Combining the first term in Eq. (A.4) with Eq. (A.5), we have the amplitude M_D corresponding to (symmetrized) scattering of the incident boson from each of the bosons in the target, in impulse approximation. The second term in Eq. (A.4) corresponds to HPT (and was called M_{HPT} for that reason).

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$$\int_0^\infty dx\, x e^{-\alpha x} j_L(\beta x) = \frac{\sqrt{\pi}\,\beta^L (L+1)!}{2^{L+1}\Gamma(L+\frac{3}{2})(\alpha^2+\beta^2)^{1+\frac{1}{2}L}} \times {}_2F_1\left(1+\tfrac{1}{2}L, \tfrac{1}{2}L;\ \tfrac{3}{2}+L;\ \frac{\beta^2}{\alpha^2+\beta^2}\right).$$

[11]Note that in elastic electron scattering the contributions of the different ^{12}C intermediate states to the form factor are coherent and give (see Ref. 9)

$$F_{el}(q) = \int_0^\infty dr\, r^2 j_0(\tfrac{3}{4}qr)\sum_{\nu J}|C_{\nu J}^{(0)}|^2 |\phi_{\nu J}^{(0)}(r)|^2.$$

Since the $|\phi_{\nu J}^{(0)}(r)|^2$ have roughly the same shape, at moderate values of q the above expression is almost indistinguishable from the result of Ref. 9.

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Precompound Decay from a Time-Dependent Point of View*

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A model is proposed for following the progress of nucleon-induced nuclear reactions as a function of time. It predicts that the majority of protons and high-energy neutrons emitted from heavy targets in these reactions appear before internal statistical equilibrium has been established. However, for the 18-MeV (p,xn) reaction with Ta^{181} the model predicts fewer high-energy neutrons than are observed experimentally. Further, there is a difference in shape between the predicted and observed spectrum. The same differences arise when a comparison is made between the experimental spectrum and that predicted by Griffin's model for decay. It is suggested that these differences arise because both models fail to consider surface reactions explicitly.

It is also suggested that this model might best be used in conjunction with Monte Carlo cascade calculations of high-energy nuclear reactions to follow the complete equilibration of the nonequilibrated excited nuclei left after these cascades are terminated. An example of using the model for this purpose is given.

I. INTRODUCTION

The experimental results from both high-energy and medium-energy nucleon-induced reactions have usually been interpreted in terms of essentially similar two-step models. At high energies where the bombarding nucleon has an energy $\gtrsim 100$ MeV, the reaction is assumed to proceed in the following fashion:

(a) In the first, or fast step, the incident nucleon develops a cascade in the target nucleus through a series of binary nucleon-nucleon collisions in which some particles escape.

(b) In the second, or slow step, the residual excited nucleus deexcites through the emission of other nucleons, clusters of nucleons, or γ rays.

In the first step the incident nucleon excites very few degrees of freedom in the target nucleus. However, in past calculations of the second step it has been assumed that many degrees of freedom in the residual nucleus are excited. In fact, it has been assumed that the residual nucleus was in internal statistical equilibrium. Hence, its deexcitation could be treated by the statistical theory of nuclear reactions. However, this last assumption requires that the residual excited nucleus equilibrate in such a manner that no further nucleons are emitted between the end of the fast step and the completion of internal equilibration. In a previous article,[1] hereafter denoted as I, this assumption was examined and it was found to be essentially valid for a highly excited one-component Fermi gas which had the gross characteristics of a nucleus. This conclusion was reached through the solution of a set of Boltzmann-like master equations for the evolution of the occupation numbers of the single-particle states of the system while at the same time allowing for the escape of particles.

The usual model for medium-energy reactions, where the bombarding nucleon has an energy between ~15 and ~50 MeV, is the following:

(a) In the first step, it is assumed that the incident nucleon interacts with one or more nucleons in the target in such a manner that some of them may escape. In analogy with the first step in the model for high-energy reactions, this step excites only a few degrees of freedom in the target nucleus and is thought to occur relatively fast. This step has usually been denoted as the direct component of these medium-energy reactions.

(b) The second step of these reactions is assumed to be completely analogous to the second step of high-energy reactions. That is, it is assumed that many degrees of the residual nucleus are excited or, equivalently, that the residual nucleus is in statistical equilibrium.

Recently, Griffin[2] proposed a slightly different model for these medium-energy reactions in which an attempt is made to calculate the decay probability of an excited nucleus at each stage of its approach to statistical equilibrium. Blann,[3] and Blann and Lanzafame[4] recently expanded on Griffin's model, and Williams[5] has examined some of the details in its original derivation.

In the following, an attempt is made to describe the progress of medium-energy reactions as a function of time by a model which, in some respects, is similar to Griffin's. That is, this model will also monitor the escape of nucleons as more and more degrees of freedom in the emitting nucleus are excited until, finally, the emitting nucleus is in statistical equilibrium. This will be accomplished by solving a set of Boltzmann-like master equations for the evolution of the single-particle-state occupation numbers of a two-component Fermi gas which again has some of the gross characteristics of a nucleus.

The main results obtained by solving these equations for targets whose mass number is greater than approximately 100 are:

(1) A portion of the experimentally observed high-energy tail in the emitted neutron spectra from medium-energy (p, xn) reactions is predicted by these equations. As is well known, a statistical-model calculation would not predict the presence of these high-energy neutrons, because of the low temperatures or excitation energies of the emitting nuclei. Further, these high-energy neutrons are emitted before statistical equilibrium has been reached.

(2) Nearly all of the protons emitted from proton- and neutron-induced reactions at these energies appear before equilibrium has been established. A statistical-model calculation would predict a very small emission probability for protons from these nuclei because of their high Coulomb barriers.

(3) The conclusion that few particles escape during the equilibration phase of high-energy nuclear reactions that was made in I does not change when a two-component Fermi system is considered.

In Sec. II the model that was used in these calculations is examined. Some of the results and conclusions from this study are presented in Sec. III.

II. MODEL EMPLOYED

The assumptions used in developing the master equations for the relaxation of a two-component Fermi gas are essentially the same as those given in I for the relaxation of a one-component gas, except for minor modifications.

The nucleus is viewed as being composed of independent proton and neutron Fermi gases. Therefore, the proton and neutron occupation numbers for the single-particle states of these gases completely specifies the internal configuration of the nucleus at any time. Further, it is assumed that the mechanism for the equilibration of the gases is through binary nucleon-nucleon collisions. These two gases are initially confined to translational states within a volume $V=\frac{4}{3}\pi r_0^3 A$, where $r_0 = 1.5\times 10^{-13}$ cm and A is equal to the total number of nucleons within the nucleus at time $t=0$.

For medium-energy reactions it is assumed that

$A-1$ of the nucleons are in their ground state (i.e., two zero-degree Fermi gases with a total of $A-1$ nucleons). It is further assumed that initially the incident nucleon occupies an internal translational state whose energy is the sum of the channel energy, the binding energy, and the Fermi energy for the incident nucleon in a nucleus of A nucleons.

The maximum energy of a bound proton or neutron state in this model is the sum of the corresponding Fermi and binding energies for that type of nucleon in the nucleus containing A nucleons. The two gases are either allowed to scatter by binary collisions into or to occupy only those states within the nucleus that are consistent with the initial excitation E^* deposited by the incident nucleon. Further, it is assumed that nucleons whose internal energies are greater than their corresponding Fermi-plus-binding energies have access to all those states outside of the nucleus which are, again, consistent with the initial excitation energy of the system. For computational convenience, the states have been grouped into 1-MeV bins, and the difference in mass between the neutron and proton has been neglected.

The total number of internal proton translational states, g_i^P, in the ith group of states with average energy ϵ_i^P is given by

$$g_i^P=\int_{\epsilon_i^P-1/2}^{\epsilon_i^P+1/2}\rho_P(\epsilon)d\epsilon\ ,$$

where $\rho_P(\epsilon)$ is the density of internal proton translation states. A similar expression is used for g_i^N, the number of internal neutron translational states with average energy ϵ_i^N. For protons, $1\leq\epsilon_i^P\leq\epsilon_f^P+E^*$, while for neutrons $1\leq\epsilon_i^N\leq\epsilon_f^N+E^*$, where ϵ_f^N and ϵ_f^P are the neutron and proton Fermi energies and E^* is the excitation energy of the nucleus. Similarly, the states outside of the nucleus which correspond to escaped particles were also grouped such that $g_{i'}^P$ and $g_{i'}^N$ are the total number of proton and neutron translational states in the i'th group with average energies $\epsilon_{i'}^P$ and $\epsilon_{i'}^N$, respectively. For example,

$$g_{i'}^P=\int_{\epsilon_{i'}^P-1/2}^{\epsilon_{i'}^P+1/2}\rho_P'(\epsilon)d\epsilon\ ,$$

where $\rho_P'(\epsilon)$ is the density of laboratory proton translational states. For protons, $1\leq\epsilon_{i'}^P\leq E^*-B_P$ and for neutrons $1\leq\epsilon_{i'}^N\leq E^*-B_N$, where B_P and B_N are the binding energies for the protons and neutrons, respectively, in the nucleus containing A nucleons. The average occupation number for the ith internal proton group, n_i^P, is then defined by $n_i^P g_i^P=N_i^P=$ the total number of occupied states in the ith group. The defining equation for n_i^N, the average occupation number for the ith internal neutron group, is similar to the above defining equation for n_i^P. The master equations describing the relaxation of the proton Fermi gas are then:

$$\begin{aligned}\frac{dn_i^P}{dt}=&\sum_{jkl}[\omega_{kl\to ij}^{PP}g_k^Pg_l^Pg_j^Pn_k^Pn_l^P(1-n_i^P)(1-n_j^P)-\omega_{ij\to kl}^{PP}g_j^Pg_k^Pg_l^Pn_i^Pn_j^P(1-n_k^P)(1-n_l^P)]\delta(\epsilon_i^P+\epsilon_j^P-\epsilon_l^P-\epsilon_k^P)\\&+\sum_{jkl}[\omega_{kl\to ij}^{PN}g_k^Pg_l^Ng_j^Nn_l^Nn_k^P(1-n_i^P)(1-n_j^N)-\omega_{ij\to kl}^{PN}g_j^Ng_l^Ng_k^Pn_i^Pn_j^N(1-n_k^P)(1-n_l^N)]\,\delta(\epsilon_i^P+\epsilon_j^N-\epsilon_k^P-\epsilon_l^N)\\&-n_i^P\omega_{i\to i'}^Pg_{i'}^P\delta(\epsilon_{i'}^P-\epsilon_i^P+\epsilon_f^P+B_P)\ ,\end{aligned}\tag{1}$$

$$\frac{dN_{i'}^P}{dt}=n_i^Pg_i^P\omega_{i\to i'}g_{i'}^P\,\delta(\epsilon_{i'}^P-\epsilon_i^P+\epsilon_f^P+B_P)\ ,\quad i=1,\ldots,\epsilon_f^P+E^*,\quad i'=1,\ldots,E^*-B_P\ ,\tag{2}$$

where $N_{i'}^P=$ the number of escaped protons with laboratory energy $\epsilon_{i'}^P$; $\omega_{i\to i'}^P$ is the probability per unit time that a proton in a particular state of the ith group escapes, with $\omega_{i\to i'}^P=0$ for $\epsilon_i^P\leq\epsilon_f^P+B_P$; and $\omega_{ij\to kl}^{PP}$ is the probability per unit time that a proton in a particular state of the ith group scatters from a proton in a particular state in the jth group such that one proton goes to the lth group and the other to the kth group. The δ functions ensure energy conservation in the transitions. $\omega_{ij\to kl}^{PN}$ is the probability per unit time that a proton in the ith group scatters from a neutron in the jth group such that the proton goes to the kth group while the neutron goes to the lth group. The master equations for the relaxation of the neutron Fermi gas are identical with the above except that everywhere that a P appears in the above it is to be replaced with an N and vice versa.

For a given set of initial conditions and a given set of transition probabilities the two sets of master equations can be solved numerically for the proton and neutron occupation numbers and the number of escaped protons and neutrons as a function of time. For medium-energy reactions we have already specified the initial conditions: two zero-degree Fermi gases plus one nucleon in an excited state. Therefore, the only quantities that still remain unspecified are the transition probabilities. As in I, the internal transition probabilities used were purely classical, i.e.,

$$\omega_{ij\to kl}^{PP}=\frac{\sigma_{PP}(\epsilon_i^P+\epsilon_j^P)[(2/M)(\epsilon_i^P+\epsilon_j^P)]^{1/2}}{V\sum_{mn}'g_m^Pg_n^P\,\delta(\epsilon_i^P+\epsilon_j^P-\epsilon_m^P-\epsilon_n^P)},\tag{3}$$

where $\sigma_{PP}(\epsilon)$ is the elementary proton-proton elastic scattering cross section after Coulomb effects have been removed and is therefore equal to $\sigma_{NN}(\epsilon)$ (see I for a further discussion). M is the neutron mass and the prime on the summation means that the summation is only taken over those states that are allowed in the proton-proton scattering process within the nucleus. Therefore, $\omega_{ij\to kl}^{PP}$ differs from $\omega_{ij\to kl}^{NN}$ only in the normalization factor, $\sum'_{mn} g_m^N g_n^N \delta(\epsilon_i^N+\epsilon_j^N-\epsilon_k^N-\epsilon_l^N)$, since, in general, a different number of states are involved in the neutron-neutron scattering process. $\omega_{ij\to kl}^{PN}$ is given by

$$\omega_{ij\to kl}^{PN}=\frac{\sigma_{PN}(\epsilon_i^P+\epsilon_j^N)[(2/M)(\epsilon_i^P+\epsilon_j^N)]^{1/2}}{V\sum'_{mn} g_m^P g_n^N \delta(\epsilon_i^P+\epsilon_j^N-\epsilon_m^P-\epsilon_n^N)}, \qquad (4a)$$

where $\sigma_{PN}(\epsilon)$ is the elementary proton-neutron scattering cross section. The elementary scattering cross sections were taken from Chen *et al.*[6] $\omega_{i\to i'}^P$ is given by

$$\omega_{i\to i'}^P=\frac{\sigma_{\mathrm{inv}}^P(\epsilon_{i'}^P)[(2/M)\epsilon_{i'}^P]^{1/2}}{g_i^P\Omega}, \qquad (4b)$$

where Ω is the laboratory volume and $\sigma_{\mathrm{inv}}^P(\epsilon_{i'}^P)$ is the inverse cross section for the absorption by a particular nucleus of a proton with channel energy $\epsilon_{i'}^P$. The quantity $\omega_{i\to i'}^N$ is defined in a similar manner. The inverse cross sections were taken from the approximate continuum-theory cross sections of Dostrovsky, Fraenkel, and Friedlander.[7]

If we neglect the escape of nucleons, then at equilibrium ($t=\infty$) the solutions to the master equations are

$$n_i^P(t=\infty)=\frac{1}{e^{\beta(\epsilon_i^P-\mu_P)}+1}, \qquad (5a)$$

$$n_i^N(t=\infty)=\frac{1}{e^{\beta(\epsilon_i^N-\mu_N)}+1}, \qquad (5b)$$

which are the expected values of n_i^P and n_i^N for a two-component Fermi gas system at equilibrium, where $\beta=(kT)^{-1}$ and the μ_N and μ_P are the chemical potentials for the neutron and proton gas, respectively. The quantities β, μ_N, and μ_P could, of course, be determined from the total energy, number of neutrons, and number of protons of the two-component Fermi systems. Further, after equilibrium has been established we expect that the neutron spectrum emitted during a short time interval will be proportional to $\epsilon e^{-\beta\epsilon}$. We shall show later in the discussion of the results that this latter requirement is obeyed. Note that in the formulation of the relaxation process in medium-energy reactions we neither consider structural details of the nucleus (i.e., diffuse edge, etc.), nor do we consider any conservation laws other than energy conservation in the individual binary collisions. We could, of course, modify the master equations to consider these other details. However, in so doing, the complexity of the equations would be increased enormously.

III. TYPICAL RESULTS OF CALCULATIONS

A. Comparison with Neutron Spectrum from 18-MeV Protons on Ta^{181}

There are a number of properties that can be derived from this proposed model of medium-energy nucleon-induced reactions. However, we shall be concerned here with only one of them: the production cross section and energy spectrum of the relatively high-energy neutrons from (p, xn) reactions on heavy elements.

Since the model only includes gross features of real nuclei, the most reasonable test of it would be for reactions on targets which are far from closed shells. An ideal candidate is Ta^{181} because it has a large atomic number, should not exhibit any closed-shell effects, and, more particularly, because Verbinski and Burrus[8] have recently measured the neutron spectrum from this target when it was bombarded by 18-MeV protons. This spectrum clearly shows the presence of high-energy neutrons. For the above reasons we shall present our results calculated for 18-MeV proton-induced reactions on Ta^{181} as being representative of some of the studies that have been made with the model.

For proton-induced reactions with Ta^{181}, the values of 7.0 MeV for B_P and 8.0 MeV for B_N, which are the proton and neutron binding energies, respectively, in W^{182} were used. In the calculations the initial values of the proton and neutron occupation numbers were established in the manner discussed previously, and then the two sets of the master equations were solved numerically using the method of Runga-Kutta-Gill. The proton and neutron occupation numbers as well as the total number of escaped protons and neutrons were recorded as a function of time. In most cases a reaction was followed only until the proton and neutron occupation numbers reached their equilibrium values.

First we shall compare the calculated high-energy neutron spectrum with that measured by Verbinski and Burrus.[8] The relevant data are tabulated in columns 2 and 5 of Table I. Although the model does indeed predict that neutrons should be emitted in this high-energy region, the calculated value is low by about a factor of 4. Further, the spectrum does not have the same shape as the experimental one: The difference between the two spectra increases as the energy of the emitted neutrons decreases.

In this context, it is also of interest to examine the neutron spectrum that is predicted by the model for pre-equilibrium decay that has been proposed by Griffin[2] and extended by Blann[3] and Blann and Lanzafame.[4] This model, the exciton model, is similar to the one used here in that it also puts particles into single-particle states and assumes that changes in configuration can only arise through two-body interactions. One of the primary differences between the two models is the manner in which the state of an excited nucleus is designated. In the exciton model the state is characterized by the sum of the number of particles excited above the Fermi level and the number of vacancies below the Fermi level: this sum is defined as the number of excitons, n. On the other hand, in the present model the state is characterized by the average occupation number of each of the single-particle states. Thus, in the exciton model a nuclear reaction induced by a nucleon incident on an even-even nucleus is considered as proceeding through $n=1$ as the nucleon enters the nucleus, going to $n=3$ when there are two excited particles and one hole resulting from an allowed two-body interaction between the incident nucleon and a nucleon in the Fermi sea, going to $n=5$ when three particles and two holes result from an interaction in the $n=3$ configuration, etc.

Ignoring differences among the many different distributions of particles and holes that correspond to the same exciton number at a given excitation energy, an excited nucleus with n excitons has three possibilities: another particle-hole pair may be created in a two-body interaction leading to exciton number $n+2$; a particle-hole pair may be annihilated in a two-body interaction leading to exciton number $n-2$; or if one of the excited

TABLE I. Calculated vs experimental cross sections in mb/MeV for the emission of high-energy neutrons in the (p,xn) reaction induced by 18-MeV protons on Ta^{181}.

Neutron energy (MeV)	$\frac{d\sigma}{d\epsilon}$ a	$\frac{d\sigma}{d\epsilon}$ b	$\frac{d\sigma}{d\epsilon}$ c	$\frac{d\sigma}{d\epsilon}$ d
10 ± 0.5	2.5	2.8	10.2 ± 1.1	$10.4\pm2.$
11 ± 0.5	2.0	2.4	8.7 ± 1.0	$9.3\pm1.$
12 ± 0.5	1.6	2.0	8.2 ± 1.0	$7.6\pm1.$
13 ± 0.5	1.3	1.6	8.8 ± 1.0	$4.9\pm1.$
14 ± 0.5	0.9	1.2	5.1 ± 0.8	2.8 ± 0.5
15 ± 0.5	0.5	0.8	3.7 ± 0.6	0.9 ± 0.3
Total	8.8	10.8	44.7 ± 2.2	35.9 ± 2.7

[a] Cross sections taken from this calculation.
[b] Cross sections calculated from the "exciton model."
[c] Cross sections taken from a Monte Carlo calculation described by Chen *et al.* (Ref. 6).
[d] Cross sections taken from the experiment by Verbinski and Burrus (Ref. 8).

particles is in an unbound state, it may be emitted, which leads to exciton number $n-1$. If, on the basis of phase-space arguments, it is assumed that the rate of interactions leading from n to $n-2$ is negligible, at least for values of n appreciably less than the most probable value after equilibrium is established, then the energy spectrum of particles that are emitted prior to equilibrium may be simply expressed as

$$I(\epsilon')=\int_0^{E^*}\sum_n P(n,E)I(\epsilon'\,|\,n,E)dE\,, \tag{6}$$

where E^* is the initial excitation energy of the nucleus, $P(n,E)$ is the probability density that the excited nucleus pass through a state characterized by exciton number n and excitation energy E on the way to equilibrium, and $I(\epsilon'\,|\,n,E)$ is the probability density that a nucleus with excitation energy E and n excitons emits a particle with laboratory kinetic energy ϵ'.

The quantity $P(n,E)$ in the integral accounts for the depletion of the original nonequilibrated nucleus through particle emission and allows for the emission of more than one particle prior to equilibration.

If, as is not necessarily correct, it is assumed that before equilibrium all configurations of a nucleus with n excitons and excitation energy E have equal *a priori* probability, the second factor in the integral may be expressed as

$$I(\epsilon'\,|\,n,E)=\frac{\rho_{n-1}(U)\rho_I(\epsilon)}{\rho_n(E)}\,\frac{\lambda(\epsilon'\,|\,\epsilon)}{\lambda(\epsilon'\,|\,\epsilon)+\lambda(n+2\,|\,n,\epsilon,U)}, \tag{7}$$

where $\rho_n(E)$ is the density of states of the nucleus with excitation E characterized by n excitons, $\rho_{n-1}(U)$ is the density of states with excitation energy U characterized by $n-1$ excitons, $\rho_I(\epsilon)$ is the density of single-particle states with kinetic energy ϵ within the nucleus, $\lambda(\epsilon'\,|\,\epsilon)$ is the probability per unit time that a particle with kinetic energy ϵ inside the nucleus escapes to the outside where its kinetic energy is ϵ', and $\lambda(n+2\,|\,n,\epsilon,U)$ is the probability per unit time that the excited nucleus with n excitons in the particular set of configurations which includes one excited particle with kinetic energy ϵ within the nucleus and the residual excitation energy U randomly distributed among the remaining $n-1$ excitons undergoes a two-body interaction which changes the exciton number to $n+2$. The first factor in Eq. (7) is just the probability that the nucleus with excitation energy E is in a configuration such that there is a particle in an unbound state with kinetic energy ϵ. The second factor is the probability that the particle in the unbound state escapes to the continuum where its kinetic energy is ϵ', rather than there be a two-body interaction which raises the exciton number to $n+2$.

The expression for $\lambda(\epsilon'|\epsilon)$ may, like Eq. (4b), be immediately written down from the law of detailed balance

$$\lambda(\epsilon'|\epsilon)=\frac{\sigma_{\text{inv}} V_R \rho_c(\epsilon')}{\rho_I(\epsilon)\Omega}, \tag{8}$$

where σ_{inv} and Ω have been defined previously, V_R is the velocity of the emitted particle relative to the residual nucleus, and $\rho_c(\epsilon')$ is the density of external translational states.

The quantity $\lambda(n+2|n,\epsilon,U)$ is much more difficult to evaluate and, hence, is a primary source of difficulty in using the exciton model. In previous work it has essentially been left as a free parameter which is proportional to the density of states with $n+2$ excitons. In the present calculation we obtain an estimate of this quantity from the solutions to the master equations. However, this estimate, as well as the previous ones, is an average over *all* configurations consistent with the excitation energy and the exciton number rather than the appropriate set of particular configurations that $\lambda(n+2|n,\epsilon,U)$ specifies in its definition.

It is worthwhile digressing at this point to show that Eqs. (7) and (8) lead to the usual expression

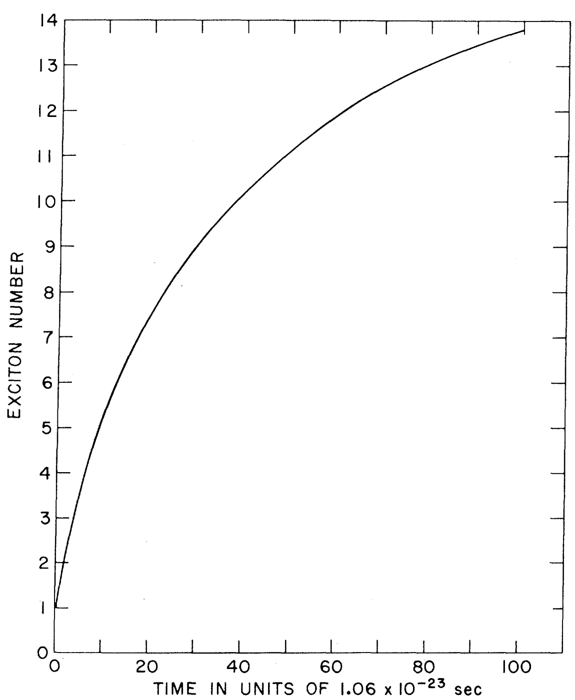

FIG. 1. The excitation number versus time for Ta^{181} bombarded with 18-MeV protons. The curve is derived from the solutions to the master equations for this system.

for the emission of particles from equilibrated nuclei. For this situation it is not necessary to know $\lambda(n+2|n,\epsilon,U)$. At equilibrium, the probability that the excited nucleus is in a state with exciton number n is, by definition of equilibrium, $\rho_n(E)/\rho(E)$, where $\rho(E)$ is the density of all states with excitation energy E, $\rho(E)=\sum_n \rho_n(E)$. The probability that the excited nucleus with n excitons be in the particular set of configurations such that there is a particle with kinetic energy ϵ in an unbound nuclear state and the residual excitation energy U is randomly distributed among the remaining $n-1$ excitons is just, as stated before, the ratio of the density of such states to the density of all states with exciton numbers n: $\rho_{n-1}(U)\rho_I(\epsilon)/\rho_n(E)$. Finally, the probability per unit time that the particle with kinetic energy ϵ escapes to outside the nucleus where its kinetic energy is ϵ' is given by Eq. (8). The combination of all these factors gives the following for the probability per unit time and energy that an equilibrated nucleus emits a particle with kinetic energy ϵ':

$$\omega(\epsilon')=\sum_n \frac{\rho_n(E)}{\rho(E)}\frac{\rho_{n-1}(U)\rho_I(\epsilon)}{\rho_n(E)}\frac{\sigma_{\text{inv}} V_R \rho_c(\epsilon')}{\rho_I(\epsilon)\Omega}. \tag{9}$$

Realizing that within the context of this model

$$\sum_n \rho_{n-1}(U)=\rho'(U),$$

where $\rho'(U)$ is the density of states of the residual nucleus with excitation U, the expression for $\omega(\epsilon')$ becomes

$$\omega(\epsilon')=\frac{\rho'(U)}{\rho(E)}\frac{\sigma_{\text{inv}} V_R \rho_c(\epsilon')}{\Omega}. \tag{10}$$

Equation (10) is precisely the usual expression for describing the emission of particles from equilibrated, excited nuclei.

We now return to the emission of particles from nonequilibrated nuclei. The precompound emission of neutrons from Ta^{181} bombarded with 18-MeV protons may be calculated from Eq. (6) if some method is found for the estimation of $\lambda(n+2|n,e,U)$. An estimate of this quantity may be obtained from the time dependence of the exciton number as illustrated in Fig. 1. This curve was taken from the solutions to the master equations, and its slope was used to estimate $\lambda(n+2|n,\epsilon,U)$. To complete the evaluation of Eq. (6) the quantities $\lambda(\epsilon'|\epsilon)$ and $\rho_I(\epsilon)$ were taken to be the same as those used in solving the master equations, and $\rho_n(E)$ and $\rho_{n-1}(U)$ were taken from the approximate expression for the level density in terms of the particle number p and hole number h that is given by Ericson[9]

$$\rho_{p,h}(E)=g_0{}^{p+h}E^{p+h-1}/p!\,h!\,(p+h-1)!\ , \qquad (11)$$

where g_0 is the density of single-particle states at the Fermi energy and, consistent both with its definition[9] and with the nuclear model that is used here, has a value of 12.8 MeV^{-1}. From Eq. (11) $\rho_n(E)$ is simply given by

$$\rho_n(E)=g_0{}^nE^{n-1}\Big/(n-1)!\left(\frac{n-1}{2}\right)!\left(\frac{n+1}{2}\right)!\ , \qquad (12)$$

and $\rho_{n-1}(U)$ is given by

$$\rho_{n-1}(U)=g_0{}^{n-1}U^{n-2}\Big/(n-2)!\left(\frac{n-1}{2}\right)!\left(\frac{n-1}{2}\right)!\ . \qquad (13)$$

Finally, Eq. (6) was evaluated by using the above, setting $P(n,E)=\delta(E-E^*)$, and summing over odd values[2] of n starting with $n=3$. In the evaluation, only particles emitted from nuclei with the initial excitation energy were considered, since our only concern was the escape of high-energy neutrons. The results calculated in this manner are presented in column 3 of Table I. As one can see, these results are again lower and, more importantly, again fall off less rapidly with increasing energy than the experimental ones. A conjecture as to why neither the results obtained from the master equation nor those obtained from the exciton model agree with experiment will be advanced shortly. In the mean time it is interesting to note that the master-equation results are very similar to the exciton model's when the quantity $\lambda(n+2|n,\epsilon,U)$ is approximated by means of the master equation.

For further comparison, there are also presented in the fourth column of Table I the results from a Monte Carlo intranuclear-cascade calculation described by Chen *et al.*[6] which uses a step potential, but does not allow for the refraction of reflection of cascade nucleons at potential boundaries. While there are serious theoretical objections that may be raised to using this semiclassical calculation at such low incident energies, it is, nevertheless, another technique for examining the relaxation through two-body interactions of a nucleus that is excited and partially equilibrated. Further, in contrast to the other methods for treating this problem, the Monte Carlo technique follows the spatial as well as the energy distribution of the excited particles. From Table I it is seen that the Monte Carlo calculation predicts the neutron spectrum fairly well; in fact, it predicts the total cross section to within ~20% of that measured experimentally.

In this Monte Carlo calculation it was found that the majority of the pre-equilibrium (p,n) reaction occurs in the diffuse edge of the nucleus. It is evidently the inclusion of explicit consideration of the spatial distribution of the excited particles, in particular the position of the first two-body interaction of the incident particle, that leads to better results with the Monte Carlo method than with the approaches that ignore geometrical factors. This result is not surprising. Clearly the probability for the escape of a particle that is in an unbound state depends on the position of that particle: those close to the surface have a greater probability than those in the interior of the nucleus. Expressions for escape probabilities such as Eqs. (4b) and (8), which do not explicitly include geometrical factors, must represent some kind of average over the position of the escaping particle. There is no reason to assume that this kind of average is necessarily the same as that computed from the spatial distribution of excited particles resulting from the bombardment of a nucleus with a nucleon. This suggests that the use of those formalisms which do not include explicit considerations of the spatial distribution of the excited particles and holes is probably most reliable when the trend toward equilibrium has progessed to a point where this distribution *in space* is fairly random; a formalism, such as the Monte Carlo method, which includes geometrical factors seems to be required for the early stages. Thus, the master equation might find its most fruitful application by using the output from a Monte Carlo intranuclear-cascade calculation as its initial nonequilibrium configuration. This approach will be used in the following section.

B. Equilibration in High-Energy Nuclear Reactions

The extension of the model to higher excitation energies and the examination of the deexcitation of a few highly excited nuclear systems will now be considered. In order to consider the relaxation of nuclei with high excitations it was not necessary to change the form of the master equations but only the range of internal and laboratory energies involved. In particular, for highly excited systems we used $1\le\epsilon_i^P\le100$, $1\le\epsilon_i^N\le100$, $1\le\epsilon_{i'}^P\le100-\epsilon_f^P-B_P$, and $1\le\epsilon_{i'}^N\le100-\epsilon_f^N-B_N$.

The first example is the deexcitation of a compound nucleus produced in a Monte Carlo cascade calculation of 200-MeV protons on U^{238}. The cascade calculation is as described by Chen *et al.*[6] except that a square rather than a step potential was used for the target nucleus. The calculation included refraction and reflection at the potential walls. The radius of U^{238} used in this calculation was $1.5\times10^{-13}(238)^{1/3}$ cm. A binding energy of 6 MeV for both protons and neutrons was used both in the cascade and relaxation calculations. As

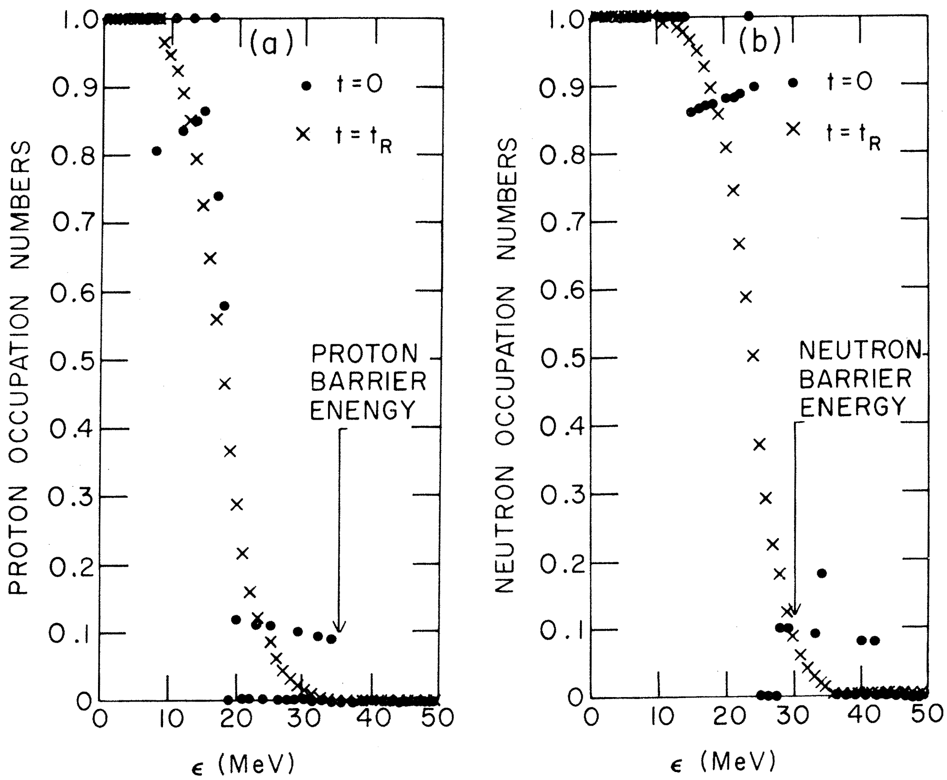

FIG. 2. (a), (b) Proton and neutron occupation numbers versus energy at $t = 0$ and $t = t_R$ in a compound nucleus produced in a Monte Carlo cascade calculation of 200-MeV protons on U^{238} using a square-well potential and including refraction and reflection.

mentioned previously, the output from the cascade calculation provided the initial configuration needed by the master equations.

The internal average proton and neutron occupation numbers of the system both after the cascade ($t = 0$) and after a time t_R when the system had internally equilibrated are presented in Figs. 2(a) and 2(b). It may be seen there that after the cascade was terminated ($t = 0$) all protons had internal energies less than that needed to escape from the nucleus, whereas there were neutrons which had internal energies greater than that needed to escape [i.e., all those nonzero occupation numbers above the neutron escape energy in Fig. 2(b)]. These neutrons were present because the cascade calculation follows the neutrons until their kinetic energies fall below a certain arbitrary energy picked for convenience which in this instance was greater than the energy required for emission.

It is also seen in Fig. 2 that by $t = t_R$ both the neutron and proton occupation numbers have assumed the form that one would ascribe to two Fermi gases in equilibrium. It is worthy of mention that even the initial distribution of particles and holes has some resemblance to that at equilibrium. This is as it should be; the Monte Carlo calculation is itself, after all, another way of simulating the equilibration of an excited nucleus. It is just not a very efficient way of following the completion of the equilibration process after most of the excited particles are below their escape energies.

Figure 3 presents the spectra of neutrons emitted from (a) $t = 0$ (the end of the cascade) to $t = t_R$, (b) $t = 0$ to $t = 5t_R$, and (c) $t = t_R$ to $t = 5t_R$. The spikes in the total neutron spectra for (a) and (b) represent the partial emission of those neutrons left after the cascade whose energies were above the neutron escape energy. These spikes do not appear in the spectrum emitted between t_R and $5t_R$, because of the requirement that preferential emission of those neutrons that were originally above the neutron escape energy must occur before equilibrium has been reached.

The calculated spectrum of the neutrons emitted between t_R and $5t_R$ is seen in Fig. 3 to be proportional to $\epsilon^N e^{-\beta\epsilon^N}$ with a value for β^{-1} of 2.5 MeV. This, of course, is the form of the spectrum that is expected for neutrons emitted from an equilibrium system. By the time t_R has been reached, the system has lost ~1 nucleon and ~17 MeV of ex-

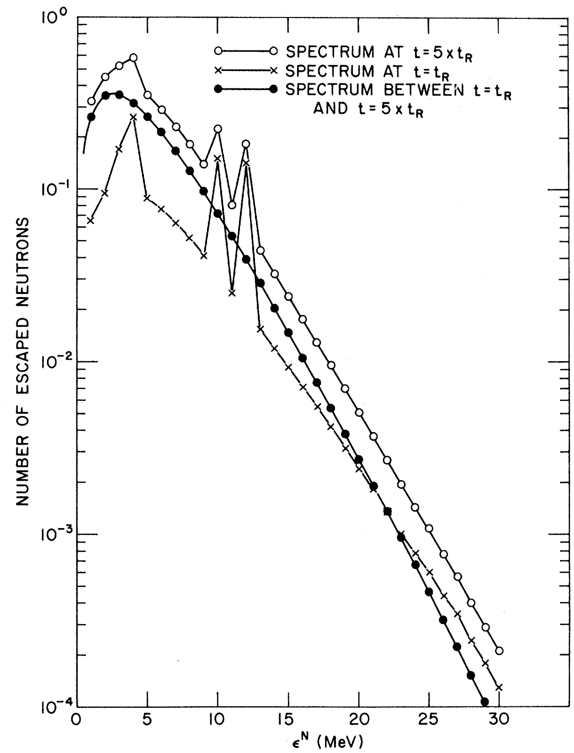

FIG. 3. The spectra of neutrons emitted from $t=0$ to $t=t_R$, from $t=0$ to $t=5t_R$, and the difference between these two spectra. The curve through the difference spectrum is proportional to $\epsilon^N e^{-\beta E^N}$ with $\beta^{-1}=2.51$ MeV. The spectra were taken from the relaxation of a compound nucleus produced in a Monte Carlo cascade calculation of 200-MeV protons on U^{238} using a square-well potential and including refraction and reflection.

citation energy; thus a temperature of 2.6 MeV is predicted for this equilibrated nucleus of mass number 238, and $E^*=189$ MeV by the usual approximate model based on a two-component Fermi gas with a constant single-particle level density.

Finally, we would like to conclude with a few remarks concerning the relaxation of a two-component Fermi system with an excitation energy greater than its total binding energy. For this purpose, the relaxation of W^{182} with an initial excitation of 1724 MeV was followed. It was found that the system lost ~14% of its excitation energy and ~5% of its constituent particles before reaching equilibrium. This is in favorable agreement with the results obtained in I from studying a one-component system of 100 fermions excited to 1054 MeV, where we found an excitation loss of ~10% and a particle loss of ~5% before equilibration was formed. Therefore, the conclusions made in I based on a one-component Fermi system are equally valid for a two-component system.

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