Comments and Addenda

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(d, p) Reaction on ^{20}Ne

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We have investigated the ^{20}Ne$(d,p)^{21}$Ne reaction at deuteron energies of 8.0, 12.0, and 14.0 MeV. Good agreement is obtained with recent results at E_d=16.4 MeV for spectroscopic factors for the low-lying levels of ^{21}Ne. These indicate appreciable mixing among the rotational bands of ^{21}Ne.

Recent work on the ^{20}Ne$(d,p)^{21}$Ne reaction has been reported by Howard, Pronko, and Whitten[1] at 16.4-MeV bombarding energy. Previous results at 3.2 MeV by Lambert *et al.*[2] were in certain respects in disagreement with the higher-energy data. We wish to report here additional studies of this reaction done at deuteron energies of 8.0, 12.0, and 14.0 MeV and are concerned only with the low-lying states of ^{21}Ne. Extensive work on the higher-lying levels has already been reported in Ref. 1.

The reaction was investigated using an enriched ^{20}Ne gas target in a thin-walled gas cell. Pressures of ~250 Torr were typically used. The deuteron beam was supplied by the Stanford FN tandem accelerator. Identification of the outgoing proton groups was accomplished using an E-ΔE counter telescope and the Stanford SCANS system.[3] The SCANS system was also used to analyze the data by fitting the peaks with a single or double Gaussian. The typical energy resolution obtained was ~60 keV. In addition a fixed detector served to monitor any changes in gas pressure, etc. The absolute cross sections were determined by comparison of the Kr(p,p) reaction at 2.0 MeV. The cross sections are believed accurate to 20%. The error is due to uncertainties in gas pressure and the normalization to the Kr(p,p) results.

TABLE I. Optical-model parameters used in the DWBA analysis.[a]

Particle	V_0 (MeV)	W_{vol} (MeV)	W_{surf} (MeV)	V_{so} (MeV)	r_0 (F)	r_C (F)	r_0' (F)	a (F)	a' (F)	Ref.
deuteron	101.9	29.56	0	8.0	1.289	1.3	1.353	0.706	0.594	5
proton	45	0	9.5	6.5	1.25	1.25	1.25	0.65	0.47	6
neutron				$\lambda=25$		1.25		0.65		

[a]The optical potential has the form

$$V(r)=U_C(r)-V_0\frac{1}{1+e^x}-iW_{vol}\frac{1}{1+e^{x'}}+i4W_{surf}\frac{d}{dx'}\frac{1}{1+e^{x'}}+\frac{1}{2}\left(\frac{\hbar}{m_\pi c}\right)^2 V_{so}(\sigma\cdot l)\frac{1}{r}\frac{d}{dx}\frac{1}{1+e^x},$$

where $x=(r-r_0A^{1/3})/a$, $x'=(r-r_0'A^{1/3})/a'$, and $U_C(r)$ is the usual Coulomb potential. V_0 for the neutron is determined in the usual manner by binding the neutron at an energy $B_n=[2.225+Q(d,p)]$ MeV.

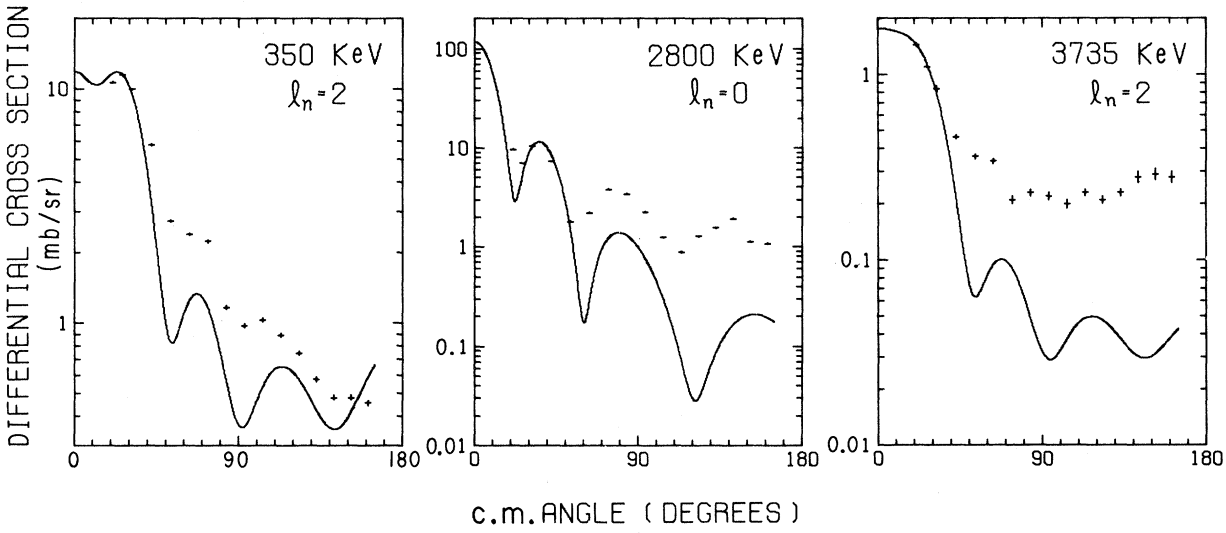

FIG. 1. The angular distributions for the ^{20}Ne$(d,p)^{21}$Ne reaction to states at 350, 2800, and 3735 keV. The incident energy is 12.0 MeV. The solid line is the NLFR DWBA fit using a finite-range parameter $R=0.65$ and nonlocality parameters of 0.54 for the deuteron and 0.85 for the proton. The quality of the fits for 8.0- and 14.0-MeV bombarding energy is essentially the same. In addition, the use of the LZR DWBA does not affect the shape or magnitude of the fitted curve appreciably.

The analysis of the data to extract spectroscopic factors was done using the distorted-wave Born-approximation (DWBA) code DWUCK.[4] The optical-model potential for the deuteron channel was taken from the work of Lutz[5] who did $d+^{22}$Ne at 12.1 MeV. For the proton channel the optical-model parameters were taken from fitting of elastic scattering of 14.0-MeV protons on ^{21}Ne.[6] These potentials are shown in Table I. The code allows both finite-range and nonlocal effects to be taken into account. We have performed the calculations for both the local zero-range (LZR) and nonlocal finite-range (NLFR) cases. The states at 0.0, 1.75, 2.87, 3.66, and 3.89 MeV did not display stripping patterns and no attempt was made to fit the angular distributions. In fitting the levels at 2.790 and 2.80 MeV it was assumed that only the $\frac{1}{2}^+$ member at 2.80 MeV was strongly populated, in accord with the previous work. Typical fits for $E_d=12.0$ MeV to the 0.350-, 2.80-, and 3.74-MeV states are shown in Fig. 1. We have assumed the transfer of a $1d_{5/2}$, $2s_{1/2}$, and $1d_{5/2}$, neutron, respectively, for the population of these states. In general, inclusion of finite-range and nonlocal effects had little effect on the shape of the angular distributions and less than ~20% effect on the magnitude of the spectroscopic factors derived from the fits are given in Table II. With the possible exception of the 8-MeV data the agreement with the previous work at high energy is excellent. The slight discrepancy for the 8-MeV data is perhaps due to the inadequacy of the optical-model potentials for the lower deuteron energy. No attempt was made to vary the potentials. The disagreement with the work at $E_d=3.0$ MeV can be understood because of the low energy of the deuterons and possible competition from other than direct-reaction modes.

The interpretation of the positive-parity levels of ^{21}Ne in terms of the Nilsson model has enjoyed

TABLE II. Spectroscopic factors for ^{20}Ne$(d,p)^{21}$Ne.

Level (MeV)	J^π	Present work $(2J+1)S$[a] 8.0 MeV	12.0 MeV	14.0 MeV	Previous work Ref. 2 3.0 MeV	Ref. 1 16.4 MeV	Theory[b] Unmixed	Mixed
0.0	$\frac{3}{2}^+$	•••	•••	•••	1.0	≤0.11	0.09	0.036
0.35	$\frac{5}{2}^+$	4.5	4.0	3.9	3.1	3.7	1.9	4.1
2.80	$\frac{1}{2}^+$	1.9	1.7	1.6	.9	1.6	0.5	0.64
3.735	$\frac{5}{2}^+$	•••	0.22	•••	•••	0.20	2.0	0.17

[a] $\sigma_{\text{expt}}=1.58(2J+1)S\sigma_{\text{DWBA}}$.

[b] The theoretical values are taken from Ref. 2.

considerable success.[7] The levels at 0.35 and 2.80 MeV can be described in terms of Nilsson orbits with the odd particle in orbits 7 and 9, respectively. It has been shown by Satchler[8] that the spectroscopic factor for stripping to a rotational band is given by

$$(2J+1)S = g^2 \langle \phi_2 | \phi_1 \rangle \langle Nlj | K\omega\alpha \rangle^2,$$

where $g^2 = 2$ for an even-even target nucleus. The factor $\langle \phi_2 | \phi_1 \rangle$ is the vibrational overlap term and is usually taken to be unity. The quantities $\langle Nlj | K\omega\alpha \rangle$ are the usual Nilsson expansion coefficients describing the rotational wave functions in terms of spherical shell-model wave functions. The above expression holds under the assumption that the rotational bands are unmixed. Lambert *et al.* have tabulated the expected spectroscopic factors using a deformation parameter $\eta = 4.3$. This value has been suggested[9] as being consistent with electromagnetic transition probabilities in ^{21}Ne. The theoretical spectroscopic factors are also listed in Table II. It is obvious the experimental results of Ref. 1 and those reported here are not in accord with the interpretation of the levels being comprised of unmixed rotational bands. Further calculations which included mixing between the rotational bands have also been carried out by Lambert *et al.* They included mixing between Nilsson orbits 7 ($K = \frac{3}{2}$), 9 ($K = \frac{1}{2}$), 5 ($K = \frac{5}{2}$), 11 ($K = \frac{1}{2}$), and 8 ($K = \frac{3}{2}$). These results indicate an appreciable admixture of orbits 7 and 5 in the wave function of the $\frac{5}{2}^+$ state at 0.350 MeV.

When the mixing is included the theoretical spectroscopic factor is given by

$$(2J+1)S_{ij} = 2\Big[\sum_{\alpha K} C_{K\alpha} \langle Nlj | K\omega\alpha \rangle\Big]^2,$$

where the $C_{K\alpha}$ are defined by

$$|JM\rangle = \sum_{K\alpha} C_{K\alpha} |JMK\alpha\rangle$$

in which $|JM\rangle$ is the wave function of the total Hamiltonian including the Coriolis mixing. We have included the spectroscopic factors which include the effects of mixing in Table II. These results indicate considerable improvement in the agreement between experiment and theory for the $\frac{5}{2}^+$ levels at 0.35 and 3.74 MeV. However, there still remains a discrepancy for the $\frac{1}{2}^+$ level at 2.80 MeV. It should be pointed out, however, that the expected spectroscopic factor for the $\alpha = 9$, $\frac{1}{2}^+$ state shows a strong dependency on η, the deformation parameter.[10] For example, with no mixing among the bands the theoretical spectroscopic factor increases by approximately 40% when η decreases approximately 20%. Thus better agreement could be obtained for a smaller prolate deformation. This observation has already been noted in Ref. 1.

In conclusion the results obtained here are in good agreement with those obtained in Ref. 1 and indicate that appreciable mixing occurs among the low-lying rotational bands in ^{21}Ne.

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