# Study of the Matrix Element for $\beta$ Decay of $^{139}$Ba, $^{141}$Ce, and $^{148}$Pm by the $(e, e'p)$ Reaction

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The ratio $|M^\beta/\xi i\int\vec{r}|$ for the first-forbidden $\beta^-$ decay has been studied to discuss the composition of $M^\beta$. Experimental values of $|M^\beta|$ and $|i\int\vec{r}|$ are estimated from $ft$ values and from the $E1$ matrix element. The latter was obtained from the $(e,e'p)$ reaction through the isobaric analog states in $^{139}$La, $^{141}$Pr, and $^{148}$Sm. The experimental value of the ratio was compared with the theory using a $j$-$j$-coupling shell model. The result shows good agreement for $^{141}$Ce but not always good agreement for $^{139}$Ba and $^{148}$Pm.

## I. INTRODUCTION

The experimental study for the composition of the matrix element of $\beta$ decay is usually difficult, because many component matrix elements contribute in complex ways to the total matrix element $M^\beta$. Some of the component matrix elements have been studied with polarization experiments, angular-correlation experiments, etc. in $\beta$ decay, but not enough information has been accumulated to explain the composition of $M^\beta$.

Another study for the composition of $M^\beta$ in the first-forbidden $\beta^-$ decay will be presented in this paper. One of the most important $\beta$ matrix elements $|i\int\vec{r}|$ has been impossible to determine directly by $\beta$ decay experiments, but it should be possible to estimate its value from the matrix element of the $E1$ transition through isobaric analog states (hereafter referred to as IAS) when the $\gamma$ transition is analogous to the $\beta$ transition.[1,2] The relation can be expressed as follows:

$$\begin{aligned}\left|i\int\vec{r}\right|^2 &= \sum_{m_f} |\langle f\,|[\vec{r}_\gamma, T_-]|P\rangle|^2\\ &= \frac{4\pi}{3}\sum_{\mu,m_f} |\langle f|[rY_{1\mu}, T_-]P\rangle|^2\\ &\simeq 2(T+1)\frac{4\pi}{3}\sum_{\mu,m_f}\left|\left\langle f\left| r\,Y_{1\mu}\frac{T_-}{[2(T+1)]^{1/2}}\right|P\right\rangle\right|^2\\ &\simeq 2(T+1)\frac{4\pi}{3}\sum_{\mu,m_f} |\langle f\,|rY_{1\mu}|\mathrm{IAS}\rangle|^2\\ &\simeq 2(T+1)(4\pi/3)|M^\gamma_{\mathrm{IAS}}(E1)|^2;\end{aligned}$$

i.e.,

$$\left|i\int\vec{r}\right| \simeq [2(T+1)(4\pi/3)]^{1/2}|M^\gamma_{\mathrm{IAS}}(E1)|,$$

where $|P\rangle$ is the parent state and $|f\rangle$ is the daughter state of the $\beta$ decay and they correspond to the IAS (isospin of $T+1$) and the final state (isospin of $T$) for an $E1$ transition. Examples are the first-forbidden $\beta$ transitions of $^{139}$Ba, $^{141}$Ce, and $^{148}$Pm feeding the ground state. The $E1$ transition between the ground IAS and the ground states in $^{139}$La, $^{141}$Pr, and $^{148}$Sm is analogous to the $\beta$ decay mentioned above. The relations are indicated in Fig. 1. The available shell-model estimate is indicated for the related state in the figure. The matrix element for the $E1$ transition can be obtained from the radiative width by measuring the integrated photoproton cross section for the IAS with the $(e,e'p)$ reaction.[2] If the $\beta$ matrix element $|i\int\vec{r}|$ can be determined in this way, the validity of the theoretical composition of $M^\beta$ can be examined with a method shown in the next section.

## II. THEORETICAL TREATMENT

One of the typical equations which have been used for the estimation of $M^\beta$ was obtained by Morita *et al.* and by Buhring with a uniform charge distribution.[3] This equation will be applied to the theory in the present paper. Additional theoretical relations between several matrix elements can be used which are known from the conserved-vector-current (CVC) theory[4]:

$$\int\vec{\alpha}\Big/ i\int\vec{r} = \xi\Lambda = W_0 - 2.5 + \xi\lambda,$$

$$\int\gamma_5\Big/ i\xi\int\vec{\sigma}\cdot\vec{r} = \Lambda_0,$$

where $\xi = \alpha Z/2R$ and $\lambda = 2.4$. The constant $\Lambda_0$ depends on the matrix element of order zero, so it vanishes except for the $\Delta j = 0$ transition, for which

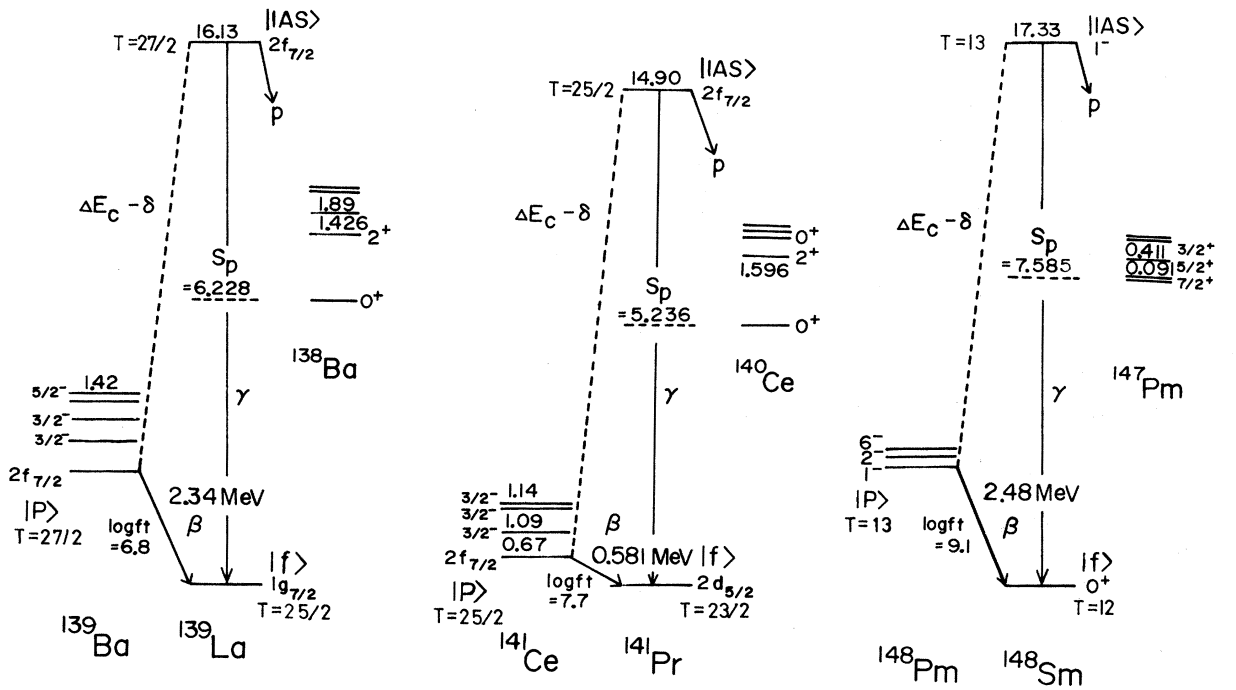


FIG. 1. The level diagrams for the relations of $E1$ IAS and $\beta$ decay. The energy levels are indicated in units of MeV. The shell-model estimations for the related levels are indicated.

the value is nearly equal to 1 or 2 according to the approximation. When the $j$-$j$-coupling shell-model theory is applied, additional relations can be given:

$$-\int \vec{\sigma}\times\vec{r}\Big/ i\int \vec{r} = \Lambda_1,$$

$$\Lambda_1 = j_i(j_i+1) - l_i(l_i+1) - j_f(j_f+1) + l_f(l_f+1),$$

$$\int \vec{\sigma}\cdot\vec{r}\Big/ i\int\vec{r} = \left(\int\vec{\sigma}\cdot\vec{r}\Big/ i\int\vec{\sigma}\times\vec{r}\right)\left(i\int\vec{\sigma}\times\vec{r}\Big/ i\int\vec{r}\right),$$

$$= -i\Lambda_1 A(j,l),$$

$$\int B_{ij}\Big/ i\int\vec{r} = \left(\int B_{ij}\Big/ i\int\vec{\sigma}\times\vec{r}\right)\left(i\int\vec{\sigma}\times\vec{r}\Big/ i\int\vec{r}\right),$$

$$= -i\Lambda_1 B(j,l),$$

$$\int \gamma_5\Big/ i\int\vec{r} = \left(\int\gamma_5\Big/ i\xi\int\vec{\sigma}\cdot\vec{r}\right)\left(i\xi\int\vec{\sigma}\cdot\vec{r}\Big/ i\int\vec{r}\right),$$

$$= \xi\Lambda_0\Lambda_1 A(j,l),$$

where $\int\vec{\sigma}\cdot\vec{r}/i\int\vec{\sigma}\times\vec{r} = A(j,l)$ and $\int B_{ij}/i\int\vec{\sigma}\times\vec{r} = B(j,l)$ are represented with only $j$ and $l$ by Lipnik and Sunier.[5] Applying these relations, $i\int\vec{r}$ can be extracted from $M^\beta$. The factor $|M^\beta/\xi i\int\vec{r}|$ can be determined with the matrix-element ratios mentioned above. When the ratios are not changed strongly, even in the case that $\int\vec{r}$ is hindered more or less according to the relevant states, the value of $|M^\beta/\xi i\int\vec{r}|$ depends only on the composition of $M^\beta$. It will be interesting to compare this proportionality factor with the theoretical and the experimental values.

## III. EXPERIMENTAL METHOD AND RESULTS

Self-supporting metal foils of $^{139}$La (99.9% natural abundance, 10 and 13 mg/cm$^2$), $^{141}$Pr(100% natural abundance, 10 and 13 mg/cm$^2$), $^{148}$Sm(95.4% enriched, 4.9 mg/cm$^2$) were bombarded by the electron beam of the Tohoku University linear accelerator. Energy distributions of protons from the $(e,e'p)$ reaction were measured with a broad range magnetic spectrometer at $\theta = 125.3°$. Examples of the results are shown in Fig. 2. The position of the maximum end-point energy of protons and that of $p_0$ through the ground IAS are indicated. The differential cross sections of the $(e,e'p)$ reaction at $\theta = 125.3°$ were obtained from the numbers of all the protons of $E_p > 5.3$ MeV. The contribution of lower-energy protons was small and neglected for the present cross sections.

The results are shown in Fig. 3. Based on the virtual-photon theory, they are analogous to the photoproton yield curves. The expected positions of $E1$ IAS are indicated by arrows with the single-particle configuration or spin-parity. The extreme left arrow for each curve corresponds to the ground IAS which is connected to the $\beta$-decay prob-

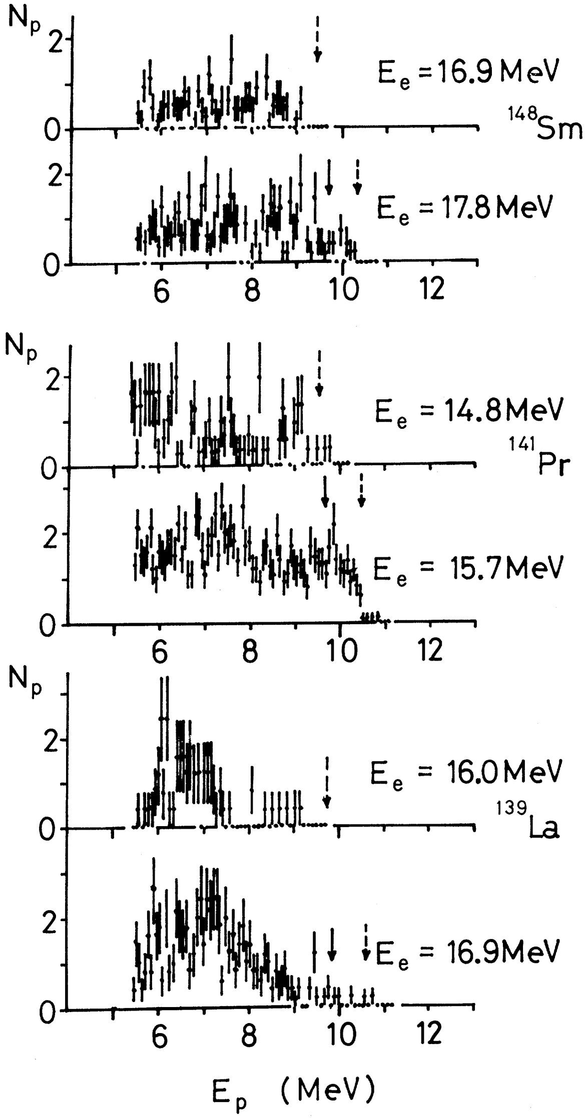


FIG. 2. Examples of the proton energy distributions. The positions expected for the maximum end-point energy of protons are shown by the dashed vertical arrows. The solid vertical arrows indicate the position of $p_0$ through the ground IAS.

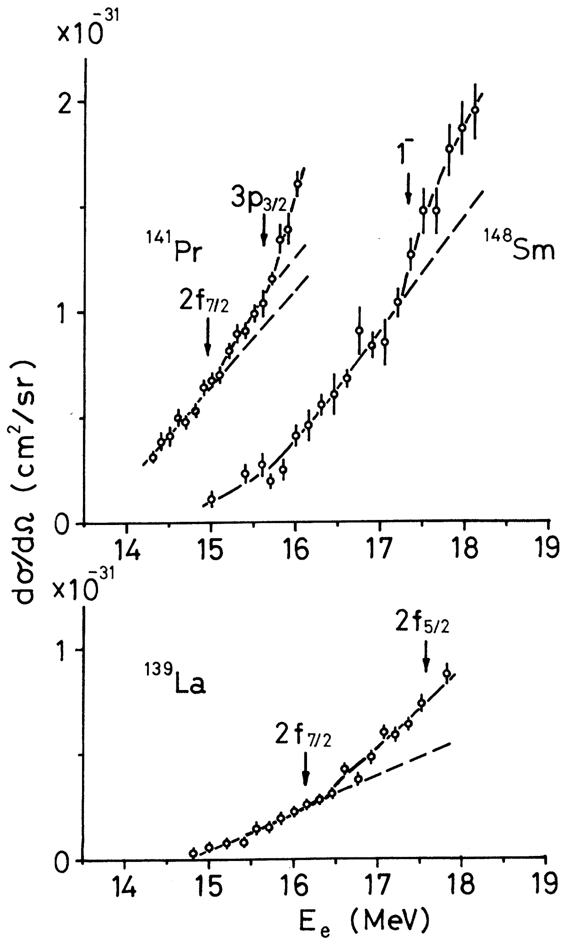


FIG. 3. Cross section of the $^{139}$La$(e,e'p)$, $^{141}$Pr$(e,e'p)$, and $^{148}$Sm$(e,e'p)$ reactions at $\theta=125.3°$. The positions of IAS are shown by arrows labeled by the single-particle shell state or spin-parity.

lem. The energies corresponding to the sharp rises in the experimental curves agree well with the expected positions, with the exception of the $^{139}$La case, which shows a small but acceptable shift within 300 keV. The integrated photoproton cross sections for these IAS were calculated from the increment of the $(e,e'p)$ cross section with a method described in the preceding paper.[2]

The radiative widths of the resonances at the IAS ($\Gamma_\gamma^R$) were calculated from the integrated cross sections. The interference between the IAS and the giant-dipole resonance was calculated, and the noninterferent component of the radiative width of the IAS ($\Gamma_\gamma^{\mathrm{IAS}}$) was estimated with a treatment described in Ref. 2. The result is shown in Table I accompanying the corresponding matrix element ($M_{\mathrm{IAS}}^{\gamma}$). The present value for $^{141}$Pr is in good agreement with a previous $(p,\gamma_0)$ result.[1] Single-particle transition widths were estimated by $2(T+1)\Gamma_\gamma^{\mathrm{IAS}}$, where $T$ is the isospin of the ground state of the target nucleus. Table I also shows their ratios to the Weisskopf unit and to a single-particle radiative width for the transition between the indicated shell states calculated with an infinite square-well potential.

Based on the pairing theory, the single-particle radiative width should be hindered by a factor $\eta^2$, which is similar to that of the corresponding $\beta^-$ decay; i.e., $\eta^2 = U_i^2(p)U_f^2(n)$ for odd-$Z$ even-$N$ target nuclei and $\eta^2 = V_f^2(p)U_f^2(n)$ for even-even target nuclei, where $V^2$ ($U^2$) is the probability of

TABLE I. $\gamma$ matrix elements for the ground $E1$ IAS. The errors include statistical uncertainties only.

| Target nucleus | Ground state | IAS | $E_x^{\mathrm{IAS}}$ (MeV) | $\Gamma_\gamma^{\mathrm{R}}$ (eV) | $\Gamma_\gamma^{\mathrm{IAS}}$ (eV) | $2(T+1)\frac{\Gamma_\gamma^{\mathrm{IAS}}}{\Gamma_{\mathrm{W}}}$ | $2(T+1)\frac{\Gamma_\gamma^{\mathrm{IAS}}}{\Gamma_{\mathrm{sp}}}$ | $F^{-1}$ $\left[=2(T+1)\frac{\Gamma_\gamma^{\mathrm{IAS}}}{\Gamma_{\mathrm{sp}}}\frac{1}{\eta^2}\right]$ | $\lvert M_{\mathrm{IAS}}^\gamma\rvert$ (F) |
|---|---|---|---|---|---|---|---|---|---|
| $^{139}$La | $1g_{7/2}$ | $2f_{7/2}$ | 16.1 | $52\pm17$ | $35\pm12$ | $0.12\ \pm0.04$ | $37\pm12$ | $90\pm30$ | $0.089\pm0.015$ |
| $^{141}$Pr | $2d_{5/2}$ | $2f_{7/2}$ | 14.9 | $32\pm5$ | $15\pm2$ | $0.059\pm0.009$ | $0.072\pm0.011$ | $0.12\ \pm0.02$ | $0.064\pm0.005$ |
| $^{148}$Sm | $0^+$ | $1^-$ | 17.3 | $68\pm7$ | $37\pm4$ | $0.096\pm0.01$ | $0.018\pm0.002$ [a] | $0.029\pm0.003$ [a] | $0.082\pm0.004$ |
| | | | | | | | $4.1\ \pm0.4$ [b] | $4.9\ \pm0.5$ [b] | |

[a]The single-particle transition ($2f_{7/2}\rightarrow 2d_{5/2}$) is assumed. [b]The single-particle transition ($2f_{7/2}\rightarrow 1g_{7/2}$) is assumed.

occupation (nonoccupation) and $f$ and $i$ denote the ground state of the target nucleus and the parent state of the IAS, respectively.[6] In the present nuclear region ($N\approx 82$), $U_f^2(n)=1$ may usually be taken for the IAS. The hindrance factor $F^{-1}$ to the single-particle transition is estimated from the equation $F^{-1}=2(T+1)(\Gamma_\gamma^{\mathrm{IAS}}/\Gamma_{\mathrm{sp}})1/\eta^2$ using Rho's value[7] for $U^2{}_i(p)$ and $V^2{}_i(p)$. The result is also shown in Table I. The result seems anomalously large for the spin-flip transition ($1g_{7/2}$-$2f_{7/2}$) in $^{139}$La. This is quite similar to the case[2] of $^{209}$Bi, though the single-particle description may be less exact for $^{139}$La than $^{209}$Bi. Precise discussions of the radiative width are difficult for $^{148}$Sm because the theoretical value of $\Gamma_{\mathrm{sp}}$ depends sensitively on the configuration mixing, which is not known exactly. Two examples of calculations are indicated in the extreme cases.

From a $\beta$-decay point of view, the matrix element $\lvert i\int\vec{r}\rvert$ can be estimated from $\lvert M_{\mathrm{IAS}}^\gamma\rvert$ as mentioned before. The result is shown in Table II. The table also shows the experimental $\log ft$ and the total $\beta$ matrix element $\lvert M^\beta\rvert$ which is calculated from the $ft$ value. The factor $\lvert M^\beta/\xi i\int\vec{r}\rvert$ was obtained from them and compared with the theoretical estimate in the table. For the theory, radial electron wave functions at the nuclear surface were used.

## IV. DISCUSSION

*(i)* $^{139}Ba\rightarrow{}^{139}La$. The single-particle transition is $2f_{7/2}\rightarrow 1g_{7/2}$. Therefore, the matrix element of order zero must contribute and depend on the value of $\Lambda_0$. This dependence of $\lvert M^\beta\rvert$ is studied for various values of $\Lambda_0$ between 0 and 2. The result shows only slight changes, within 6%, and shows a minimum at $\Lambda_0=0.95$. The value shown in the table is taken for $\Lambda_0=0.95$. The theoretical value of $\lvert M^\beta/\xi i\int\vec{r}\rvert$ is six times as large as the experimental estimate. This discrepancy might indicate an incorrect composition in the theoretical $\beta$ matrix element and/or impropriety of the estimate of $\lvert i\int\vec{r}\rvert$ from the $\gamma$ transition in this nucleus. The correction for the electron wave function which was neglected in the present estimate of the $\beta$ matrix element $\lvert i\int\vec{r}\rvert$ might improve the disagreement but it does not seem to be enough.

*(ii)* $^{141}Ce\rightarrow{}^{141}Pr$. The single-particle transition is $2f_{7/2}\rightarrow 2d_{5/2}$. Agreement of $\lvert M^\beta/\xi i\int\vec{r}\rvert$ between theory and experiment is very good. Ejiri *et al.*[1] studied this transition using the $^{140}$Ce($p,\gamma_0$) reaction. They obtained the value $\lvert M^\beta/\xi i\int\vec{r}\rvert=0.48\pm0.07$, which is in good agreement with the present value.

TABLE II. Comparison of $\beta$ matrix elements. The errors include statistical uncertainties only. The error of log$ft$ is not included.

| Transition | log$ft$ | $\lvert M^\beta\rvert$ from $ft$ value (F) | $\lvert i\int\vec{r}\rvert$ $\left\{=\left[\frac{4\pi}{3}2(T+1)\right]^{1/2}\lvert M_{\mathrm{IAS}}^\gamma\rvert\right\}$ (F) | $\lvert M^\beta/\xi i\int\vec{r}\rvert$ Experiment | Theory: Eq. from Ref. 3 | Theory: $\xi$ approximation |
|---|---|---|---|---|---|---|
| $^{139}$Ba($2f_{7/2}$)$\rightarrow$ $^{139}$La($1g_{7/2}$) | 6.8 | 12.2 | $0.95\pm0.16$ | $1.0\ \pm0.2$ | 6.5 | 8.0 |
| $^{141}$Ce($2f_{7/2}$)$\rightarrow$ $^{141}$Pr($2d_{5/2}$) | 7.7 | 4.32 | $0.66\pm0.05$ | $0.50\ \pm0.04$ | 0.57 | 0.2 |
| $^{148}$Pm($1^-$) $\rightarrow$ $^{148}$Sm($0^+$) | 9.1 | 0.880 | $0.86\pm0.04$ | $0.075\pm0.004$ | 0.79 [a] | 0.5 [a] |
| | | | | | 10.8 [b] | 8.0 [b] |
| | | | | | 0.52 [c] | |

[a]Complete single-particle transition ($2f_{7/2}\rightarrow 2d_{5/2}$) is assumed.
[b]Complete single-particle transition ($2f_{7/2}\rightarrow 1g_{7/2}$) is assumed.
[c]The ratios between matrix elements obtained from shape factor (see Ref. 9) were used. The contribution of $B_{ij}$ is neglected.

The shape factor $C$ is defined by

$$f_0|M^\beta|^2=\int_1^{W_0} pW(W_0-W)^2F(Z,W)C\,dW\,.$$

This factor is usually expressed by a convenient type with a conversion $C=\kappa C'$.[8] The factor $C'$ is approximated as $C'=1+aW$ in the present discussion. The shape factor in the present calculation was obtained as $C'=1-0.22W$, which agrees fairly well with the experimental result[9] $C'=1-(0.28\pm0.02)W$.

*(iii)* $^{148}Pm \rightarrow {}^{148}Sm$. The spin-parity of the ground state of $^{148}$Pm was estimated as $1^-$ by Baba, Ewan, and Suarez.[10] This estimate is confirmed with the existence of $E1$ IAS of this state in the present result. For the comparison of $|M^\beta/\xi i\int\vec{r}|$, two calculations were made for examples of the complete single-particle transitions ($2f_{7/2}\rightarrow 2d_{5/2}$ and $2f_{7/2}\rightarrow 1g_{7/2}$) because the configuration is not known exactly.

The theoretical estimate is larger by about two orders of magnitude than the experimental one for $2f_{7/2}\rightarrow 1g_{7/2}$. In the case of $2f_{7/2}\rightarrow 2d_{5/2}$, the disagreement is about an order of magnitude.

Baba, Ewan, and Suarez[10] measured the shape factor of this $\beta$ decay. They deduced some ratios of the $\beta$ matrix elements from the fits to the shape factor. They obtained $\int\vec{\alpha}/i\int\vec{r}=34$ (from CVC theory) and $\int\vec{\sigma}\times\vec{r}/i\int\vec{r}=-2.2\pm0.4$. Table II also shows $|M^\beta/\xi i\int\vec{r}|$ calculated by using their values. The result is about an order of magnitude greater than the experimental result. They estimated the value of $i\int\vec{r}$ as 1% of the nuclear radius, which is about 10 times as small as the present experimental value.

The shape factors obtained in the present calculation are $C'=1-0.11W$ for $2f_{7/2}\rightarrow 2d_{5/2}$ and $C'=1+0.34W$ for $2f_{7/2}\rightarrow 1g_{7/2}$. The experimental value is estimated as $C'=1+0.18W$ based on the data by Baba, Ewan, and Suarez.[10] From a shape-factor point of view, the main contribution seems to be $2f_{7/2}\rightarrow 1g_{7/2}$ as shown by Baba, Ewan, and Suarez.[10] However, this configuration does not explain well the factor $|M^\beta/\xi i\int\vec{r}|$. This discrepancy may be similar to the situation in the $^{139}$Ba case.

The $\beta$ matrix elements $M^\beta$ in the present transitions can be more crudely expressed on the basis of the $\xi$ approximation. The calculation of $|M^\beta/\xi i\int\vec{r}|$ was also made using the $\xi$ approximation and the Ahrens-Feenberg approximation[11] ($\Lambda_0\approx1$) and $j$-$j$ coupling shell model. The results are included in Table II.

The present result shows that agreement with the single-particle estimation and with the $\beta$-decay experiment is found to be good in $^{141}$Pr but not always good in $^{139}$La and $^{148}$Sm. This suggests that the important effects appear in the $1g_{7/2}\rightarrow 2f_{7/2}$ spin-flip transition.

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