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Note on Short-Range Correlations and Elastic Electron Scattering

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It is proved exactly that short-range correlations enter into diagonal matrix elements of any one-body operator only in 2nd and higher order if the single-particle wave functions have been chosen self-consistently as solutions of the Hartree-Fock equations. Deviations between experimental and theoretical single-particle form factors for elastic electron scattering due to short-range correlations can*not* be described by the *usual* Jastrow functions and generally are much smaller than hitherto assumed.

Several attempts have been made to account for the deviations between the single-particle (s.p.) and experimental form factors by short-range correlations.[1-3] It is the purpose of this note to show that their conclusions cannot be quite correct. We first prove the following theorem (which is almost trivial): Short-range correlations enter into diagonal matrix elements of a s.p. operator (between nondegenerate states) only in second order if one chooses self-consistent Hartree-Fock (HF) s.p. wave functions.

Proof: The nondegenerate wave function can be written without loss of generality as[4-6]

$$|\Psi\rangle = e^S|\Phi\rangle\,, \tag{1}$$

where the state $|\Phi\rangle$ is a determinant of s.p. states chosen such that $\langle\Phi|\Psi\rangle \neq 0$ and $S=\sum_{n=1}^{A} S_n$. S_n is an operator which excites n particle-hole pairs.

The quantity S_1 can be transformed away by choosing a self-consistent HF set of s.p. states.[6] Indeed, $e^{S_1}|\Phi\rangle = |\Phi'\rangle$ itself is a new determinant of new s.p. states – as is well known.[7] One has with a one-particle operator O_I

$$\begin{aligned}\langle\Psi|O_I|\Psi\rangle &= \langle\Phi'|e^{S^\dagger}O_I e^S|\Phi'\rangle \\ &= \langle\Phi'|O_I|\Phi'\rangle + \langle\Phi'|S^\dagger O_I|\Phi'\rangle \\ &\quad + \langle\Phi'|O_I S|\Phi'\rangle + \langle\Phi'|S^\dagger O_I S|\Phi'\rangle + \cdots\,.\end{aligned} \tag{2}$$

If $S=S_2+S_3+\cdots$ the two terms linear in S and $S^\dagger$ vanish, since then O_I excites or deexcites one particle, whereas S and $S^\dagger$ excite or deexcite at least two. This completes the proof.

The last term with $S\to S_2$ is the lowest- (second-) order correlation contribution. Indeed, in the first-order Born approximation the form factor for elastic electron scattering turns out to be

$$\begin{aligned}F(\vec q) = \frac{1}{A}\Bigg[&\sum_\nu \langle\nu|e^{-i\vec q\cdot\vec r}|\nu\rangle - 2\sum_{\nu\rho}\frac{\langle\nu|e^{-i\vec q\cdot\vec r}|\rho\rangle}{\epsilon_\rho-\epsilon_\nu-c_\nu} \\ &\times(\langle\rho|U|\nu\rangle - \langle\rho|U_0|\nu\rangle) + \tfrac{1}{2}\sum_{\mu,\nu}\langle S_{\nu\mu}|e^{-i\vec q\cdot\vec r}|S_{\nu\mu}\rangle \\ &-\tfrac{1}{2}\sum_{\mu,\nu,\lambda}\langle S_{\nu\mu}|S_{\nu\lambda}\rangle\langle\lambda|e^{-i\vec q\cdot\vec r}|\mu\rangle\Bigg]\,;\end{aligned} \tag{3}$$

ρ means all unoccupied states,

ν,μ,λ means all occupied states.

U_0 is the "input" s.p. potential defining $|\Phi\rangle$, and U is the HF potential defining $e^{S_1}|\Phi\rangle$. $S_{\nu\mu} = S_{\nu\mu}(\vec x_1, \vec x_2)$ is the amplitude for exciting two particles out of occupied states $|\nu\rangle$ and $|\mu\rangle$. Here the partial self-consistency condition of Kallio and Day[8] has been used, which amounts to putting

$$U = U_0 + \sum_\nu c_\nu |\nu\rangle\langle\nu|\,. \tag{4}$$

Choosing a HF self-consistent input s.p. potential, the two terms with U and U_0 cancel. With a Woods-Saxon potential it becomes rather small. One obtains two second-order correlation terms in addition to the s.p. contribution [1st term in the right-hand side of (3)]. These terms can be represented by the two diagrams of Fig. 1.

We have computed the correlation contributions using $S_{\nu\mu}$ from the Bethe-Goldstone equation[9] for ^{16}O. This has been done by us with partial fulfillment of the self-consistency conditions.[10] The results of those preliminary calculations clearly exhibit the smallness of the effect. The bulk of deviations can*not* be explained by short-range correlations. This is in line with Faessler's result[11] that he could rather well describe ^{40}Ca by his HF s.p. wave functions and with the observation by Donnely and Walker[12] that the cross sections sensitively depend on the s.p. wave functions.

It is in contrast to the results by the authors

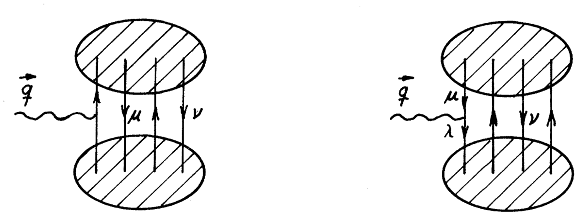

FIG. 1. Lowest order correlation contributions to the elastic form factor.

using Jastrow functions. It is clear that the Jastrow method as an *approximation* must be measured against the rigorous theory sketched here. In Refs. 1–3 the Jastrow contributions have been used in 1st order, which should vanish according to our results here. This requirement could be satisfied by modifying the Jastrow function by a projection operator projecting out of the occupied states. In Refs. 1 and 3, unfortunately, there also is an error in the sign due to choosing a positive definite $g(1, 2)$ in the correlation function defined by

$$|\Psi|^2 = |\Phi|^2[1 - \sum_{i<j} g(r_{ij})], \quad (5)$$

which is in disagreement with the correct result [last term in (2)]. Thus, even seemingly good fits of experimental results have nothing to do with correlations, at least if the latter are defined as deviations from the "best" shell-model wave function Φ. The experimental form factors may indicate, however, that here the "classical" nuclear theory breaks down and that one is no longer allowed to treat the nucleus as a many-body system with potential interaction. In this case the Jastrow as well as our method breaks down, however.

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Mean Lifetime of the 1.61-MeV Level in ^{37}Ar†

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The mean lifetime of the 1.61-MeV level of ^{37}Ar has been measured by applying the recoil-distance method to the ^{34}S$(\alpha, n)^{37}$Ar reaction. The mean lifetime was found to be 6.02 ± 0.29 nsec.

INTRODUCTION

The present investigation is a continuation of a study[1] of the low-lying levels of ^{37}Ar in which the lifetimes of the 1.41-, 2.22-, 2.49-, and 2.80-MeV levels were measured using the Doppler-shift attenuation method (DSAM). The $\frac{7}{2}^-$ level of ^{37}Ar at 1.61 MeV exhibited no Doppler shift in the DSAM work. Goosman and Kavanagh[2] obtained a mean lifetime of 5.15 ± 0.70 nsec for this level using the recoil-distance method[3] (RDM) with the stopper at only one distance from the target. In the present study a more precise value of the mean life of the 1.61-MeV level has been obtained using the RDM. This level is of interest, since it contains essentially all of the $1f_{7/2}$ single-particle strength.[4] Recent calculations by Harris[5] of radiative widths for $M2$-$E3$ transitions of the type $1f_{7/2} \rightarrow 1d_{3/2}$ for $33 \leq A \leq 41$ used existing lifetimes to obtain effective charges and moments. Improved measurements of radiative widths for transitions of this type are needed.