Comments and Addenda

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Distribution of Intensities of Thermal-Neutron-Capture Gamma Rays*

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The distribution of the intensities of thermal-neutron-capture γ rays is discussed within the framework of the statistical model. It is shown that if initial states of only one spin and parity are involved, the γ-ray intensities for most nuclides approximately satisfy the Porter-Thomas distribution, independent of how many resonances contribute to the thermal-neutron-capture cross section.

During the past decade the distribution of the widths of radiative transitions to a single low-energy state from a set of initial states formed by resonance neutron capture has been intensively studied, and for many nuclides the widths are found to be in good agreement with the Porter-Thomas distribution.[1] A related subject is the distribution of the intensities of radiative transitions to a set of low-energy states from the fixed initial energy (not necessarily an eigenvalue) of the system formed by thermal-neutron capture. Judging from what little has been written and spoken on this subject, there appears to be a widespread belief that the relative variance of the intensity distribution of the thermal-neutron-capture γ rays is inversely proportional to the effective number of resonances that contribute to the thermal cross section. Presumably this belief stems from the casual assumption that thermal-neutron capture involving the tails of several resonances is equivalent to capture in an energy interval in which one experimentally averages over several resonances. The purpose of this note is to point out that these two situations are not the same. The point is simply that the amplitude for a transition following thermal-neutron capture is proportional to the sum of *amplitudes* for the various contributing resonances, whereas the intensity (average width) of a transition following capture in an energy interval containing several resonances is proportional to the sum of *widths* of the resonances. Note that this distinction between the summing of amplitudes and widths is the same one involved in Erickson[2] fluctuations.

To derive explicitly the intensity distribution for thermal-neutron capture, let us consider transitions of a given kind (e.g., $E1$) to a single final state j from a set of initial states r having the same spin and parity. Also, assume that the width Γ_{rj} of these transitions is a random variable that satisfies the Porter-Thomas distribution,[1] and assume that all partial widths of the initial states are uncorrelated. Now let several of the initial states (neutron resonances) contribute to the thermal-neutron-capture cross section. Then, at the thermal-capture energy E the amplitude A_j for the transition to a final state j is

$$\begin{aligned} A_j &\propto \sum_r \frac{(\Gamma^0_{nr}\Gamma_{rj})^{1/2}}{E_r - E + \frac{1}{2}i\Gamma_r} \\ &= \sum_r \frac{(\Gamma^0_{nr})^{1/2}}{(E_r-E)^2 + \frac{1}{4}\Gamma_r{}^2}\left[(E_r - E) - \tfrac{1}{2}i\Gamma_r\right]\Gamma_{rj}{}^{1/2}, \end{aligned} \tag{1}$$

where Γ_r is the total width, $E_r{}^{1/2}\Gamma^0_{nr}$ is the neutron width, and E_r is the energy of neutron resonance r.

The assumed Porter-Thomas distribution of the width Γ_{rj} is equivalent to the assumption that $\Gamma_{rj}{}^{1/2}$ is a random variable with a normal distribution and zero mean. Hence, the form of Eq. (1) and a well-known theorem of statistics imply that A_j (consisting of sums of normally distributed random variables) is of the form

$$A_j \propto f_j + ig_j\,, \tag{2}$$

where f_j and g_j are correlated normally distributed random variables with zero means; and if the average width (average over many initial states) is independent of the final state,[3] the amplitudes of the transitions from the thermal-neutron-capture state to any set of final states will also satisfy a relationship with the same form as Eq. (2). That is, the intensities I of transitions to a set of final states satisfy the relation

$$I \propto f^2 + g^2 . \tag{3}$$

The implications of Eq. (3) may be discussed in terms of three special cases.

Case I. $\langle f^2 \rangle \gg \langle g^2 \rangle$, which applies when $E_r \gg \frac{1}{2}\Gamma_r$ for all resonances. Here the intensity $I \to f^2$ satisfies the Porter-Thomas distribution, independent of how many resonances contribute to the capture cross section. The approximation $\langle f^2 \rangle \gg \langle g^2 \rangle$ is accurate for most nuclides, since typically $\langle f^2 \rangle / \langle g^2 \rangle \approx D^2/\Gamma_r{}^2 \approx 10^4$, where D is the average level spacing. Any correlation between f and g strengthens this conclusion.

Case II. Complete correlation between f and g. This is true when one resonance (usually with $E_r < D$) makes the dominant contributions to both f and g. Here again the intensity satisfies the Porter-Thomas distribution.

Case III. $\langle f^2 \rangle \approx \langle g^2 \rangle$, and f and g are independent. This condition is mathematically possible but physically unlikely. It could happen if a weak resonance near the thermal energy dominates g, and strong resonances farther from the thermal energy dominate f. If the conditions are satisfied, then the intensity distribution is a χ^2 distribution with two degrees of freedom, i.e., an exponential function.

The above analysis shows that, for all except the unlikely case III, the intensities of the thermal-neutron-capture γ rays approximately satisfy the Porter-Thomas distribution when only one spin state is involved. However, for all real nuclides except those with target spin zero, two spin states can contribute to the thermal-neutron-capture cross section. Since the amplitudes for these two classes of states do not add coherently, the transition intensity is equal to the sum of two intensities, each of which satisfies a relationship such as Eq. (3). Thus, depending on the relative magnitudes of the two components, for most nuclides (those covered by cases I and II above) the distribution of the intensities of the observed γ-ray lines should be intermediate between χ^2 distributions with one and two degrees of freedom. The thermal-neutron-capture γ-ray intensities tabulated by Bartholomew[4] many years ago are consistent with this expectation.

There may be a few nuclides for which the special relationships between f and g specified by case III cause the intensity distribution to be significantly narrower than is indicated in the preceding paragraph. In the extreme situation, where both spin states contribute equally to the cross section, and the f and g functions for both spin states satisfy the conditions of case III, the distribution would be a χ^2 distribution with four degrees of freedom. Perhaps this kind of effect is at least partly responsible for the unusual degree of uniformity in the intensities[5] of the high-energy transitions of ^{167}Er$(n, \gamma)^{168}$Er.

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[1]C. E. Porter and R. G. Thomas, Phys. Rev. 104, 483 (1956).

[2]T. Erickson, Ann. Phys. (N.Y.) 23, 390 (1960).

[3]This assumption is supported by the results of L. M. Bollinger and G. E. Thomas, Phys. Rev. C 2, 1951 (1970). For simplicity, in the present discussion it is assumed that the average width is independent of γ-ray energy.

[4]G. A. Bartholomew, National Academy of Sciences Report No. NAS-NRC 974, 1962 (unpublished), p. 209.

[5]W. Michaelis, H. Ottmar, and F. Weller, Nucl. Phys. A150, 161 (1970).