Phase-Equivalent Ambiguities in an Optical Model: Coupled-Channel Analysis of $\alpha + {}^{24}$Mg Scattering at 17 MeV*

K. A. Eberhard† and D. Robson‡
Department of Physics, Florida State University, Tallahassee, Florida 32306
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The existence of phase-equivalent ambiguities in the generalized optical model is investigated using a coupled-channel analysis of $\alpha + {}^{24}$Mg at 17 MeV. When compound-nucleus contributions are included in the analysis it is found that the phase-equivalent ambiguities typical of the spherical optical potential still occur when the potential is nonspherical. The values obtained for the deformation parameter which are appropriate for the rotational states of ^{24}Mg are found to be relatively insensitive to the choice of a phase-equivalent potential. A simple explanation of this insensitivity has not yet been found.

I. INTRODUCTION

The phenomenon of potential ambiguities in optical-model analyses of complex-particle scattering such as α-particle scattering has been of particular interest through the last few years. Some insight into these ambiguities and an understanding of their origin has been gained by various optical-model analyses of α-scattering data for a great number of target nuclei. Attempts, however, to derive an optical potential with a unique set of parameter values (U, W, R, a) have not yet been conclusive. For a detailed discussion "on the uniqueness of the real optical potential" for α scattering, we refer to the recent work of Watson *et al.*[1]

In this paper particular interest is given to the question of resolving optical-model ambiguities by using coupled-channel calculations, as suggested by Thompson, Crawford, and Davis.[2] We, therefore, have analyzed the scattering of α particles from ^{24}Mg to the first three ground-state rotational levels 0^+, 2^+, and 4^+ between 15.4- and 19.1-MeV bombarding energy in terms of optical-model and coupled-channel calculations. A reason for choosing this reaction is that the target nucleus ^{24}Mg is strongly deformed, which leads to strong couplings between the 0^+, 2^+, and 4^+ levels. In addition, the ambiguities involved in this reaction have already been studied in great detail at 10.8-MeV bombarding energy.[2]

The $\alpha + {}^{24}$Mg scattering data used here were measured by Eberhard, Klages, and Mayer-Böricke[3] with the Heidelberg tandem Van de Graaff accelerator. In the energy range from 15.4 to 19.1 MeV angular distributions between 25 and 165° (lab) were obtained every 20 keV. For the analysis presented in this paper we have averaged the angular distributions over the entire energy range of 3.7 MeV; thus the mean bombarding energy is 17.25 MeV.

The analysis is complicated by the presence of a large compound-nucleus contribution in the experimental cross sections, which is clearly revealed and characterized by strong variations of the cross section with energy (Ericson fluctuations). A detailed study of the statistical fluctuations in the excitation functions for this reaction has been given elsewhere.[3] The energy-averaged compound contribution has been taken into account explicitly in the analysis by adding a compound-nucleus cross section calculated from a Hauser-Feshbach-type formula to each "direct" cross section calculated either by the optical-model or the coupled-channel code.

As a by-product the so-called width correlation factor[4] W_{cc} has been determined which accounts for the fact that the decay of the compound nucleus into the elastic channel is enhanced by the factor W_{cc} relative to the inelastic channels. This factor heretofore has not been determined experimentally for α scattering.

II. OPTICAL-MODEL ANALYSIS

In this section the energy-averaged angular distribution for the elastic $\alpha + {}^{24}$Mg scattering at a mean bombarding energy of 17.25 MeV is analyzed with the optical model using a four parameter Woods-Saxon potential

$$V(r) = -(U + iW)[1 + e^{(r-R)/a}]^{-1} + V_{\text{Coul}}. \quad (1)$$

To account for the compound-nucleus-reaction contribution in the averaged data the same procedure used before by John *et al.*[5] and Bisson, Eberhard, and Davis[6] is followed. A compound-nucleus cross section [hereinafter referred to as Hauser-Feshbach (HF) cross section] for the elastic scattering of spin-zero particles

$$\left\langle \frac{d\sigma}{d\Omega}(\theta) \right\rangle = \frac{W_{cc}}{4\rho k^2} \sum_l (2l+1) \frac{(T_c^l)^2 P_l^2(\cos\Theta)}{\exp[-l(l+1)/2\sigma^2]} \quad (2)$$

was added to every optical-model cross section. The quantities σ and ρ in Eq. (2) were treated as free parameters and were adjusted simultaneously with the optical-model parameters. The significance of the values for σ and ρ obtained in this way can be controlled by relating σ and ρ to Fermi-gas-model expressions.[7,8]

The values for σ and ρ as obtained in this analysis and listed in Table I can be well reproduced by Fermi-gas calculations.

Seven discrete phase-equivalent sets of (U, W_V, a, R) with values for U ranging from 20 to 200 MeV were found; four of them which are later used in the coupled-channel analysis are listed in the table. One can see that as the real well depth U increases, the volume absorption W_V increases, too, and the interaction radius R decreases. The values of σ and ρ do not change considerably for the various potentials, indicating that always about the same compound-reaction contribution is found, which is, of course, expected. The quantity χ^2 is the usual fitting criterion

$$\chi^2=\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\sigma^i_{\rm exp}-\sigma^i_{\rm th}}{\Delta_i\sigma_{\rm exp}}\right), \qquad (3)$$

where $\sigma_{\rm exp}$ and $\sigma_{\rm th}$ are the experimental and calculated differential cross sections, respectively. The error $\Delta\sigma_{\rm exp}$ includes the experimental error of about 10% (Ref. 3) plus an additional error of a few percent which is due to the averaging procedure and which takes into account the stability of the angular distribution at a certain angle as a function of the averaging interval. One can see from the χ^2 values in the table that the fits corresponding to different potentials are of the same quality, demonstrating the expected discrete phase-equivalent ambiguities in the optical model.

III. COUPLED-CHANNEL ANALYSIS

The energy-averaged data for the elastic and inelastic scattering to the first three ground-state rotational levels 0^+, 2^+, and 4^+ were analyzed using Tamura's coupled-channel computer code JUPITOR.[9] Axial symmetry of the ^{24}Mg target nucleus was assumed and the potential was expanded in Legendre polynomials. Quadrupole and hexadecapole deformations were allowed. The results are shown in Fig. 1. The solid circles with error bars (see Sec. II) represent the experimental angular distributions for the elastic scattering (0^+) and for the inelastic scattering to the 2^+ state at -1.37 MeV and to the 4^+ state at -4.12 MeV. The solid lines represent the computed coupled-channel cross section plus the compound-nucleus cross section. For these calculations the OM potential with the smallest real well depth of $U=22$ MeV (see the table) was used. One can see that the agreement between the experimental and theoretical values is good for all three angular distributions. The deformations β_2 and β_4 varied

TABLE I. Phase-equivalent optical-model potentials (U, W_V, a, R), Hauser-Feshbach parameters σ and ρ, and the fitting criterion χ^2.

U (MeV)	W_V (MeV)	a (fm)	R (fm)	σ	ρ	χ^2
22	7.5	0.52	5.3	2.85	52	2.89
79	15.0	0.52	4.8	2.85	54	2.96
123	17.5	0.52	4.5	2.90	51	2.95
171	19.0	0.52	4.4	2.90	50	2.90

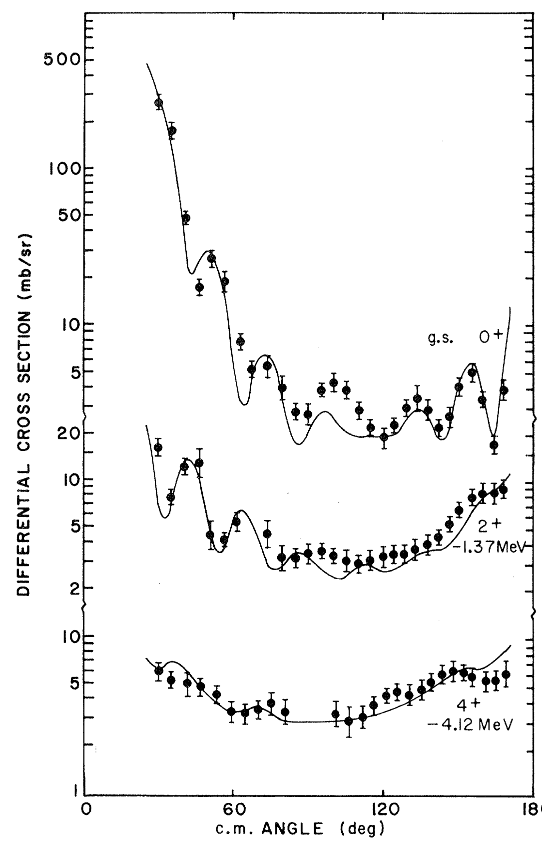

FIG. 1. Experimental and calculated cross sections for the $\alpha+{}^{24}$Mg elastic (g.s., 0^+) and inelastic scattering to the first two excited states (-1.37 MeV, 2^+) and (-4.12 MeV, 4^+). The experimental data are averaged over the energy range 15.4 to 19.1 MeV. The solid lines represent the computed coupled-channel cross section plus the compound-nucleus cross section (see Sec. II). For the elastic scattering both cross sections are shown separately in Fig. 2. The potential used for the coupled-channel calculation is the one with the smallest real potential depth of $U=22$ MeV of the table along with quadrupole and hexadecapole deformations of $\beta_2=0.40$ and $\beta_4=0.05$, respectively.

to obtain the best fit are in this case $\beta_2=0.40$ and $\beta_4=0.05$.[10] In Fig. 2 the calculated direct and compound cross sections for the elastic scattering are shown separately. One can see that the resulting cross section shown in Fig. 1 is mainly given at forward angles by the direct term and at backward angles by the compound term. Also included in Fig. 2 is the compound-nucleus cross section obtained by an Ericson-type fluctuation analysis of the fine-energy-resolution excitation functions (taken from Ref. 3). The agreement within the error bars of these "experimentally" derived compound cross sections with the calculated HF cross sections is a further indication that the HF term used in the analysis is a reasonable parametrization of the compound-nucleus part of the reaction.

A further remark on the calculations of the compound cross sections is necessary. Since for the inelastic scattering nonspin-zero particles are involved in the reaction, the more general HF formula from Ref. 8 was used.[11] In order to get an approximate set of transmission coefficients for the inelastic scattering needed in the HF formula, we have calculated the transmission coefficients from the optical model keeping all parameters fixed except the incident center-of-mass energy which was decreased by 1.37 MeV for the 2^+ transition and by 4.12 MeV for the 4^+ transition. The same values for σ and ρ were used throughout the analysis. There is one more quantity which enters the HF calculations, namely the correlation factor W_{cc}. For inelastic scattering $(c \neq c')$ $W_{cc'}$ is defined to be $W_{cc'}=1$, and for the elastic scattering $(c=c')$ we find for the reaction studied here $W_{cc}=2$. This value is in agreement with the one for proton scattering[12] and also with

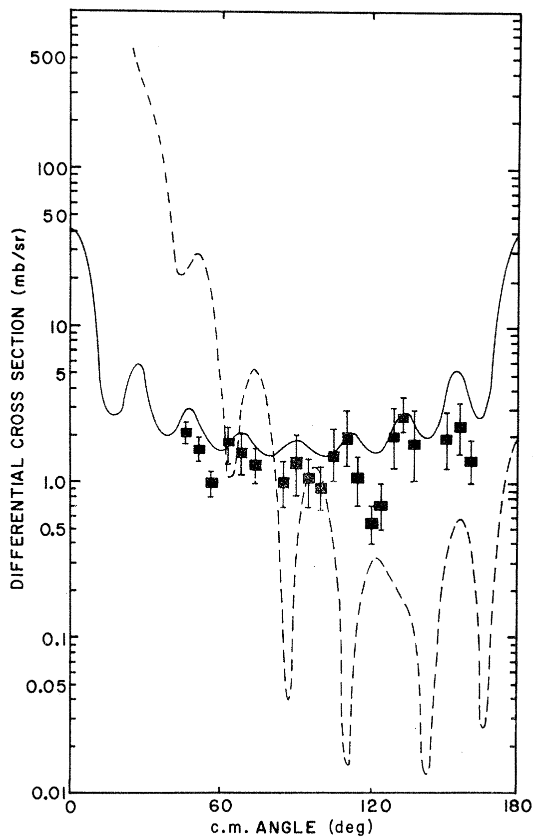

FIG. 2. Direct (dashed line) and compound-nucleus cross section (solid line) for the elastic scattering $\alpha + {}^{24}Mg$ as calculated from the coupled-channel computer code and the Hauser-Feshbach formula, respectively. The same parameter values as in Fig. 1 were used and the sum of the direct and the compound part is shown in Fig. 1. The solid squares represent compound-nucleus cross sections obtained by an Ericson-type fluctuation analysis (taken from Ref. 3); they are included for a comparison with the compound-nucleus cross sections as obtained and used in this paper.

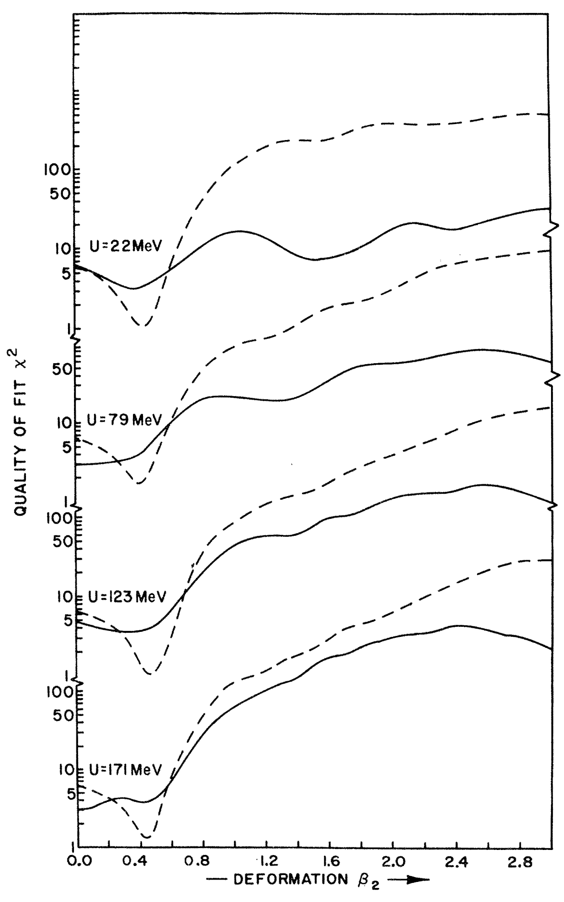

FIG. 3. The quality of the fits for the $\alpha + {}^{24}Mg$ elastic (solid line) and first-inelastic-state scattering (dashed line) as obtained by the coupled-channel plus Hauser-Feshbach analysis is shown in terms of the fitting criterion χ^2 as a function of the quadrupole deformation β_2. The calculations were performed for the four phase-equivalent potentials listed in the table and the real well depth U of the corresponding potential is indicated in the figure. No higher-order deformations are included.

theoretical estimates.

It is demonstrated in Fig. 1 that a good description of the experimental data can be obtained with the shallow potential ($U = 22$ MeV) and the question is: Do the other phase-equivalent potentials listed in the table describe the data as well? Figure 3 shows the results for the four potentials listed in the table, and the real potential depths are indicated in the figure. The quality of the fits in terms of the fitting criterion χ^2 is plotted as a function of the quadrupole deformation β_2 for β_2 ranging from 0 to 3. No higher-order deformations were included. The solid line represents the elastic scattering and the dashed one the first inelastic state. One can see that for all four potentials with quite different potential depths a clear minimum is found for deformations between 0.4 and 0.5. At these deformations χ^2 values are close to unity for the 2^+ curve; for the elastic scattering they are about equal to the regular optical-model χ^2 values, that is for $\beta_2 = 0$. The similarity of the fits for different potentials indicates that in the present work there is no preference for one of the optical-model potentials by the coupled-channel calculations.

An almost unique value for β_2 is obtained in the present analysis for all four optical potentials used in the coupled-channel calculations. This suggests that the determination of deformation parameters such as β_2 in coupled-channel analyses does not seem to be ambiguous as far as discrete phase-equivalent ambiguities in the optical potential are concerned.

IV. CONCLUSIONS

As a main result of this paper it is shown that the well-known discrete phase-equivalent ambiguities in the optical model can not be reduced or resolved in terms of coupled-channel calculations; at least not for the $\alpha + {}^{24}Mg$ scattering at 17 MeV studied here. The good description of the energy-averaged α-scattering data to the first three ground-state rotational states in ^{24}Mg indicates that a coupled-channel calculation using quadrupole and hexadecapole deformation along with taking into account the compound-nucleus contribution explicitly is a realistic way to analyze the data.

It is gratifying to note that some of the problems in an earlier study of this reaction at 10.8 MeV by Thompson, Crawford, and Davis[2] are understood and resolved in this paper. In Ref. 2 the absorption W, obtained from fitting the forward angles only, was found to be much larger than that one obtained from fitting the entire angular region. This paradox is due to the neglect of the compound-reaction contribution in the reaction, which is particularly large at backward angles, and is completely resolved by taking this contribution into account explicitly as in the analysis prescribed here. Another paradox which is also resolved by accounting for the compound contribution is the unreasonably small absorption of $W = 0$ or at most 0.4 MeV (the real potential depth being $U = 160$ MeV) which was needed[2] in the coupled-channel calculations to get the right magnitude of the inelastic cross section to the first excited 2^+ state but which gave much too large cross sections for the elastic scattering.

The major unresolved feature of the present work is that we have not found a simple-minded explanation for the value of β_2 being essentially independent of the choice of phase-equivalent potentials. Some simpler coupled-channel models are being considered in an attempt to understand this problem.

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†Present address: Sektion Physik, Universität München, Amalienstr 54, 8 München 13, Germany.

‡Alfred P. Sloan Research Fellow.

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[10]We like to note that the value of $\beta_4 = 0.05$ reported in this paper is in disagreement with the one of $\beta_4 = 0.16$ of J. S. Eck and W. J. Thompson [in *Proceedings of the Fifth International Conference on Nuclear Reactions Induced by Heavy Ions, Heidelberg, Germany, 1969,* edited by R. Bock and W. R. Hering (North-Holland Publishing Company, Amsterdam, The Netherlands, 1970), p. 85] found from $^{16}O + ^{24}Mg$ scattering. This difference could be due to an error concerning the β_4 coupling term in the computer code used by Eck and Thompson, which is supported by the fact that our earlier calculations using the same uncorrected computer code as in Eck and Thompson indeed gave best description of the data with $\beta_4 = 0.20$.

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Spins of Levels in Cu^{64}†

C. C. Wellborn,* R. P. Williams,‡ and S. G. Buccino
Tulane University, New Orleans, Louisiana 70118
(Received 4 June 1970)

The method-I angular correlation technique of Litherland and Ferguson has been applied to the reaction $Ni^{64}(p, n\gamma)Cu^{64}$. Spin assignments were obtained for the following levels in Cu^{64}: 1 (0 keV), 2 (159 keV), (1, 2) (278 keV), 1 (344 keV), 3 (362 keV), and 1 (663 keV). $E2/M1$ mixing ratios for the 159- and 203-keV transitions were determined. A weak transition from the 362-keV level to the ground state was observed.

INTRODUCTION

The level scheme of Cu^{64} up to 1322-keV excitation has been investigated and discussed with regard to energies and γ-ray branching ratios by Shera and Bolotin.[1] The tentative assignments of these authors represented the most convincing data on the spins of these levels until the recent stripping reaction studies of Park and Daehnick.[2] Several authors had previously and have subsequently performed angular correlation[3,4] and circular polarization[3,5-8] measurements in order to deduce the spins of several levels, but the results have been conflicting. The stripping measurements,[2] gave in many cases ambiguous results and in several cases unlikely results.

In the present work, angular correlation and distribution measurements were performed on three γ-ray cascades from final states of the $Ni^{64}(p, n)$-Cu^{64} reaction. The general technique is that denoted "method I" by Litherland and Ferguson.[9] The products of the reaction are unobserved, leaving the initial bound state of the residual nucleus in a state of polarization aligned with respect to the beam axis. Analysis of the angular correlations is made in terms of the relative populations of the magnetic substates, requiring no *a priori* knowledge of the reaction process.

In general, such measurements require the use of large NaI(Tl) scintillators to obtain acceptable coincidence count rates. As a consequence, complex spectra, such as that exhibited by Cu^{64}, cannot be studied in this way because of the lack of sufficient energy resolution. The $Ni^{64}(p, n)Cu^{64}$ reaction populates several interesting levels with yields high enough to permit measurements utilizing Ge(Li)-NaI(Tl) and even Ge(Li)-Ge(Li) coincidence systems, with which sufficient energy resolution is readily attainable.

The cascades studied in this way were $(E_{\gamma_1}, E_{\gamma_2}) = (185, 159)$; $(203, 159)$; and $(385, 278)$ keV, depopulating the 344-, 362-, and 663-keV levels, respectively (see Fig. 1). Information concerning these γ transitions was obtained, and the spins of the levels at 0-, 159-, 278-, 344-, 362-, and 663-keV excitation were determined.

EXPERIMENT

1. Apparatus. The Tulane University 3-MV Van de Graaff accelerator was the source of protons for the reaction. The angular correlation apparatus is shown in Fig. 2 with one 30-cm^3 Ge(Li) detector and one 7.6-cm × 7.6-cm NaI(Tl) scintillator mounted. Most of the coincidence measurements were performed with this detector configuration. Another detector mount to accomodate a 2-cm^3 Ge(Li) detector was employed for the angular distributions and for two simultaneous correlations requiring energy separation of γ peaks in both de-