Coriolis Coupling in the Odd Tungsten Isotopes

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The (d,p) and (d,t) reactions into the five odd tungsten isotopes from ^{179}W to ^{187}W were studied with 12-MeV incident deuterons. It was found that many gross discrepancies between the experimental cross sections and those calculated using the Nilsson model and the distorted-wave Born approximation could be eliminated or reduced by inclusion in the cross-section calculations of the Coriolis interaction with reduced coupling matrix elements. In particular, the highly distinctive systematics of the cross sections in different isotopes could be reproduced. An energy-dependent systematic reduction of the matrix elements which satisfactorily accounts for the experimental observations is suggested.

The Nilsson model has had a wide and successful application in the study of nuclei with stable quadrupole deformations. More recently, the inclusion of various couplings among the single-particle states has considerably widened the range of applicability of the model. One of the more important of these couplings is the Coriolis interaction,[1] which is the subject of this paper.

We have studied the (d,p) and (d,t) reactions at 12-MeV bombarding energy leading into states in the five odd tungsten isotopes from ^{179}W to ^{187}W. The reaction products were analyzed in a magnetic spectrograph, and absolute cross sections were extracted to an accuracy of $\pm 20\%$ for intensities above 10 μb/sr. From 5 to 11 rotational bands comprising from 10 to about 35 levels were identified in each nucleus. Figure 1 shows the level scheme for ^{181}W as an example of the spectroscopic results obtained. Earlier works[2,3] contain details of the experiments and a more complete discussion of the Coriolis mixing.

The differential cross section in a single-neutron transfer reaction on a spin-zero even-even deformed target nucleus, proceeding to a level of spin $I=j$ in the final nucleus is given[4] by $d\sigma/d\Omega = 2\Phi_l C_{jl}^2 P^2$. Here, Φ_l is the normalized distorted-wave Born-approximation (DWBA) cross section, $P=U$ or V is the pairing factor, and C_{jl} is the coefficient of the component with angular momentum j in the Nilsson wave function. The cross sections to the successive members of a rotational band built on a given Nilsson orbital thus exhibit an intensity pattern which is characteristic of that orbital. Although this feature has led to the identification of many rotational bands in deformed nuclei, the measured cross-section patterns are often only in qualitative agreement with the model, and the observed intensities frequently differ by orders of magnitude from those expected. It is of considerable interest to determine the source of these discrepancies and to study whether they can be simply explained within the general framework of the Nilsson model.

With the inclusion of Coriolis mixing, the factor $C_{jl}^2 P^2$ in the cross-section formula is replaced by a coherent summation over the admixed levels $(\sum_i a_i C_{jl}^i P_i)^2$, where the amplitudes of the admixed wave functions are denoted by a_i. This often gives rise to drastically altered cross sections. The Coriolis-coupling matrix element between states of angular momentum I differing by one unit in the K quantum number is

$$\langle I, K | V_{\mathrm{Cor}} | I, K+1\rangle = -\frac{\hbar^2}{2\mathcal{I}}\eta_K \eta_{K+1}\langle K | j_- | K+1\rangle \times [(I-K)(I+K+1)]^{1/2}(U_K U_{K+1} + V_K V_{K+1}), \quad (1)$$

where $\hbar^2/2\mathcal{I}$ is the inertial parameter. It is taken here as the *mean of its values in the two neighboring even-even nuclei.* The quantity $\langle K | j_- | K+1\rangle$ is

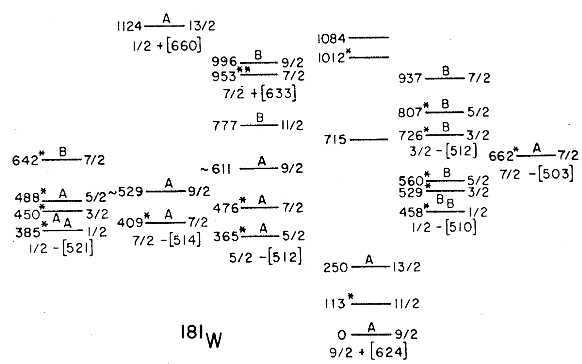

FIG. 1. Level scheme deduced for ^{181}W. To the left are hole excitations, to the right particle excitations. The letters A and B indicate greater and slightly lesser confidence in the assignments although all the identifications shown here are considered firm. Unassigned levels are shown above the ground-state band. Energies marked with an asterisk are more accurate values taken from a concurrent radioactivity study (see Ref. 8). (The 953-keV level was not observed in the reactions.)

calculated from the Nilsson wave functions, and the last factor reflects the effects of pairing. The attenuation factors η_K and η_{K+1} are equal to unity in the Nilsson model. In the present analysis they are adjusted according to an empirical prescription.

The Coriolis calculations were performed with Nilsson wave functions ($\kappa = 0.0637$, $\mu = 0.42$, $\delta = 0.22$) to calculate the Coriolis matrix elements between all pairs of negative-parity orbitals identified. The reductions due to pairing were included corresponding to an energy-gap parameter $\Delta = 750$ keV. A fit was made to the experimental excitation energies by varying, for each band, the band-head energy and the rotational energy parameters A_K and $a_{K=1/2}$. In general, these fits were highly overdetermined. Test calculations showed that the inclusion of unobserved orbitals had negligible effects on the levels of interest.

Mixing calculations with the full Nilsson-model coupling strength ($\eta_K \eta_{K+1} = 1$) are unable to reproduce the experimental energies. For example, the $\frac{7}{2}$ and the $\frac{9}{2}$ rotational states of the $\frac{5}{2}-|512|$ and $\frac{7}{2}-|514|$ orbitals in ^{183}W and ^{185}W are too closely spaced to be compatible with the full Nilsson-model coupling strength. In the tungsten nuclei, these two orbitals predominately mix with one another, and the ratio of transfer-reaction cross sections for their $\frac{7}{2}$ states is strongly dependent upon the amount of mutual mixing. We have therefore used the cross sections for these two $\frac{7}{2}$ states to empirically adjust the coupling strength $\eta_{5/2}\eta_{7/2}$. The results are shown in Fig. 2(a) along with some results of energy and cross-section fits for two other pairs of Nilsson bands which predominantly mix with each other. Somewhat arbitrarily, η_K was limited to ≤ 1 in these fits.

Figure 2(a) suggests that the same matrix element requires different attenuations in different nuclei. At present, the origin of this attenuation and its variation in different nuclei is not understood. Guided by the results shown in Fig. 2(a), we therefore performed coupling calculations in which each wave function was attenuated with an η_K decreasing linearly with the excitation energy of the band head. An identical energy dependence was used for $^{179,183-187}$W; a somewhat stronger decrease was optimal for ^{181}W [see Fig. 2(b)]. The same empirical rule for η_K also gave optimal results for the positive-parity states in these nuclei.[3] Other prescriptions for the η_K factors also yield improved agreement with experiment although the best agreement was obtained with that described above. In any case, it is clear that certain matrix elements must be substantially attenuated in order to reproduce the observed energies and cross sections.

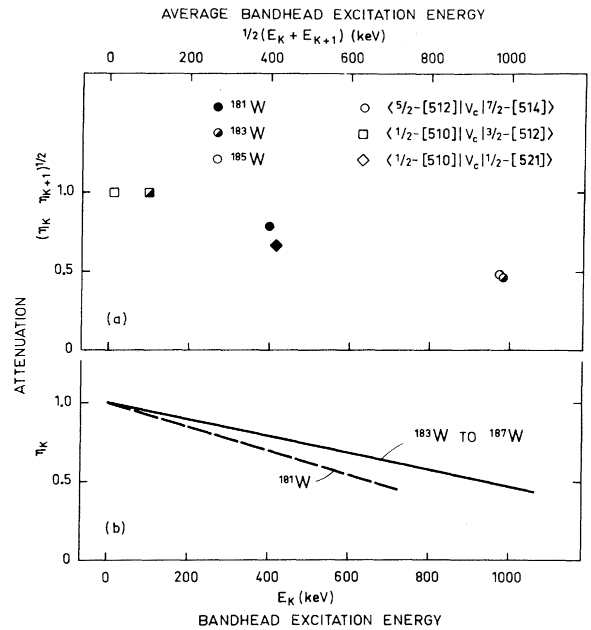

FIG. 2. (a) Empirical Coriolis matrix-element attenuation factors in different tungsten nuclei shown as a function of the average excitation energy of the two admixed bands. The ordinate scale is a convenient measure of the attenuation of the coupling of the orbitals K and $K+1$ [Compare Eq. (1)]. (b) Energy dependence of the attenuation factors η_K adopted in the coupling calculations.

Once the attenuation prescription was chosen, the coupling calculations were performed to reproduce the observed excitation energies only. Using the wave functions obtained from this energy fit, and with no further variation of parameters, the cross sections were calculated and compared with the experimental values. Some results are given in Figs. 3 and 4.

Figure 3(a) shows the (d,t) results for the $\frac{1}{2}-|510|$ and $\frac{3}{2}-|512|$ orbitals in ^{185}W. The mixing changes the entire pattern of intensities to the two bands and brings the perturbed and experimental cross-section patterns into close agreement. Figure 3(b) shows the (d,p) results for the $\frac{1}{2}-|521|$ band in ^{181}W which is a hole excitation in this nucleus. The spin $\frac{1}{2}$ and $\frac{5}{2}$ members of the band have approximately the (small) cross section expected for this hole excitation in the (d,p) reaction. However, the $\frac{3}{2}$ and $\frac{7}{2}$ members show very large (d,p) intensities. The simple Nilsson model with pairing cannot account for this but the mixing calculation does so quite well. This somewhat unusual behavior is due to strong mixing with the nearby $\frac{1}{2}-|510|$ particle excitation. It is a straightforward consequence of the magnitudes of the C_{jl} coefficients and the mixing amplitudes that in this case only certain rotational states receive a strong (d,p) cross section.

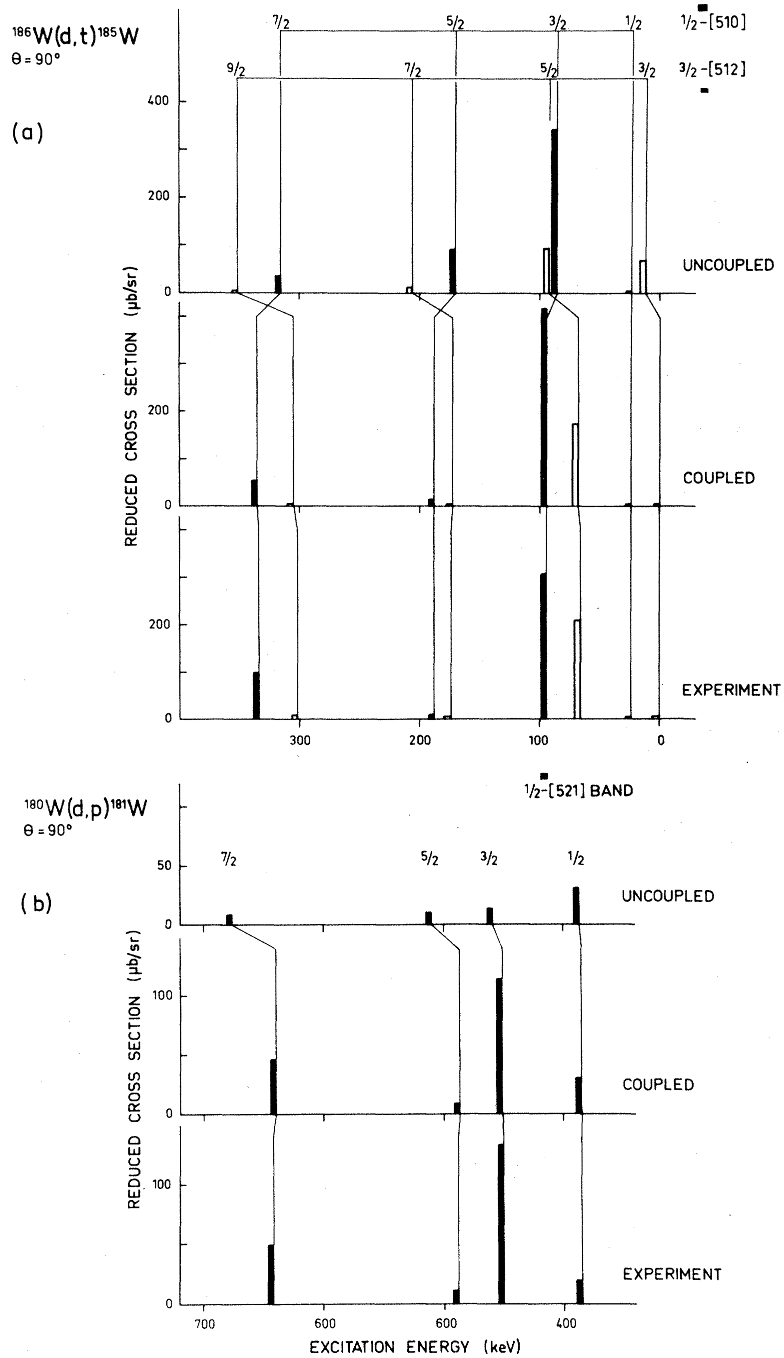

FIG. 3 (a) Calculated and experimental (d,t) cross sections for the $\frac{1}{2}-|510|$ and $\frac{3}{2}-|512|$ bands in ^{185}W. The cross sections correspond to a Q value of −2 MeV; the experimental cross sections are reduced to this Q value. The smallest states shown have ≤ 5 μb cross section. The results shown are those of a mixing calculation including seven Nilsson bands. However, the mixing effects are essentially caused by the mutual mixing of the two bands shown. The coupled cross sections are calculated with $\eta_{1/2}\eta_{3/2}=1$. (b) Calculated and experimental (Q-reduced) (d,p) cross sections for the $\frac{1}{2}-|521|$ band in ^{181}W. Six Nilsson bands were included in calculating the coupled cross sections. ($\eta_{1/2-|521|}=0.70$, $\eta_{1/2-|510|}=0.65$.)

Figure 4 displays some cross-section systematic for strongly populated levels with cross sections that vary by more than a factor of 5 throughout these isotopes. Figures 4(a) to 4(d) give the (d,p) or (d,t) systematics for several levels of the $\frac{1}{2}-|510|$ and $\frac{3}{2}-|512|$ bands. The pairing theory predicts a smooth cross-section variation of about a factor 2 through the tungsten nuclei, whereas the observed cross sections vary by factors 10 to 100. The sudden change in the cross sections shown from ^{183}W to ^{185}W is caused by the interchange of the positions of the $\frac{3}{2}-|512|$ orbital and its prominent mixing partner, the $\frac{1}{2}-|510|$ orbital. In ^{181}W and ^{183}W, the $\frac{1}{2}-$ band lies below the $\frac{3}{2}-$ band, while in ^{185}W and ^{187}W their positions are reversed. A similar interchange of the relative excitation energies of the $\frac{7}{2}-|514|$ and $\frac{5}{2}-|512|$ bands from ^{181}W to ^{183}W causes the drastic cross-section variation for the $\frac{9}{2}\,\frac{5}{2}-|512|$ state shown in Fig. 4(e). The calculations reproduce very well the observed cross-section variations, although the absolute cross sections sometimes differ from the predicted values (see Fig. 4). Systematic deviations in absolute cross section were also found for certain other states not shown here. This might be attributed to deficiencies of the Nilsson wave functions or of the DWBA, although the absolute cross sections in general are surprisingly well matched by the calculations.

A similar pattern as shown in Figs. 3 and 4 applies for almost all states which exhibit strong cross-section variations. Variations of orders of magnitude are not uncommon, but the calculations with Coriolis coupling reproduce the experimental results satisfactorily. For example, in the (d,t) reaction data for ^{183}W and ^{185}W there are 22 states (with $\sigma \geq 10$ μb/sr) which differ by more than 30% from the calculated unperturbed cross sections, 14 of these by more than a factor of 2. 20 of these 22 states are affected appreciably by the mixing, and in every case the deviation from experiment is reduced. With mixing included, only 11 of the original 22 states differ by more than 30% from the experimental value, 6 of these by a factor of 2 or more. It should finally be noted that many cross sections are hardly affected by mixing. For most of these states the calculated and observed cross sections are in satisfactory agreement.

The Coriolis coupling has been known for a long time to be of importance for the understanding of the spectra of the odd tungsten nuclei.[1,5,6] The results presented here consider a larger amount of data than has been available before and stress the fact that Coriolis coupling in these nuclei drastically affects the single-particle transfer cross sections. The inclusion of the coupling in the cross-section calculations very substantially improves the agreement with the observed values and hence, the confidence in the orbital assignments made from the reaction data.

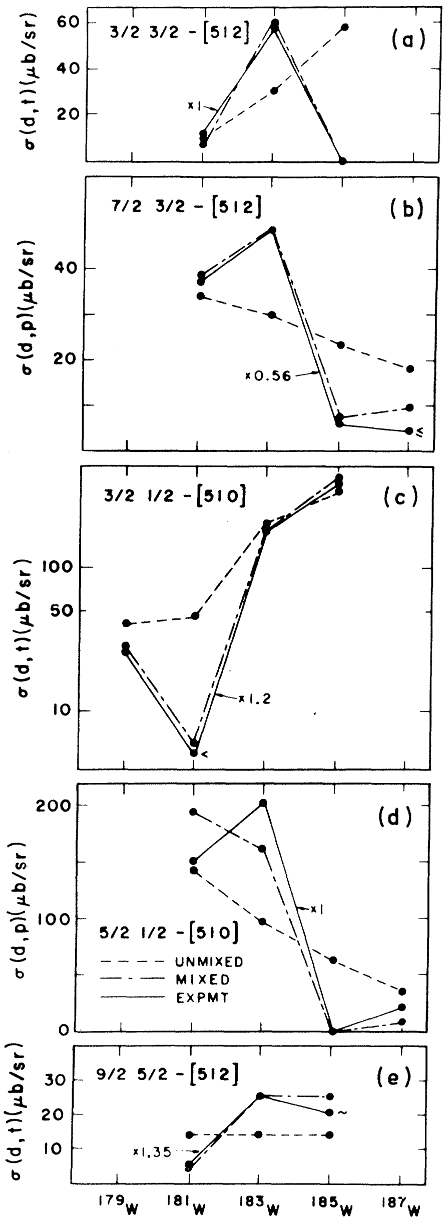

FIG. 4. Systematics of the unperturbed (dashed line), Coriolis coupled (dot-dashed line), and experimental (solid line) cross sections to several states in the tungsten nuclei. (All cross sections are reduced to the standard Q values.) The experimental cross sections are normalized with the indicated multiplicative numbers so as to emphasize the systematics of the cross-section variations.

It is found that in many cases the full Nilsson-model coupling matrix elements produce excessive mixing, especially for higher-lying states in the tungsten nuclei. An empirical attenuation factor

η_K for each orbital which is unity near the ground state and decreases linearly with excitation energy has been found to describe the observations, but other attenuation schemes seem to be feasible. The origin of the matrix-element attenuation is at present not understood. It has been suggested[7] that it might be due to dilution of the wave functions for the high-lying states. The present analysis does not support this suggestion, since the transfer cross sections indicate spectroscopic factors near unity for the uncoupled states.[8]

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Quasirotational Levels in ^{152}Gd and Excited Levels of ^{152}Tb from the Decay of 4.2-min ^{152m}Tb†

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In the decay of the 4.2-min isomeric state in ^{152}Tb, 8 γ rays are found to be associated with the deexcitation of levels in ^{152}Tb and 25 γ rays with that in ^{152}Gd. Experimental studies were made chiefly with Ge(Li) γ-ray detectors in singles and coincidence modes. Conversion-electron spectra were obtained with a Si(Li) spectrometer. The isomeric state, believed to be an 8^+ state, is at 501.8 keV, and the isomeric transition is a 159.59-keV $E3$ transition to a level at 342.2 keV. Other levels in ^{152}Tb are proposed at 58.9, 65.1, and 106.7 keV. An alternative but less likely level scheme is also possible. The ground state (2^-) and levels at 58.9, 342.2, and 501.8 keV appear to be best described in terms of Nilsson states. The electron-capture decay of the 501.8-keV isomeric state is by an allowed unhindered transition ($\log ft$ 4.7) which leads almost entirely to a level at 2394.5 keV in ^{152}Gd. The deexcitation of the latter level populates 6^+ and 8^+ members of the quasiground-state and quasi-β-vibrational bands. The quasirotational character of levels in ^{152}Gd which has been proposed by Sakai and coworkers appears to be amply confirmed. A previously unreported level in ^{152}Gd at 1861.7 keV (5^+) is proposed. In contrast to previous work, no α-decay branch was observed.

I. INTRODUCTION

Isomerism in Tb nuclides is very common, especially among the odd-odd isotopes.[1] Since the neutron number of known Tb nuclides ranges from the 82-neutron region well into the region of stable deformation, a systematic study of the odd-odd Tb nuclides provides a basis for analyzing the coupling of the odd neutron and odd proton as a function of deformation. In this work we describe the results obtained from studying the decay of 4.2-min ^{152m}Tb.

Since the decay of the 18-h ground state[2,3] of ^{152}Tb populates chiefly low-spin levels in ^{152}Gd, we hoped that the isomeric state would undergo β decay to higher-spin states. The quasirotational character of ^{152}Gd levels[4] which has been observed in reaction spectroscopy, at least insofar as the high-spin levels are concerned, has not been apparent from the decay of either ^{152}Eu[5] or 18-h ^{152}Tb.[2,3]