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Elastic Scattering of 1.33-MeV Photons from Lead and Uranium

Gerald Hardie, Joan S. DeVries, and Chwan-Kang Chiang

Physics Department, Western Michigan University, Kalamazoo, Michigan 49001

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Differential cross sections for the elastic scattering of 1.33-MeV photons from lead and uranium have been measured at angles ranging from 12 to 60°, using a lithium-drifted germanium detector. Available calculations of the various elastic scattering amplitudes are briefly reviewed and an argument is presented to establish the relative phases between the two polarization components of the real scattering amplitudes. Those relative phases and the relative phases among the amplitudes of the various scattering processes permit the addition of the amplitudes to calculate theoretical cross sections. Discrepancies exist between the measured and theoretical cross sections. Suggestions by several authors for resolving these discrepancies are discussed and the need for further theoretical work is indicated.

INTRODUCTION

Continuing attempts to improve the accuracy of differential elastic scattering cross-section measurements are motivated by the desire to observe a contribution from the real part of the Delbrück scattering amplitude. In the intermediate state of the Delbrück process an electron-positron pair is created in the static Coulomb field surrounding the nucleus. Since the whole atom recoils, subsequent annihilation of this pair produces a photon of essentially the same energy as the incident photon. The contributions from real and virtual pairs are contained in the imaginary and real parts, respectively, of the scattering amplitude. Experimental verification of the importance of virtual pairs to real processes has already been established (see the review article by Kane and Basavaraju¹). Further the detection of scattering attributable to the imaginary part of the Delbrück scattering amplitude (see Jackson and Wetzel² and references contained therein) provides indirect evidence for a real part because the two are connected by a dispersion relation. Nevertheless, elastic scattering measurements are still made, not only to attempt to provide very direct evidence for effects attributable to virtual pairs, but also because of increasing awareness that discrepancies exist between these measurements and theoretical calculations.

Before the Delbrück amplitudes were calculated, a discrepancy between the measurements and the coherent addition of the Rayleigh and nuclear

Thomson processes could be taken as evidence for the existence of Delbrück scattering. Since Ehlitzky and Sheppey³ performed exact numerical calculations of the Delbrück scattering amplitudes, a much more stringent comparison between theory and experiment is possible. Any significant disagreement between the measurements and a theoretical cross section which includes all the elastic scattering processes indicates a systematic error in the measurements, an incorrect theoretical calculation, or both.

As pointed out by Nath⁴ and by Hardie, Mellow, and Schwandt⁵ a discrepancy exists between theory and experiment for the case of 2.62-MeV photons scattered from lead. In an earlier paper (Ref. 5) results were presented for the scattering of 1.33-MeV photons from lead and uranium through large angles, and disagreements between the experimental results and theoretical calculations were discussed. The present paper reports the extension of these measurements to smaller angles and continues the discussion of discrepancies between theory and experiment. This disagreement was first considered in detail by Dixon and Storey⁶ and more recently by Basavaraju and Kane.⁷ These authors suggest that agreement can be secured by a suitable choice of relative phases among the various scattering amplitudes. Hence it is important to determine the theoretical restrictions which can be placed on the possible choices of relative phases. The theory section below slightly amplifies an earlier discussion⁵ of this question and also summarizes the calculations of the various

scattering processes.

Following the section on theory is a brief description of the procedure for making the measurements. The results are then summarized and compared with those of other workers. Finally, the discrepancies between the experimental results and theoretical calculations are discussed and it is suggested that more extensive calculations are required before firm conclusions can be drawn.

THEORY

To calculate an elastic scattering cross section the amplitudes for the contributing processes must be known, as well as their relative phases. Processes of importance at a photon energy of 1.33 MeV are nuclear Thomson, Rayleigh, and Delbrück scattering. Another possibility, nuclear resonance scattering, can be shown^{8,9} to contribute negligibly at this energy for targets of lead and uranium.

The nuclear Thomson amplitudes are given by¹⁰

$$a^T = -r_0 Z^2 (m/M) \vec{e}_f \cdot \vec{e}_i,$$

with Z representing the atomic number, m the electron mass, M the nuclear mass, r_0 the classical electron radius, and \vec{e}_f and \vec{e}_i unit polarization vectors for the scattered and incident beams, respectively.

The Rayleigh amplitudes are not easy to calculate. Early attempts to obtain the real amplitudes used the form-factor approximation

$$a^R = r_0 F \vec{e}_f \cdot \vec{e}_i.$$

The form factor F is defined as

$$F = \int_0^\infty |\psi|^2 e^{i\vec{q} \cdot \vec{r}} d\vec{r},$$

where $|\psi|^2$ represents the charge density and \vec{q} the momentum change of the photon. In this approximation Bethe¹¹ has calculated, using Dirac wave functions, the Rayleigh scattering from K -shell electrons. Later Brown and Mayers¹² accurately calculated the Rayleigh scattering of 1.31-MeV photons from the K -shell electrons of mercury. They found that the exact spin-flip amplitude is well approximated by the corresponding form-factor amplitude as derived by Bethe.¹¹ However, the form-factor amplitude is a very poor approximation to the nonspin-flip amplitude. Brown and Mayers¹² proposed a modified form factor G given by

$$G = \int_0^\infty \frac{mc^2}{E+V} |\psi|^2 e^{i\vec{q} \cdot \vec{r}} d\vec{r},$$

with E representing the total energy of the scattering electron, V the Coulomb potential, m the electron mass, and c the speed of light. The nonspin-flip amplitude is reasonably well represented by

$$a_{RK}^{NSF} = r_0 G (1 + \cos\theta)/2,$$

when the momentum transferred to the nucleus is small. The form factor and modified form factor are useful in extrapolating the amplitudes obtained by Brown and Mayers¹² to energies and targets not covered in their calculations and also in estimating the Rayleigh contribution from the L -shell electrons.

Delbrück amplitudes can be calculated in closed form only for scattering in the forward direction. This was done, using two different approaches, by Rohrlich and Gluckstern.¹³ Numerical calculations for energies between 1 and 20 MeV and for scattering angles from 0 to 120° have been performed by Ehlötzky and Sheppey.³ Sannikov¹⁴ gives an approximate treatment of Delbrück scattering valid for high energies and large scattering angles. His calculations are in serious disagreement with those of Ehlötzky and Sheppey.³ However, there are several reasons for preferring the results of Ehlötzky and Sheppey.³ First, their results for a scattering angle of 0° agree with those of Rohrlich and Gluckstern.¹³ Secondly, the results of Sannikov¹⁴ are incompatible with the experimental results of Bösch *et al.*¹⁵ at 9 MeV and Jackson and Wetzel² at 10.3 MeV. Thirdly, at least part of Sannikov's theoretical treatment appears to be incorrect (see the discussion in Ref. 3).

To calculate cross sections the relative phases among the various scattering amplitudes must be determined. The use of dispersion relations to establish these relative phases has been summarized by Hardie, Merrow, and Schwandt.⁵ As correctly pointed out by Bernstein and Mann¹⁶ and Basavaraju and Kane,⁷ the spin-flip amplitudes vanish at 0° and hence dispersion relations do not directly establish the relative phases among these amplitudes. However, circular polarization is only one of a number of possible choices for polarization states. Another choice is plane polarization, one unit vector being chosen parallel (\parallel), and the other perpendicular (\perp), to the scattering plane. The nonspin-flip and spin-flip amplitudes are related to the parallel and perpendicular amplitudes by the equations⁶

$$a^{NSF} = (a^{\parallel} + a^{\perp})/2,$$

$$a^{SF} = (a^{\parallel} - a^{\perp})/2.$$

Assuming these equations to be correct, the a^{\parallel} and a^{\perp} for any given process must have the same sign at 0°. Furthermore, dispersion relations

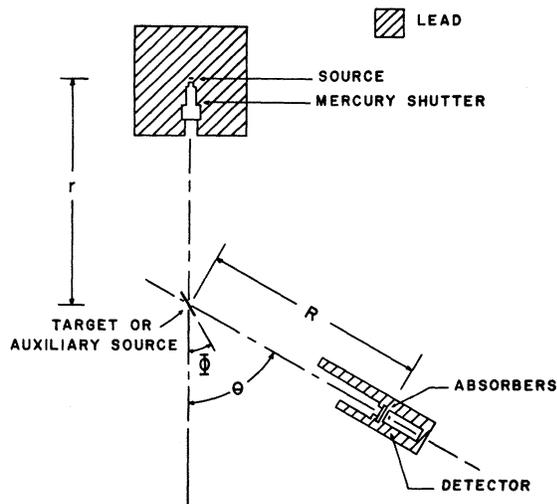


FIG. 1. A cross section of the apparatus (median plane) as seen from above. The source is approximately 100 Ci of ^{60}Co . A lithium-drifted germanium detector of 12 cm^3 active volume is used to detect the scattered photons.

determine whether or not this sign agrees with that of the a^{\parallel} and a^{\perp} at 0° for some different process. Thus all relative phases associated with these four amplitude components can be determined at 0° . This result fixes all relative phases at all angles, if the calculative techniques can be trusted not to introduce spurious sign changes in proceeding from one angle to the next.

APPARATUS AND PROCEDURE

Only a brief description of the apparatus and procedures will be given, as they are similar to those described in other papers (see, for example, Hardie, Merrow, and Schwandt,⁵ Dixon and Storey,⁶ and Standing and Jovanovich¹⁷). Figure 1 is a drawing of the experimental arrangement. The ^{60}Co source had a strength of about 100 Ci. Mer-

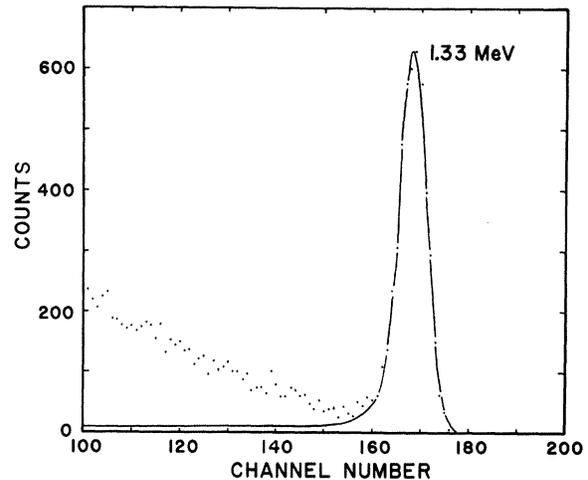


FIG. 2. The spectrum of ^{60}Co photons elastically scattered through an angle of 12° from a lead target. A bi-asied amplifier was used to select and expand the region containing the 1.33-MeV photons. The solid curve is the shape of the 1.33-MeV full-energy peak obtained with the main source closed and the target replaced by the weak auxiliary source.

cury, which normally completes the shielding around the source, can be forced into a reservoir, permitting a beam of photons to strike a lead or uranium target. Targets were 14.2 cm by 16.5 cm and had thicknesses ranging from 0.33 to 2.02 g/cm² for lead and 1.0 to 1.7 g/cm² for uranium. An angle Φ (see Fig. 1) was chosen to minimize the spread in scattering angles due to the finite size of the scatterer.⁵ Photons scattered from the target were detected with a lithium-drifted germanium semiconductor detector of 12-cm³ sensitive volume and an energy resolution of 3.0-keV (full width at half maximum) at 1.33 MeV. Lead was placed in front of this detector to discriminate against the numerous lower-energy photons Compton-scattered from the target.

The distance r between the source and target

TABLE I. Differential cross sections for elastic scattering of 1.33-MeV photons from lead (mb/sr).

Scattering angle (deg)	Present results	Dixon and Storey (Ref. 6)	Standing and Jovanovich (Ref. 17)	Bernstein and Mann (Ref. 16)	Hara <i>et al.</i> ^a
12	121.6 ± 7.2		130 ± 20		
15				133 ± 22	
20	32.1 ± 1.9		32.2 ± 3.3		43 ± 13
30	7.17 ± 0.37	6.20 ± 0.40	7.7 ± 0.7	10.0 ± 1.5	
35					3.81 ± 0.76
45	0.791 ± 0.042	0.760 ± 0.040	0.90 ± 0.06	1.5 ± 0.4	
50					0.58 ± 0.12
60	0.190 ± 0.010	0.185 ± 0.013	0.206 ± 0.011	0.47 ± 0.07	

^aE. Hara, J. Banaigs, P. Eberhard, and L. Goldzahl, *J. Phys. Radium* **19**, 668 (1958).

TABLE II. Differential cross sections for elastic scattering of 1.33-MeV photons from uranium (mb/sr).

Scattering angle (deg)	Present results	Goldzahl and Eberhard (Ref. 18) (%)	Bernstein and Mann (Ref. 16)
12	203 ± 12		
15			205
20	37.5 ± 2.3		
30	11.3 ± 0.68		17
35		6.14 ± 20	
45	1.72 ± 0.10		2.0
50		1.23 ± 20	
60	0.430 ± 0.025		0.80

was 148.7 ± 1.0 cm and the distance R between target and detector was set equal to either r or $2r$.

To avoid determining the efficiency of the detector and the solid angle subtended by the target at the detector, two measurements were performed. We first determined the rate n_a at which the elastically scattered photons were detected in the full-energy peak when the beam strikes the target. For the second measurement, the mercury shutter was closed and the target was replaced by a weak auxiliary source made by spreading a liquid containing ^{60}Co uniformly on a piece of cardboard which has the same length and width as the targets. The rate n_b at which the 1.33-MeV photons from this source were detected in the full-energy peak was then determined. The differential elastic scattering cross section $d\sigma/d\Omega$ is calculated using the formula¹⁷

$$d\sigma/d\Omega = (n_a/n_b)(b/a)(r^2/N),$$

in which N represents the number of target atoms, and a and b represent the strength of the main and auxiliary source, respectively. A correction must be made for the reduction in counting rate due to absorption in the target of both the incident beam and elastically scattered photons.

A biased amplifier was used to select and expand the part of the spectrum containing the 1.33-MeV full-energy peak. This part of the spectrum, from a lead target and a scattering angle of 12° , is shown in Fig. 2. The line in this figure is the shape of the full-energy peak obtained with the auxiliary source.

RESULTS

Listed in Table I are the present results, as well as those of others, for the elastic scattering of 1.33-MeV photons from lead. The uncertainties quoted for the present work are standard deviations and include a contribution from the source-ratio measurement. Bernstein and Mann¹⁶ presented their results graphically: The numbers

given in Table I are Standing and Jovanovich's¹⁷ estimates from this graph. The cross sections of Standing and Jovanovich¹⁷ given in Table I are those obtained with their "two-photomultipliers" method. Not included in their uncertainties is that due to the source-ratio measurement, b/a , which they estimate as $\pm 5\%$. Our results, which lie between those of Standing and Jovanovich¹⁷ and Dixon and Storey,⁶ agree fairly well with the cross sections of these two groups, except at 30° .

Table II gives the present cross sections, as well as those of other groups, for the elastic scattering of 1.33-MeV photons from uranium. The results of Bernstein and Mann¹⁶ were estimated from the graph presented in their paper. Our cross sections agree reasonably well with those of Goldzahl and Eberhard¹⁸ but are considerably lower than those of Bernstein and Mann.¹⁶

DISCUSSION

To calculate cross sections with which to compare the present experimental results, the mercury K -shell Rayleigh scattering amplitudes of Brown and Mayers¹² must be extrapolated from a photon energy of 1.31 to 1.33 MeV and to targets of lead and uranium. Further, for the range of angles considered in the present work, the L -shell contribution cannot be neglected and so must be estimated.

Extrapolation from 1.31 to 1.33 MeV can be done by noting that, on a semilogarithmic plot, the amplitudes exhibit an almost linear energy dependence at a fixed scattering angle. For energies less than 1.5 MeV and for the scattering angles considered in this paper the complication of a sign change in the scattering amplitudes does not arise. Alternatively, form factors can be used:

$$a_{RK}^{\text{NSF}}(1.33 \text{ MeV}) = a_{RK}^{\text{NSF}}(1.31 \text{ MeV}) \frac{G(1.33 \text{ MeV})}{G(1.31 \text{ MeV})}$$

and

$$a_{RK}^{SF}(1.33 \text{ MeV}) = a_{RK}^{SF}(1.31 \text{ MeV}) \frac{F(1.33 \text{ MeV})}{F(1.31 \text{ MeV})},$$

where a_{RK}^{NSF} and a_{RK}^{SF} represent, respectively, the Rayleigh K -shell nonspin-flip and spin-flip amplitudes for mercury. Both methods of extrapolating to the slightly higher energy give the same amplitudes to within 1% and so the form-factor method was arbitrarily chosen.

Since the Z dependence of the Rayleigh K -shell amplitudes is not known, the accuracy of the extrapolation from the exact calculations for mercury

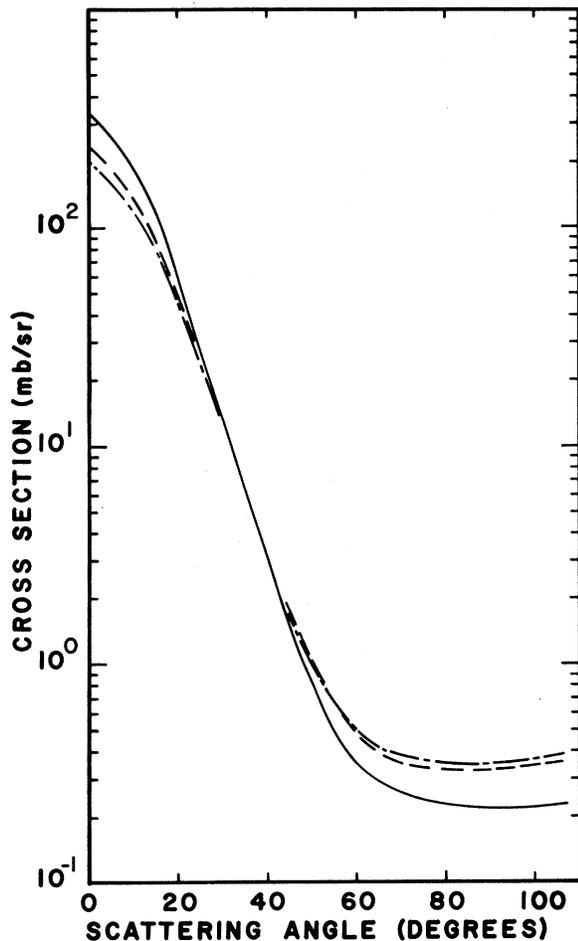


FIG. 3. Theoretical cross-section curves for the elastic scattering of 1.33-MeV photons from uranium. The solid curve is obtained by extrapolating the coherent sum of nuclear Thomson, Rayleigh K -shell, and real spin-flip Rayleigh L -shell amplitudes from mercury using the Z dependence given in Ref. 19. The dash curve contains a nuclear Thomson contribution, a Rayleigh K -shell contribution obtained by using form factors to extrapolate the exact calculations for mercury, and a Rayleigh L -shell real spin-flip contribution estimated using form factors. The dot-dash curve differs from the dash curve because the Delbrück amplitudes have been included.

to lead and uranium is open to question. The method used in the present work is as follows:

$$a_{RK}^{NSF}(Z=82) = a_{RK}^{NSF}(Z=80) \frac{G(Z=82)}{G(Z=80)},$$

$$a_{RK}^{SF}(Z=82) = a_{RK}^{SF}(Z=80) \frac{F(Z=82)}{F(Z=80)}.$$

The Rayleigh L -shell amplitudes are estimated from the K -shell amplitudes in the following manner:

$$a_{RL}^{NSF} = a_{RK}^{NSF} G_L / G_K,$$

and

$$a_{RL}^{SF} = a_{RK}^{SF} F_L / F_K.$$

The imaginary parts of the Rayleigh amplitudes were extrapolated in the same manner as their corresponding real parts.

Another method for estimating the theoretical elastic scattering cross sections has been used by Anand and Sood.¹⁹ They employ an experimentally determined Z dependence of the total elastic scattering cross section. The solid curve in Fig. 3 is the theoretical cross section for uranium based on

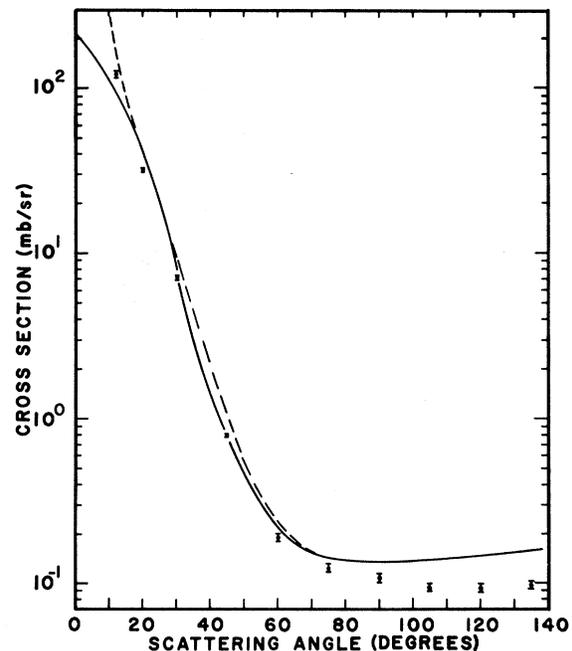


FIG. 4. The dots at angles less than or equal to 60° are the present results, while those for angles greater than 60° are from Ref. 5, for a lead target scattering 1.33-MeV photons. The solid curve is the coherent sum of the nuclear Thomson, Delbrück, Rayleigh K -shell and Rayleigh L -shell (spin-flip only) amplitudes. The dash curve results from also including a nonspin-flip Rayleigh L -shell amplitude.

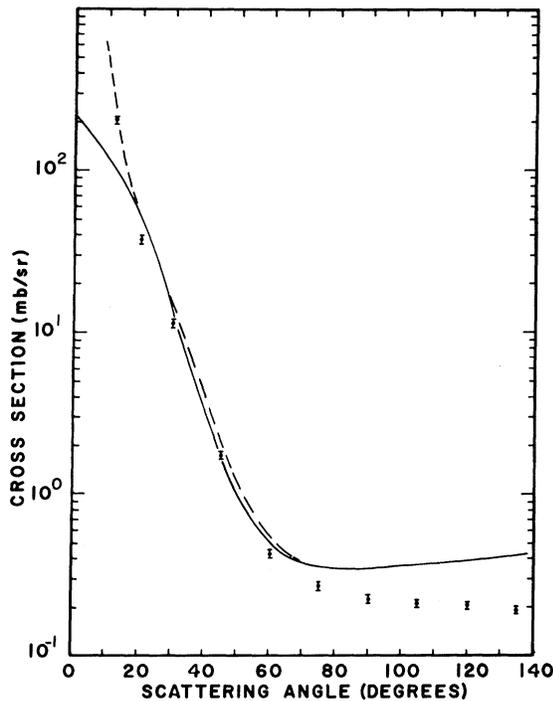


FIG. 5. The dots at angles less than or equal to 60° are the present results, while those for angles greater than 60° are from Ref. 5, for a uranium target scattering 1.33-MeV photons. The solid curve is the coherent sum of the nuclear Thomson, Delbrück, Rayleigh K -shell and Rayleigh L -shell (spin-flip only) amplitudes. The dash curve results from also including a nonspin-flip Rayleigh L -shell amplitude.

their method. The calculation included the nuclear Thomson amplitudes, the Rayleigh K -shell amplitudes, and a real spin-flip L -shell amplitude, approximated using the form factor. The dash curve in Fig. 3 is obtained by using the form factor and modified form factor, as discussed above. It includes the nuclear Thomson and Rayleigh K -shell amplitudes, as well as a real spin-flip contribution from the L -shell electrons. If the Delbrück amplitudes are also included, the dot-dash curve results. Since different methods of extrapolation result in different cross sections, care must be exercised in drawing conclusions from a discrepancy between the theoretical calculations and experimental results. In particular, it should be noted that at some angles the discrepancy between the two methods of extrapolation is greater than the contribution from Delbrück scattering. In Figs. 4 and 5 the present results for lead and uranium, respectively, are compared with the theoretical cross sections obtained using the above equations to extrapolate the Rayleigh K -shell amplitudes and to estimate the L -shell amplitudes. Also included are the experimental results of Hardie, Merrow,

and Schwandt⁵ for angles greater than 60° . The solid curve contains nuclear Thomson, Delbrück, Rayleigh K -shell and Rayleigh L -shell (spin-flip only) contributions. The dash curve is the result of also including the nonspin-flip Rayleigh L -shell amplitude. Agreement between theory and experiment is reasonable at scattering angles of 30° , 45° , and 60° . It is clear that a meaningful comparison between theory and experiment for scattering angles less than about 15° will require a reasonably accurate knowledge of the Rayleigh L -shell nonspin-flip amplitude.

Since both lead and uranium targets were used, our results can be compared with the Z dependence determined by Anand and Sood.¹⁹ The power n is defined by

$$\frac{d\sigma}{d\Omega}(Z') = \frac{d\sigma}{d\Omega}(Z) \left(\frac{Z'}{Z}\right)^n.$$

Anand and Sood¹⁹ have investigated the variation of n with q , the momentum transferred to the atom, given by

$$q = 2(E/mc^2) \sin(\theta/2)$$

in units of mc with E representing the incident photon energy, m the rest mass of the electron, c the speed of light, and θ the scattering angle. The dot-dash curve in Fig. 6, taken from the paper by Anand and Sood,¹⁹ is a smooth curve through their experimentally determined points. To obtain these experimental results they used sources with energies of 0.280, 0.412, and 0.662 MeV; targets of lead, tungsten, tin, and silver; and scat-

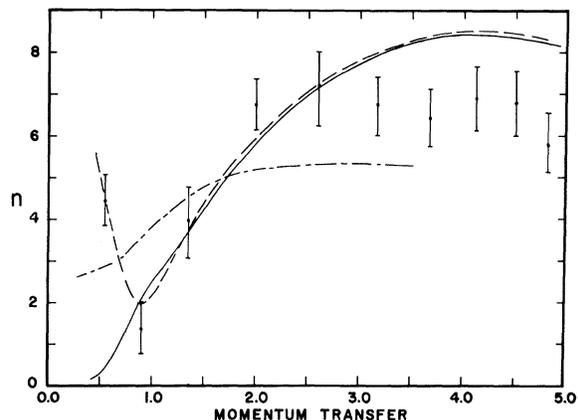


FIG. 6. The Z dependence of the elastic scattering cross section as a function of the momentum transferred to the atom. The momentum transfer is in units of mc (see text). The points are from the present work and from Ref. 5. The dot-dash curve is from Ref. 19. The solid and dash curves are calculated, respectively, from the cross sections given by the solid and dash curves in Figs. 4 and 5.

tering angles equal to or greater than 30° . The points in Fig. 6 were extracted from the present experimental work. The solid and dot-dash curves were obtained from the theoretical cross sections shown as solid and dot-dash curves, respectively, in Figs. 4 and 5. Again the importance of the L -shell nonspin-flip contribution is evident at small angles. If our results contain no undetected systematic error then the Z dependence given by Anand and Sood¹⁹ cannot be used to accurately extrapolate the elastic scattering cross sections from mercury to the targets and to the angular range covered in the present work.

The discrepancy between the calculations and results for angles greater than 60° has been discussed by a number of authors. Dixon and Storey⁶ pointed out that much better agreement is obtained by assuming either a 180° phase shift between the parallel polarization components of the Rayleigh K -shell and nuclear Thomson amplitudes without a corresponding phase shift between the perpendicular polarization components or by assuming that the Rayleigh and nuclear Thomson processes are incoherent at large scattering angles. Another way to remove this discrepancy, destructive interference at large angles between Rayleigh scattering from the K shell and from the L shell, was presented by Hardie, Merrow, and Schwandt.⁵ Recently Basavaraju and Kane⁷ have suggested that the discrepancy is due to destructive interference between the real Delbrück and Rayleigh spin-flip amplitudes. To secure agreement for 1.33-MeV photons scattered from lead through an angle of 124.5° a real Delbrück spin-flip amplitude approxi-

mately four times greater than calculated by Ehlötzky and Sheppey³ is necessary. If this is the correct explanation, then our arguments regarding relative phases, presented in the theory section, must be wrong. Further, the calculations of Ehlötzky and Sheppey³ must be wrong. However, as discussed in the present paper, there are no compelling reasons to suspect these calculations.

The cross sections for the elastic scattering of 1.33-MeV photons from lead through angles greater than 60° , measured by a number of workers, are in good agreement. However, the present authors feel that more theoretical work is required. Brown and Mayers¹² method should be used to calculate the scattering of 1.33-MeV photons from lead and uranium, thus eliminating extrapolations of uncertain accuracy. An attempt should be made to accurately calculate the Rayleigh scattering from L -shell electrons. In view of the comments by Basavaraju and Kane,⁷ Ehlötzky and Sheppey's³ numerical calculations of Delbrück scattering amplitudes should be checked. Finally, arguments purporting to establish the relative phases between the two polarization components of a given process and among the various processes should be critically evaluated.

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