-α scattering: (a) fits with the single-pole terms corresponding to the impulse approximation; (b) fits with single-pole + triangular terms.

the two theoretical fits to the data are also shown. It is evident that a large difference in magnitude as well as in structure exists between the two curves. This result will be discussed in the conclusions.

III. ANALYSIS OF THE RESULTS

The analysis of ^{6}Li$(\alpha, 2\alpha)$D is made in the framework of the three-body reaction theory, using the graph formalism.[7] In our calculations, the ^{6}Li is described by the single α + D configuration, and the other configurations, such as ^{5}He + p, ^{5}Li + n, . . . , which are more strongly bound and which lead to more than a three-body problem, are neglected. With this hypothesis we can write the amplitude of the reaction, using the graphs shown in Fig. 4, where the vertex part of the ^{6}Li decay is described by the form factor $g(\vec{k})$, and each circle between the propagation lines of particles a and b corresponds to the off-energy-shell scattering amplitude $t_{ab}(\vec{p}, \vec{q}; \epsilon)$ in the presence of the third particle c. In the amplitude t_{ab}, $\vec{p}$ and $\vec{q}$ are the initial and final relative momenta, respectively, and ϵ the relative energy in the c.m. system. If the total energy is E in the system where the total momentum is $\vec{k}_0$, we have

$$E = \epsilon + k_c^2/2m_c + (\vec{k}_0 - \vec{k}_c)^2/2(m_a + m_b) .$$

In our calculations we make the approximation of replacing the unknown off-energy-shell scattering amplitude $t(\vec{p}, \vec{q}; \epsilon)$ by the well-known on-energy-shell amplitude $t(\theta; \epsilon)$, where θ is the scattering angle $(\cos\theta = \vec{p}\cdot\vec{q}/pq)$.

To calculate the form factor $g(\vec{k})$ of the ^{6}Li, it is necessary to know the Fourier transform $\phi(\vec{k})$ of ^{6}Li in its $\alpha + d$ configuration. The ground state of ^{6}Li is $J^\pi = 1^+$, and we assume that it is mainly formed by s waves. We have

$$\phi(\vec{k}) = 2\mu g(\vec{k})/(k^2 + \lambda^2) , \qquad (1)$$

where $g(\vec{k}) \simeq g_0 = \sqrt{2\pi\lambda}/\mu = 43.5$ fm$^{3/2}$ MeV in the zero-range approximation. In Eq. (1), $\vec{k}$ is the relative momentum of the two α-d particles and $\lambda^2/2\mu = 1.47$ MeV is the binding energy of α-d.

In the lab system the ^{6}Li particle is at rest and the incoming particle has momentum k_0. Let $\vec{k}_1$, $\vec{k}_2$, $\vec{k}_3$ be the lab momenta of the particles α_1, α_2, and d, respectively. We easily compute the first four graphs (single-pole), which correspond to the reaction amplitude of an initial state of spin m_s $(s = 1)$ to a final state with the spin $m_{s'}$. After symmetrization, this amplitude $S_{m_s m_{s'}}$ is

$$S_{m_s m_s'} = \sqrt{\tfrac{1}{2}}(S^{\alpha}_{m_s m_s'} + S^{d_1}_{m_s m_s'} + S^{d_2}_{m_s m_s'}) , \qquad (2)$$

with

$$S^{\alpha}_{m_s m_s'} = g_0 t_{\alpha\alpha}\left(\frac{\vec{k}_0+\vec{k}_3}{2}, \vec{k}_{12}; \frac{k_0^2}{2m_\alpha} - \frac{\lambda^2}{2\mu} - \frac{(\vec{k}_0-\vec{k}_3)^2}{4m_\alpha} - \frac{k_3^2}{2m_d}\right)\frac{2\mu}{k_3^2+\lambda^2}\delta_{m_s m_s'},$$

$$S^{d_1}_{m_s m_s'} = g_0 t^{m_s m_s'}_{\alpha d}\left(\frac{\vec{k}_0+2\vec{k}_1}{3}, \vec{k}_{23}; \frac{k_{23}^2}{2\mu}\right)\frac{2\mu}{k_1^2+\lambda^2}, \quad (3)$$

$$S^{d_2}_{m_s m_s'} \to S^{d_1}_{m_s m_s'} \quad \text{with } (1 \longleftrightarrow 2),$$

where $\vec{k}_{ij} = [1/(m_i+m_j)]\,(m_i\vec{k}_j - m_j\vec{k}_i)$ is the relative momentum of the two particles i and j.

To describe the α-d and α-α scattering, we have parametrized the corresponding scattering amplitude with the phase shift δ_1, determined from experiments (Ref. 8 for α-d and Ref. 9 for α-α).

The connection between the total amplitude T and the cross section for detection of the two particles in the lab directions Ω_1 and Ω_2 with an energy E_1 for one α particle is given by the following formula:

$$\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_1} = \gamma^2_{\alpha d}\frac{\mu_0 m_\alpha m_d}{(2\pi)^5\hbar^7 k_0(2s+1)} \times \sum_{m_s m_s'} |T_{m_s m_s'}|^2 \frac{k_1 k_2^2}{\frac{3}{2}k_2 - k_0\cos\theta_2 + k_1\cos\theta_{12}}, \quad (4)$$

where k_0/μ_0 is the relative velocity of α-^{6}Li, and θ_i and θ_{ij} are, respectively, the angles between $\vec{k}_0$, $\vec{k}_i$ and $\vec{k}_i$, $\vec{k}_j$. We have denoted by $\gamma^2_{\alpha d}$ the structure factor which describes the probability of the α-d configuration in ^{6}Li.

A. Single-Pole Terms

In the approximation of single-pole terms we have $T_{m_s m_s'} = S_{m_s m_s'}$, and the calculations corresponding to our measurements for $T_0 = 37.5$ MeV and $T_0 = 43.5$ MeV, with deuterons at rest, are shown in Fig. 3(a). The (χ^2/N) values were 24 and 10, respectively, for these two fits. The absolute cross sections of the reaction have not been measured but the two angular distributions are normalized between themselves. All the theoretical points are multipled by the same factor η, determined by

$$\eta = \frac{\sum_{\alpha i}(N_{\alpha i}C_{\alpha i}/\sigma_{\alpha i}^2)}{\sum_{\alpha i}(C_{\alpha i}^2/\sigma_{\alpha i}^2)}, \quad (5)$$

where the index α represents the two energies (37.5 and 43.5 MeV), and $C_{\alpha i}$, $N_{\alpha i}$, and $\sigma_{\alpha i}$ correspond, respectively, to the theoretical calculation, experimental data, and their errors.

We can make the following remarks about our fits:

(i) The α-d interaction is smaller than the α-α interaction ($|S^\alpha| \sim 10|S^{d_1}|$) so the theoretical curves result almost exclusively from the α-α interaction and correspond, in fact, to the impulse approximation.

(ii) The behavior of the experimental curves is roughly reproduced using the impulse approximation, but the forward parts of the angular distributions are not correct.

B. Triangular Graphs

The previous remarks suggest that the higher-order graphs are not negligible. Because of the importance of the α-α interaction, we have calculated only the graph represented in Fig. 4(c), this graph being the first correction term of the graph

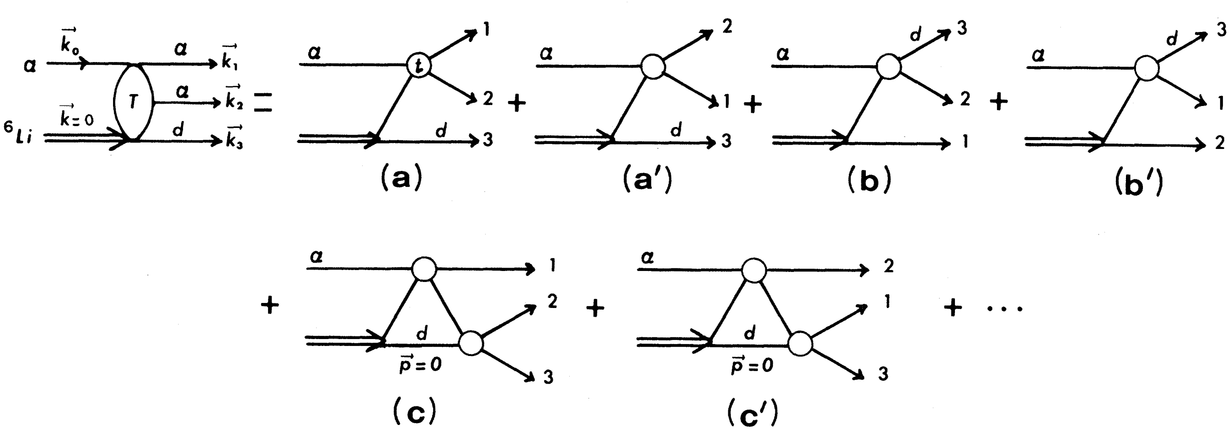

FIG. 4. Graphic development of the ^{6}Li(α, 2α)D reaction after symmetrization between the two α particles. (a), (a′), (b), (b′) correspond to the single-pole terms. (c), (c′) to the triangular terms.

(a). To calculate (c) it is necessary to integrate over-all values of $\vec{p}$ from 0 to ∞. However, the integrand has a term in $1/(p^2+\lambda^2)$, and λ is very small (~0.3 fm^{-1}), so it has a pole near $p=0$. The amplitudes $t_{\alpha\alpha}$, $t_{\alpha d}$ are then extracted from the integrand and their values are taken at $\vec{p}=0$. The calculation of the integral is then easy, and after symmetrization we find

$$\Delta_{m_s m_s'} = \sqrt{\tfrac{1}{2}}(\Delta^1_{m_s m_s'} + \Delta^2_{m_s m_s'}) \tag{6}$$

with

$$\Delta^2_{m_s m_s'} \to \Delta^1_{m_s m_s'} \quad \text{with } (1 \longleftrightarrow 2)\ ,$$

where

$$\Delta^1_{m_s m_s'} = g_0 t_{\alpha\alpha}\left(\tfrac{1}{2}\vec{k}_0,\ \vec{k}_1 - \tfrac{1}{2}\vec{k}_0;\ \frac{k_0{}^2}{4m_\alpha} - \frac{\lambda^2}{2\mu}\right) \times t_{\alpha d}^{m_s m_s'}\left(\tfrac{1}{3}\vec{q}_1, \vec{k}_{23}; \frac{k_{23}{}^2}{2\mu}\right) P_1 \tag{7}$$

with

$$P_1 = \frac{i m_\alpha \mu}{2\pi\hbar^3 q_1} \ln \frac{\lambda - i(k_{23} + \frac{1}{3} q_1)}{\lambda - i(k_{23} - \frac{1}{3} q_1)},$$

where

$$\vec{q}_1 = \vec{k}_0 - \vec{k}_1\ .$$

When $k_3 = 0$, $\vec{k}_{i3} = \frac{1}{3}\vec{q}_i$ ($i = 1$ or 2). Under these conditions, the scattering angle $\theta_{\alpha d}$ is constant and near zero. So, for every value of k_1, $t_{\alpha d}$ has a value which is almost constant and very large because of its Coulomb term $1/\sin^2\frac{1}{2}\theta_{\alpha d}$. This suggests writing $t_{\alpha d} = C_1/k_{\alpha d}$, because $\theta_{\alpha d} = 0$ is physically unacceptable. In this approximation, the total amplitude is

$$T_{m_s m_s'} = S_{m_s m_s'} + \Delta_{m_s m_s'} + C_2\ , \tag{8}$$

where

$$t_{\alpha d}^{m_s m_s'} = C_1/k_{\alpha d} \quad \text{in } \Delta_{m_s m_s'}\ .$$

The neglected graphs are taken into account by the complex constant C_2. The constants C_1 and C_2 are determined by a χ^2 fit with Hooke's method. The best fit is shown in Fig. 3(b), where $C_1 = -612 + 1472i$ and $C_2 = 2.5 - 57.2i$; the χ^2/N was, respectively, 4 and 2 for the curves at 37.5 and 43.5 MeV.

One can observe the following:

(i) C_1 is as large as can be expected because $\theta_{\alpha d} \simeq 0$.

(ii) The value of C_1 is such that the contributions of simple and triangular graphs are almost the same.

(iii) C_2 is small and the corresponding cross section represents approximately 5% of the cross section relative to the single-pole approximation:

$$\sum_{m_s m_s'} |C_2|^2 / \sum_{m_s m_s'} |S_{m_s m_s'}|^2 \approx 5\%\ .$$

(iv) In our measurements a very particular kinematic condition has been selected corresponding to deuterons with zero energy. This special condition seems to be in favor of the triangular graphs, which is, *a priori*, important (because of the approximation $\vec{p} = 0$ associated with the fact that $k_3 = 0$).

IV. CONCLUSIONS

The study of the reaction ^{6}Li(α, 2α)D at the two bombarding energies of 37.5 and 43.5 MeV, below and above the anomaly in the α-α free scattering amplitude around 40 MeV leads to the observations which follow:

(i) The experimental angular distributions of the quasifree scattering are notably different in magnitude and structure at the two energies. The distributions show a deep minimum at nearly the same angle as in the free collision.

(ii) Our analysis shows that the impulse approximation at these energies cannot correctly describe the experimental results.

(iii) Our calculations based on the graph formalism show that the α-α interaction is mainly responsible for the observed structure. We notice that the two angular distributions are correctly reproduced by the same set of parameters. Consequently, the differences observed experimentally are only associated with a rapid change of the α-α free scattering near 40 MeV.

This experiment and the theoretical analysis support the existence of an α particle in the ^{6}Li ground state.

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Effective *s*-Wave Interaction in Finite Nuclei

R. Muthukrishnan*
North Texas State University, Denton, Texas 76203
(Received 8 February 1971)

It is shown that a simple s-wave separable interaction whose parameters are determined by a fit to the binding energy and density of nuclear matter, when modified by making the strength a smooth function of the mass number A (such that the strength approaches that for nuclear matter as $A \rightarrow \infty$), gives the binding energy and radii of spherical nuclei throughout the Periodic Table in excellent agreement with experimental values. This behavior of the strength as a function of A is discussed in the light of the results of calculations which use realistic interactions.

I. INTRODUCTION

In recent years there has been great progress in attempts to relate nuclear properties to interactions that are derived from fits to the two-nucleon data.[1] These attempts have contributed considerably to the understanding of the various mechanisms involved in bringing about the basic properties of nuclei, such as saturation, the density dependence of the effective interaction, and the role of the various parts of the interaction in producing binding. These calculations are mainly based on the Brueckner-Goldstone many-body theory.[2]

Another approach to the above problem is to begin with an effective interaction which is suitable for Hartree-Fock calculations. This approach has the advantage of providing the means to calculate many properties of nuclei which can only be calculated with a prohibitive amount of work if one starts with realistic interactions. One of the attempts along this line is by Muthukrishnan and Baranger,[3] who used an s-wave separable interaction whose parameters were determined by fitting to the binding energy and equilibrium density of nuclear matter. The interaction is of the form

$$V(\vec{r}_1\vec{r}_2;\ \vec{r}_1'\vec{r}_2') = -\delta(\vec{R}-\vec{R}')(4\pi\alpha^2)^{-2}(\hbar^2\gamma/m\alpha) \times e^{-(r^2+r'^2)/4\alpha^2},$$

where m is the nucleon mass. The parameters α and γ (which is dimensionless) are

$$\alpha=\alpha_S=\alpha_T=1.175\ \text{F}, \quad \gamma=\gamma_S+\gamma_T=6.17,$$

with the subscripts S and T representing the singlet and triplet parts of the s-wave interaction, respectively. Since $\alpha_S=\alpha_T$, the expression for the binding energy of nuclear matter in first order involves only the sum of the strengths $\gamma_S+\gamma_T$. Thus the two parameters α and γ are determined by fitting to the binding energy per nucleon (15.5 MeV) and the Fermi momentum ($k_F=1.42\ \text{F}^{-1}$). The second-order correction to the energy of nuclear matter for the above interaction is $\simeq 0.2$ MeV/nucleon.[4] This was one of the stringent requirements imposed on the interaction so that one can use this interaction in Hartree-Fock calculations for finite nuclei. A detailed discussion of this point is given in Ref. 4. The results obtained for finite nuclei using this interaction are given in Table I and Fig. 1. The binding energies are too low and the radii are too large in comparison with the experimental values. However, as seen from Fig. 1, the binding energies (excluding Coulomb energy) can be fitted to the formula,

$$E/A=-a_V+a_S A^{-1/3},$$

with $a_V=14.7$ MeV and $a_S=24$ MeV. This is the familiar Bethe-Weizsäcker mass formula including only the volume and surface terms, which is appropriate for the present case, as the calculations did not include Coulomb energy and the neutron numbers are equal to the proton numbers. The empirical values for a_V and a_S are

$$a_V=15.85\ \text{MeV}, \quad a_S=18\ \text{MeV}.$$