trix-element extractions in other nuclei. However, the general expression [Eq. (3)] can be useful in matrix-element extractions.

Given a value of $\int \vec{\alpha}$ and λ, the values of $\int i\vec{r}$ and $\int i\vec{r}(r/\rho)^2$ are fixed by Eq. (3) and the definition of λ. Since $\int \vec{\sigma}x\vec{r}$ and $\int i\vec{r}$ have the same radial dependence, it also is reasonable to use the following approximation:

$$\int \vec{\sigma}x\vec{r}\left(\frac{r}{\rho}\right)^2 \approx \lambda \int \vec{\sigma}x\vec{r}\,.$$

The matrix elements $\int i\vec{r}(r/\rho)^2$ and $\int \vec{\sigma}x\vec{r}(r/\rho)^2$ are the most important higher-order matrix elements because they enter the expression for the transition probability with relatively large coefficients. Therefore, the general expression for Λ can be used to include important high-order terms in the matrix-element extraction without making the extraction more difficult. $\int \vec{\alpha}$ and λ can be treated as unknown rather than $\int \vec{\alpha}$ and $\int i\vec{r}$.

A brief analysis of the matrix elements using the shell model was presented in our earlier work on this transition.[6] A detailed study of Rb^{86} has been made by S. Wahlborn.[11] The better limits of error obtained in this work would not alter the descriptions provided in either of these earlier papers. The present results confirm our earlier observation[6] that the experimental limits of error used by Wahlborn for the matrix-element parameter w are too restrictive. The limits placed on λ provide new nuclear-structure information. The higher-order matrix element $\int i\vec{r}(r/\rho)^2$ must be less than or equal in magnitude to $\int i\vec{r}$, and the two matrix elements may have equal or opposite sign.

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*Present address: IBM Space Guidance Center, Owega, New York.

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Shell-Model Calculations for Cu^{60} and Zn^{60} with Reaction Matrix Elements*

R. P. Singh and M. L. Rustgi
Department of Physics, State University of New York, Buffalo, New York 14214
(Received 31 August 1970)

Employing Ni^{56} as core, and Kuo and Brown renormalized matrix elements for the Hamada-Johnston nucleon-nucleon potential as the residual interaction, the level structure of both Cu^{60} and Zn^{60} is studied within the framework of the shell model. The four active nucleons are allowed to populate the $1p_{3/2}$, $0f_{5/2}$, and $1p_{1/2}$ single-particle orbitals. A comparison with the observed level spectra is made, and good agreement with those levels whose spins and parity are definitely known is found. Other levels whose spins and parity assignments are not definite have corresponding theoretical levels which should help in determining their spins in the future measurements.

1. INTRODUCTION

Until a few years ago, nuclear structure calculations could be performed only by resorting to purely phenomenological models. This was partly due to the lack of the knowledge of the nucleon-nucleon interaction and partly due to the lack of development of theoretical techniques needed to carry out such calculations. However in recent years because of the availability of high-speed computers nuclear physics has entered its quantitative phase. Detailed shell-model calculations are feasible if

the configuration space chosen is such that it does not lead to extremely large matrices. It is well known that only few active orbits and very few valence nucleons are needed to produce enormously large dimensions. For example if one tried to describe Ni^{60} in terms of the configurations $\{0f_{7/2}, 1p_{3/2}, 1p_{1/2}, 0f_{5/2}, 0g_{9/2},\}^{20}$, the dimensions of the matrices involved run into the hundreds of thousands.[1] It is for this reason that almost all of the calculations of Ni isotopes have been carried out assuming a Ni^{56} core.[1-5] Instead of choosing an effective interaction arbitrarily, with just simplicity and ease of calculation in mind, it is now possible to derive such interactions[6-8] from free nucleon-nucleon interactions. In recent years there has been a considerable advance in our understanding of the two-nucleon problem.[9-11] Once the nucleon-nucleon interaction is chosen, at least in principle, the effective interaction to be employed in any shell-model calculation will be fixed, subject only to the approximations involved in the calculation of the interaction matrix elements. The main difficulty in these calculations is caused by the presence of hard cores and is overcome by employing the separation method of Scott and Moszkowski.[12]

The effective matrix elements calculated by Kuo and Brown in this way are found to be weak, as they give rather condensed shell-model spectra. The renormalization of reaction matrix G via the core excitation must be investigated. Kuo and Brown find these two processes to be generally comparable for most nuclei. The effect of ignoring the nearby orbits is treated as a perturbation by considering the two-particle or two-hole diagrams. Another approach employing the unitary transformation to realistic interactions has been followed by many others, the basis of which has been justified by Bell.[13]

Employing the renormalized matrix elements of Kuo and Brown, shell-model calculations of those nuclei whose wave functions predominantly contain two valence nucleons plus an inert core have been carried out by Kuo and Brown.[6-8] Similar calculations with the Tabakin potential have been also performed by Kuo, Baranger, and Baranger.[14] The agreement is good enough to encourage more complicated calculations with several nucleons outside the closed core. Such calculations are more sensitive to the matrix elements, and their adequacy in describing such nuclei will readily show up.

There are several reasons for doing such calculations. First if the results were to disagree violently with experiments, then the whole Hartree-Fock approach will be in trouble. But as stated above, the results agree reasonably well with experiments[4,5] and the whole approach seems valid. The calculations have enormous advantage over phenomenological shell-model calculations as the origin of the effective interaction does not remain obscure. It may be even desirable to make such calculations with different realistic potentials because it has been recently explicitly shown by Rustgi, Barman Roy, and Raj[15] that the transition rates from excited to ground state can be extremely sensitive to even small variations in the values of the matrix elements

A major drawback of our calculation, as also of all other similar ones, is that the single-particle energies are not determined in a self-consistent manner. We take these energies from the Ni^{57} spectrum.

The shell-model and quasiparticle results on Ni isotopes employing these unpublished matrix elements of Kuo and Brown[16] for $T=1$ have been already reported in Refs. 4 and 5. These matrix elements are different from those published by Kuo and used by Lawson, MacFarlane, and Kuo[17] in the following sense. In the notation of Kuo and Brown, the bare matrix element is represented by the relation

$$G = G_s + V_L + V_L(Q/e)\,V_L + 2G_s(Q/e)\,V_L\,. \qquad (1)$$

The matrix elements of Ref. 17 do not include the contribution due to G_s while the present matrix elements do. This leads to appreciable differences in the values of some of the matrix elements specially for $J=0$ states which in turn leads to significant improvement in the agreement with the observed level spectra for Ni isotopes.

In this paper we undertake the study of Cu^{60} and Zn^{60} nuclei and develop the required formalism in isospin space for calculating the nuclear spectra for four nucleons outside a closed core. As in the calculations for nickel isotopes, a Ni^{56} core is assumed and the four nucleons are distributed in all possible ways in the available orbits $1p_{3/2}$, $0f_{5/2}$, and $1p_{1/2}$. For the residual interaction, Kuo and Brown's[16] renormalized two-body matrix elements are used. The resulting spectrum obtained by diagonalizing the shell-model Hamiltonian is compared with the experimental spectrum for both the nuclei.

2. FOUR-PARTICLE BASIS STATES

For four nucleons, five different types of configurations arise on populating the particles in all possible ways between the various available orbitals:

(a) $|j_1^4 \nu J T\rangle$,

(b) $|j_1^3 \nu' J' T', j_2;\ J T\rangle$,

(c) $|j_1^2 J_1 T_1, j_2^2 J_2 T_2;\ J T\rangle$,

(d) $|j_1^2 J_1 T_1, j_2 j_3 J_{23} T_{23};\ JT\rangle$,

(e) $|j_1 j_2 J_{12} T_{12}, j_3 j_4 J_{34} T_{34};\ JT\rangle$, (2)

where ν and ν' specify the seniorities for the four and three particles; J and T, the total angular momentum and isospin, respectively, for four particles; J' and T', the total angular momenta and isospins, respectively, for three particles; and the subscripted J's and T's are intermediate angular momenta and isospins. These configurations will be referred to as types (a), (b), (c), (d), and (e) henceforth. Again as protons and neutrons behave as identical particles in charge space, the wave functions (a), (b), (c), (d), and (e) of (2) must be antisymmetrized.

The matrix elements between the various states of (2) were worked out and are not being published in the interest of brevity.

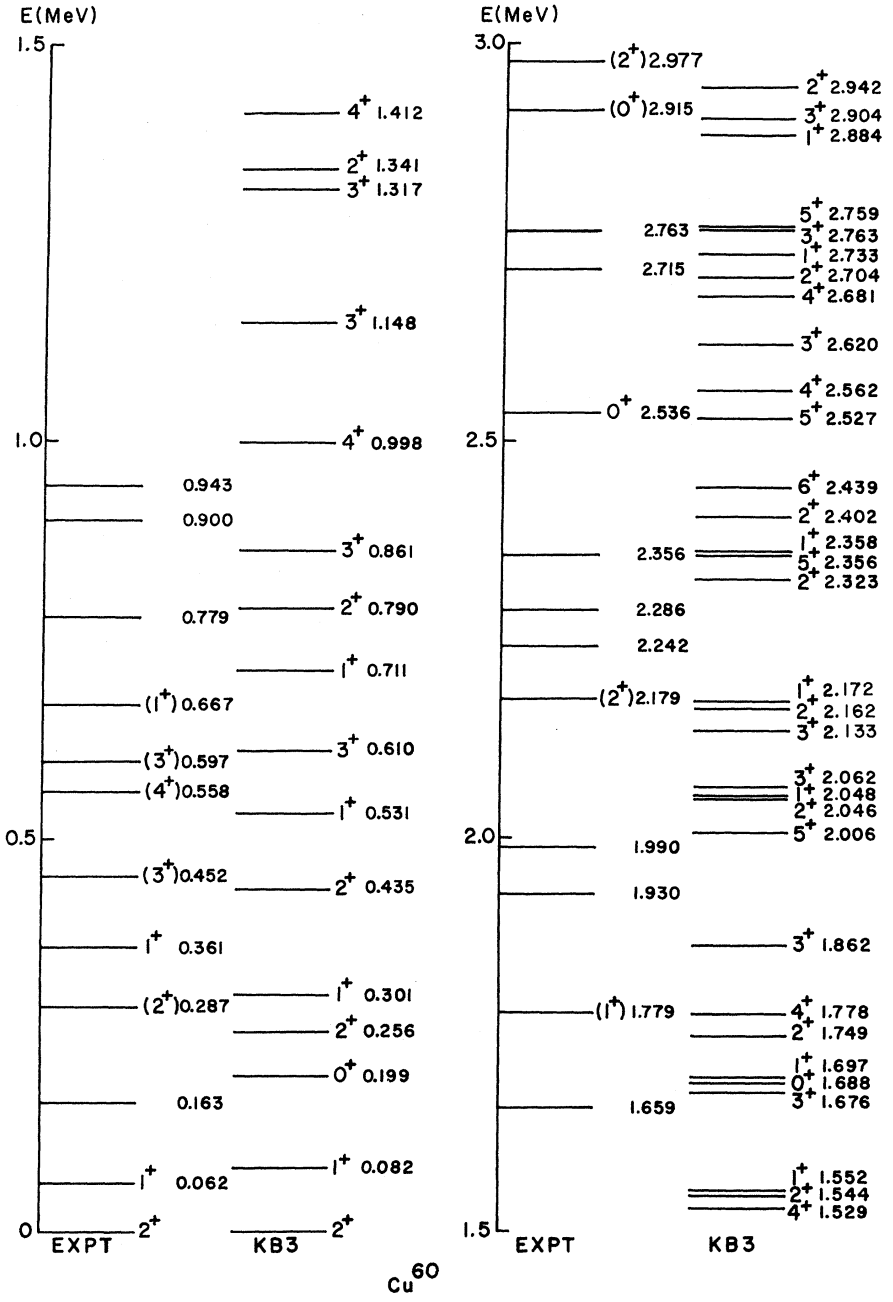

FIG. 1. Comaprison of experimental (EXPT) and shell-model spectrum for Cu^{60}. KB3 denotes results of calculations with three orbitals and Kuo and Brown reaction matrix elements (Ref. 16).

TABLE I. Lowest-energy eigenvalues and eigenfunctions for Cu^{60}. See Sec. 2 for labeling the configurations. Eigenfunctions whose magnitudes are less than 0.1 are not listed. Approximation is KB3 ($1p_{3/2}$, $0f_{5/2}$, and $1p_{1/2}$).

J^π	Eigenvalues (MeV)	Eigenfunctions	
2_0^+	−4.749	$0.5410[\frac{3}{2}^4; 1]$	$-0.1550[\frac{3}{2}^3(\frac{3}{2}\ \frac{1}{2}\ 1), \frac{1}{2}]$
		$-0.1773[\frac{3}{2}^3(\frac{5}{2}\ \frac{1}{2}\ 1), \frac{1}{2}]$	$-0.3969[\frac{3}{2}^3(\frac{3}{2}\ \frac{3}{2}\ 1), \frac{1}{2}]$
		$+0.1206[\frac{3}{2}^3(\frac{3}{2}\ \frac{3}{2}\ 1), \frac{5}{2}]$	$+0.3529[\frac{1}{2}^2(01), \frac{3}{2}^2(21)]$
		$-0.1085[\frac{3}{2}^2(01), \frac{5}{2}^2(21)]$	$-0.4287[\frac{3}{2}^2(21), \frac{5}{2}^2(01)]$
		$+0.1040[\frac{3}{2}^2(21), \frac{1}{2}\ \frac{5}{2}(20)]$	$+0.1447[\frac{3}{2}^2(30), \frac{1}{2}\ \frac{5}{2}(21)]$
1_0^+	−4.668	$0.2724[\frac{3}{2}^4; 1]$	$+0.2223[\frac{1}{2}^3(\frac{1}{2}\ \frac{1}{2}\ 1), \frac{3}{2}]$
		$-0.1877[\frac{3}{2}^3(\frac{1}{2}\ \frac{1}{2}\ 1), \frac{1}{2}]$	$+0.1670[\frac{3}{2}^3(\frac{3}{2}\ \frac{1}{2}\ 1), \frac{1}{2}]$
		$-0.2127[\frac{3}{2}^3(\frac{3}{2}\ \frac{1}{2}\ 1), \frac{5}{2}]$	$-0.1521[\frac{3}{2}^3(\frac{7}{2}\ \frac{1}{2}\ 1), \frac{5}{2}]$
		$+0.6210[\frac{3}{2}^3(\frac{3}{2}\ \frac{3}{2}\ 1), \frac{1}{2}]$	$-0.1084[\frac{5}{2}^3(\frac{1}{2}\ \frac{1}{2}\ 1), \frac{3}{2}]$
		$-0.1199[\frac{5}{2}^3(\frac{5}{2}\ \frac{3}{2}\ 1), \frac{3}{2}]$	$-0.1654[\frac{3}{2}^2(21), \frac{5}{2}^2(21)]$
		$-0.1151[\frac{3}{2}^2(30), \frac{5}{2}^2(21)]$	$-0.1050[\frac{1}{2}^2(01), \frac{3}{2}\ \frac{5}{2}(11)]$
		$-0.1279[\frac{3}{2}^2(21), \frac{1}{2}\ \frac{5}{2}(20)]$	$-0.2252[\frac{3}{2}^2(21), \frac{1}{2}\ \frac{5}{2}(21)]$
		$-0.1820[\frac{3}{2}^2(30), \frac{1}{2}\ \frac{5}{2}(21)]$	$-0.2347[\frac{5}{2}^2(01), \frac{1}{2}\ \frac{3}{2}(10)]$
		$+0.2535[\frac{5}{2}^2(01), \frac{1}{2}\ \frac{3}{2}(11)]$	$-0.1026[\frac{5}{2}^2(10), \frac{1}{2}\ \frac{3}{2}(21)]$
0_0^+	−4.550	$-0.2509[\frac{3}{2}^3(\frac{1}{2}\ \frac{1}{2}\ 1), \frac{1}{2}]$	$-0.1483[\frac{5}{2}^3(\frac{3}{2}\ \frac{1}{2}\ 1), \frac{3}{2}]$
		$+0.4636[\frac{1}{2}^2(01), \frac{3}{2}^2(01)]$	$-0.6038[\frac{3}{2}^2(01), \frac{5}{2}^2(01)]$
		$-0.2700[\frac{3}{2}^2(21), \frac{5}{2}^2(21)]$	$-0.2440[\frac{1}{2}^2(10), \frac{3}{2}\ \frac{5}{2}(11)]$
		$-0.3810[\frac{3}{2}^2(21), \frac{1}{2}\ \frac{5}{2}(21)]$	$+0.1112[\frac{5}{2}^2(21), \frac{1}{2}\ \frac{3}{2}(20)]$
		$-0.1800[\frac{5}{2}^2(21), \frac{1}{2}\ \frac{3}{2}(21)]$	
2_1^+	−4.494	$0.2810[\frac{3}{2}^4, 1]$	$-0.1495[\frac{1}{2}^3(\frac{1}{2}\ \frac{1}{2}\ 1), \frac{3}{2}]$
		$-0.1817[\frac{3}{2}^3(\frac{3}{2}\ \frac{1}{2}\ 1), \frac{5}{2}]$	$-0.1267[\frac{3}{2}^3(\frac{5}{2}\ \frac{1}{2}\ 1), \frac{5}{2}]$
		$+0.1000[\frac{3}{2}^3(\frac{7}{2}\ \frac{1}{2}\ 1), \frac{5}{2}]$	$+0.4510[\frac{3}{2}^3(\frac{3}{2}\ \frac{3}{2}\ 1), \frac{1}{2}]$
		$-0.3779[\frac{3}{2}^3(\frac{3}{2}\ \frac{3}{2}\ 1), \frac{5}{2}]$	$+0.1893[\frac{5}{2}^3(\frac{5}{2}\ \frac{1}{2}\ 1), \frac{3}{2}]$
		$+0.2102[\frac{1}{2}^2(01), \frac{3}{2}^2(21)]$	$+0.1509[\frac{1}{2}^2(10), \frac{3}{2}^2(21)]$
		$-0.3160[\frac{3}{2}^2(21), \frac{5}{2}^2(01)]$	$-0.1000[\frac{3}{2}^2(21), \frac{5}{2}^2(20)]$
		$-0.1665[\frac{3}{2}^2(21), \frac{5}{2}^2(21)]$	$-0.1416[\frac{1}{2}^2(01), \frac{3}{2}\ \frac{5}{2}(21)]$
		$-0.1120[\frac{3}{2}^2(01), \frac{1}{2}\ \frac{5}{2}(21)]$	$-0.1864[\frac{3}{2}^2(21), \frac{1}{2}\ \frac{5}{2}(20)]$
		$-0.1946[\frac{3}{2}^2(21), \frac{1}{2}\ \frac{5}{2}(21)]$	$+0.1156[\frac{5}{2}^2(01), \frac{1}{2}\ \frac{3}{2}(20)]$
		$-0.2096[\frac{5}{2}^2(01), \frac{1}{2}\ \frac{3}{2}(21)]$	
1_1^+	−4.448	$-0.2694[\frac{3}{2}^4, 1]$	$+0.3415[\frac{3}{2}^3(\frac{1}{2}\ \frac{1}{2}\ 1), \frac{1}{2}]$
		$-0.2301[\frac{3}{2}^3(\frac{3}{2}\ \frac{1}{2}\ 1), \frac{1}{2}]$	$-0.4711[\frac{3}{2}^3(\frac{3}{2}\ \frac{1}{2}\ 1), \frac{5}{2}]$
		$+0.1120[\frac{3}{2}^3(\frac{3}{2}\ \frac{3}{2}\ 1), \frac{5}{2}]$	$-0.1505[\frac{5}{2}^3(\frac{5}{2}\ \frac{1}{2}\ 1), \frac{3}{2}]$
		$-0.1189[\frac{5}{2}^3(\frac{5}{2}\ \frac{3}{2}\ 1), \frac{3}{2}]$	$-0.1853[\frac{1}{2}^2(01), \frac{3}{2}^2(10)]$
		$-0.3124[\frac{1}{2}^2(10), \frac{3}{2}^2(01)]$	$-0.1971[\frac{3}{2}^2(01), \frac{5}{2}^2(10)]$
		$-0.2544[\frac{3}{2}^2(10), \frac{5}{2}^2(01)]$	$-0.1959[\frac{1}{2}^2(01), \frac{3}{2}\ \frac{5}{2}(11)]$
		$+0.1427[\frac{1}{2}^2(10), \frac{3}{2}\ \frac{5}{2}(11)]$	$+0.1879[\frac{3}{2}^2(10), \frac{1}{2}\ \frac{5}{2}(21)]$
		$-0.1832[\frac{3}{2}^2(21), \frac{1}{2}\ \frac{5}{2}(30)]$	$-0.1205[\frac{5}{2}^2(01), \frac{1}{2}\ \frac{3}{2}(10)]$
		$+0.1044[\frac{5}{2}^2(21), \frac{1}{2}\ \frac{3}{2}(20)]$	$+0.1024[\frac{5}{2}^2(30), \frac{1}{2}\ \frac{3}{2}(21)]$

TABLE I *(Continued)*

J^π	Eigenvalues (MeV)	Eigenfunctions	
3_0^+	−4.140	$-0.1665[\frac{3}{2}^3(\frac{4}{2}\frac{1}{2}1),\frac{5}{2}]$	$+0.2550[\frac{3}{2}^3(\frac{3}{2}\frac{1}{2}1),\frac{5}{2}]$
		$+0.1827[\frac{3}{2}^3(\frac{5}{2}\frac{1}{2}1),\frac{1}{2}]$	$+0.1706[\frac{3}{2}^3(\frac{5}{2}\frac{1}{2}1),\frac{5}{2}]$
		$-0.1972[\frac{3}{2}^3(\frac{7}{2}\frac{1}{2}1),\frac{1}{2}]$	$-0.1272[\frac{3}{2}^3(\frac{7}{2}\frac{1}{2}1),\frac{5}{2}]$
		$+0.6266[\frac{3}{2}^3(\frac{3}{2}\frac{3}{2}1),\frac{5}{2}]$	$+0.3592[\frac{5}{2}^3(\frac{5}{2}\frac{1}{2}1),\frac{3}{2}]$
		$+0.2410[\frac{1}{2}^2(10),\frac{5}{2}^2(21)]$	$+0.1928[\frac{3}{2}^2(21),\frac{5}{2}^2(10)]$
		$-0.1185[\frac{3}{2}^2(21),\frac{5}{2}^2(41)]$	$+0.1020[\frac{1}{2}^2(01),\frac{3}{2}\frac{5}{2}(30)]$
		$+0.2001[\frac{1}{2}^2(01),\frac{3}{2}\frac{5}{2}(31)]$	$-0.1000[\frac{3}{2}^2(30),\frac{1}{2}\frac{5}{2}(21)]$
		$+0.1231[\frac{5}{2}^2(21),\frac{1}{2}\frac{3}{2}(21)]$	

3. SHELL-MODEL CALCULATIONS FOR Cu^{60}

The success of the previous calculations by Barman Roy, Raj, and Rustgi[4] and by Singh, Raj, Rustgi, and Kung[5] in describing the even and odd Ni isotopes with the Kuo and Brown effective interaction matrix elements has prompted us to undertake this extensive calculation for Cu^{60}. The shell-model Hamiltonian matrix between the various allowed configurations is calculated for each J, and is diagonalized to find the energy levels and wave functions. The reliability of the wave functions can be tested by calculating the spectroscopic factors for the transfer reaction $Ni^{58}(He^3, p)Cu^{60}$. Only $1p_{3/2}$, $0f_{5/2}$, and $1p_{1/2}$ orbitals are considered active.

Both the $T=0$ and $T=1$ two-body matrix elements are needed for the present calculation. They are the sum of three components: the bare interaction uncorrected for polarization of the Ni^{56} core, corrections due to the excitation of a single-core nucleon to an empty level, and corrections due to the promotion of a pair of nucleons. A complete discussion of the calculations of these Hamada-Johnston matrix elements can be found in the papers of Kuo and Brown.[6] Kuo and Brown find that in the Ni region the perturbative corrections are more important for $T=1$ matrix elements than for those with $T=0$; the corrections being about 100% for $T=1$, but only 15% for $T=0$.

In order to perform the calculation the two-body reaction matrix elements of Kuo and Brown are fed into the computer as the primary input data. The matrix for the shell-model Hamiltonian is set up for configurations (2) employing the matrix elements. The dimensions of the matrices involving all the possible four-nucleon configurations with $1p_{3/2}$, $0f_{5/2}$, and $1p_{1/2}$ orbitals are, respectively, for $J^\pi=0^+$, 16×16; for 1^+, 54×54; for 2^+, 66×66; for 3^+, 69×69; for 4^+, 50×50; for 5^+, 34×34; for 6^+, 16×16; for 7^+, 7×7; and for 8^+, 1×1. The unperturbed single-particle energies for $1p_{3/2}$, $0f_{5/2}$, and $1p_{1/2}$ are the same as were taken in the cases of Ni isotopes. A computer program to generate four-nucleon configurations for $T=0$, 1, and 2 with all possible values of J^π was used for setting up orthonormal and anitsymmetric states. In order to obtain the Cu^{60} spectrum the matrix for the shell-model Hamiltonian was diagonalized for $T=1$ and the energy spectrum obtained as shown in Fig. 1. In the figure, KB3 means that $1p_{3/2}$, $0f_{5/2}$, and $1p_{1/2}$ single-particle orbitals were included in the calculation. On excluding $1p_{1/2}$ the energy spectrum shows poor agreement with experiments and levels below 0.45 MeV are not produced. Many theoretical levels have corresponding experimental ones,[18,19] but in most cases complete identification is indeed impossible. This is because at some places there are several theoretical levels corresponding to an observed one and in most cases either the spins are doubtful or not known at all. However, below 0.361 MeV the order of the levels seems to be reproduced except for the 0.163-MeV level whose spin is not known. The observed levels at 0.062 (1^+), 0.287 (2^+), and 0.361 (1^+) MeV can perhaps be identified with the calculated ones at 0.082, 0.256, and 0.301 MeV, respectively. The observed level at 0.163 MeV has not been assigned any spin; this may perhaps correspond to the theoretical 0^+ level at 0.199 MeV. Again the observed levels at 0.597 (3^+), 0.667 (1^+), 1.779 (1^+), 2.179 (2^+), and 2.977 (2^+) MeV have doubtful spin and parity assignments but may perhaps correspond to the calculated ones at 0.610 (3^+), 0.711 (1^+), 1.697 (1^+), 2.172 (1^+), and 2.942 (2^+) MeV, respectively. The region from 3 to 5 MeV has high density of levels theoretically and it is not possible to make a one-to-one correspon-

TABLE II. Lowest-energy eigenvalues and eigenfunctions for Zn^{60}. See Sec. 2 for labeling the different configurations. Eigenfunctions whose magnitudes are less than 0.1 are not listed. Approximation is KB3 ($1p_{3/2}$, $0f_{5/2}$, and $1p_{1/2}$).

J^π	Eigenvalues (MeV)	Eigenfunctions	
0_0^+	−9.148	$0.1135[\frac{1}{2}^4, 1]$	$+0.4026[\frac{3}{2}^4, 1]$
		$+0.2765[\frac{5}{2}^4, 1]$	$+0.2508[\frac{3}{2}^3(\frac{1}{2}\,\frac{1}{2}\,1), \frac{1}{2}]$
		$-0.1399[\frac{5}{2}^3(\frac{1}{2}\,\frac{1}{2}\,1), \frac{1}{2}]$	$-0.1670[\frac{5}{2}^3(\frac{3}{2}\,\frac{1}{2}\,1), \frac{3}{2}]$
		$+0.3783[\frac{1}{2}^2(01), \frac{3}{2}^2(01)]$	$+0.1079[\frac{1}{2}^2(10), \frac{3}{2}^2(10)]$
		$+0.2499[\frac{1}{2}^2(01), \frac{5}{2}^2(01)]$	$-0.1009[\frac{1}{2}^2(10), \frac{5}{2}^2(10)]$
		$+0.4606[\frac{3}{2}^2(01), \frac{5}{2}^2(01)]$	$+0.1120[\frac{3}{2}^2(10), \frac{5}{2}^2(10)]$
		$+0.1810[\frac{3}{2}^2(21), \frac{5}{2}^2(21)]$	$+0.2270[\frac{1}{2}^2(10), \frac{3}{2}\,\frac{5}{2}(10)]$
		$+0.2218[\frac{3}{2}^2(21), \frac{1}{2}\,\frac{5}{2}(21)]$	$-0.1949[\frac{5}{2}^2(21), \frac{1}{2}\,\frac{3}{2}(21)]$
2_0^+	−8.312	$-0.1862[\frac{3}{2}^4, 1]$	$+0.2194[\frac{3}{2}^4, 2]$
		$-0.1058[\frac{5}{2}^4, 1]$	$+0.1319[\frac{5}{2}^4, 3]$
		$-0.1950[\frac{1}{2}^3(\frac{1}{2}\,\frac{1}{2}\,1), \frac{3}{2}]$	$+0.1461[\frac{1}{2}^3(\frac{1}{2}\,\frac{1}{2}\,1), \frac{5}{2}]$
		$+0.1525[\frac{3}{2}^3(\frac{1}{2}\,\frac{1}{2}\,1), \frac{5}{2}]$	$+0.3790[\frac{3}{2}^3(\frac{3}{2}\,\frac{1}{2}\,1), \frac{1}{2}]$
		$+0.1578[\frac{3}{2}^3(\frac{3}{2}\,\frac{1}{2}\,1), \frac{5}{2}]$	$+0.1201[\frac{5}{2}^3(\frac{1}{2}\,\frac{1}{2}\,1), \frac{3}{2}]$
		$+0.2260[\frac{5}{2}^3(\frac{5}{2}\,\frac{1}{2}\,1), \frac{1}{2}]$	$-0.1381[\frac{5}{2}^3(\frac{5}{2}\,\frac{1}{2}\,1), \frac{3}{2}]$
		$+0.2112[\frac{1}{2}^2(01), \frac{3}{2}^2(21)]$	$-0.1045[\frac{1}{2}^2(10), \frac{3}{2}^2(10)]$
		$+0.1228[\frac{1}{2}^2(01), \frac{5}{2}^2(21)]$	$+0.2115[\frac{3}{2}^2(01), \frac{5}{2}^2(21)]$
		$+0.2391[\frac{3}{2}^2(21), \frac{5}{2}^2(01)]$	$+0.1102[\frac{1}{2}^2(01), \frac{3}{2}\,\frac{5}{2}(21)]$
		$+0.1104[\frac{1}{2}^2(10), \frac{3}{2}\,\frac{5}{2}(20)]$	$+0.3141[\frac{3}{2}^2(01), \frac{1}{2}\,\frac{5}{2}(21)]$
		$-0.1216[\frac{3}{2}^2(10), \frac{1}{2}\,\frac{5}{2}(20)]$	$+0.1603[\frac{3}{2}^2(21), \frac{1}{2}\,\frac{5}{2}(21)]$
		$-0.2851[\frac{5}{2}^2(01), \frac{1}{2}\,\frac{3}{2}(21)]$	$+0.1174[\frac{5}{2}^2(10), \frac{1}{2}\,\frac{3}{2}(20)]$
		$-0.1406[\frac{5}{2}^2(21), \frac{1}{2}\,\frac{3}{2}(21)]$	$-0.1000[\frac{5}{2}^2(41), \frac{1}{2}\,\frac{3}{2}(21)]$
4_0^+	−7.1055	$-0.1175[\frac{5}{2}^4, 1]$	$+0.1119[\frac{5}{2}^4, 2]$
		$+0.4715[\frac{3}{2}^3(\frac{3}{2}\,\frac{1}{2}\,1), \frac{5}{2}]$	$-0.2229[\frac{3}{2}^3(\frac{7}{2}\,\frac{1}{2}\,1), \frac{1}{2}]$
		$-0.1351[\frac{3}{2}^3(\frac{7}{2}\,\frac{1}{2}\,1), \frac{5}{2}]$	$-0.4171[\frac{5}{2}^3(\frac{5}{2}\,\frac{1}{2}\,1), \frac{3}{2}]$
		$-0.1434[\frac{5}{2}^3(\frac{9}{2}\,\frac{1}{2}\,1), \frac{1}{2}]$	$+0.1036[\frac{5}{2}^3(\frac{9}{2}\,\frac{1}{2}\,1), \frac{3}{2}]$
		$-0.1891[\frac{1}{2}^2(10), \frac{3}{2}^2(30)]$	$+0.1000[\frac{1}{2}^2(01), \frac{5}{2}^2(41)]$
		$-0.1220[\frac{1}{2}^2(10), \frac{5}{2}^2(50)]$	$+0.1770[\frac{3}{2}^2(01), \frac{5}{2}^2(41)]$
		$+0.1342[\frac{3}{2}^2(21), \frac{5}{2}^2(21)]$	$-0.1254[\frac{3}{2}^2(30), \frac{5}{2}^2(10)]$
		$+0.3080[\frac{1}{2}^2(01), \frac{3}{2}\,\frac{5}{2}(41)]$	$+0.1511[\frac{1}{2}^2(10), \frac{3}{2}\,\frac{5}{2}(40)]$
		$-0.1761[\frac{3}{2}^2(10), \frac{1}{2}\,\frac{5}{2}(30)]$	$+0.2162[\frac{3}{2}^2(21), \frac{1}{2}\,\frac{5}{2}(21)]$
		$-0.1411[\frac{3}{2}^2(30), \frac{1}{2}\,\frac{5}{2}(20)]$	$-0.1750[\frac{5}{2}^2(21), \frac{1}{2}\,\frac{3}{2}(21)]$
		$-0.1233[\frac{5}{2}^2(30), \frac{1}{2}\,\frac{3}{2}(10)]$	$+0.1025[\frac{5}{2}^2(30), \frac{1}{2}\,\frac{3}{2}(20)]$
		$-0.1053[\frac{5}{2}^2(41), \frac{1}{2}\,\frac{3}{2}(21)]$	$+0.1080[\frac{5}{2}^2(50), \frac{1}{2}\,\frac{3}{2}(20)]$

TABLE II *(Continued)*

J^π	Eigenvalues (MeV)	Eigenfunctions	
0_1^+	−6.1567	$-0.5007[\frac{3}{2}^4, 1]$	$+0.4815[\frac{5}{2}^4, 1]$
		$-0.1107[\frac{5}{2}^4, 2]$	$-0.4247[\frac{5}{2}^3(\frac{1}{2}\frac{1}{2}1), \frac{1}{2}]$
		$-0.2198[\frac{5}{2}^3(\frac{1}{2}\frac{1}{2}1), \frac{1}{2}]$	$-0.2169[\frac{5}{2}^3(\frac{3}{2}\frac{1}{2}1), \frac{3}{2}]$
		$-0.2689[\frac{1}{2}^2(01), \frac{3}{2}^2(01)]$	$-0.1068[\frac{1}{2}^2(10), \frac{3}{2}^2(10)]$
		$+0.2379[\frac{1}{2}^2(01), \frac{5}{2}^2(01)]$	$-0.1321[\frac{1}{2}^2(10), \frac{5}{2}^2(10)]$
		$+0.1670[\frac{3}{2}^2(01), \frac{5}{2}^2(01)]$	$-0.1463[\frac{5}{2}^2(21), \frac{1}{2}\frac{3}{2}(21)]$

dence with the observed levels.

The wave functions obtained in the calculation are listed in Table I and would be useful in predicting the other properties of Cu^{60} and strengths for the reaction $Ni^{58}(He^3, p)Cu^{60}$.

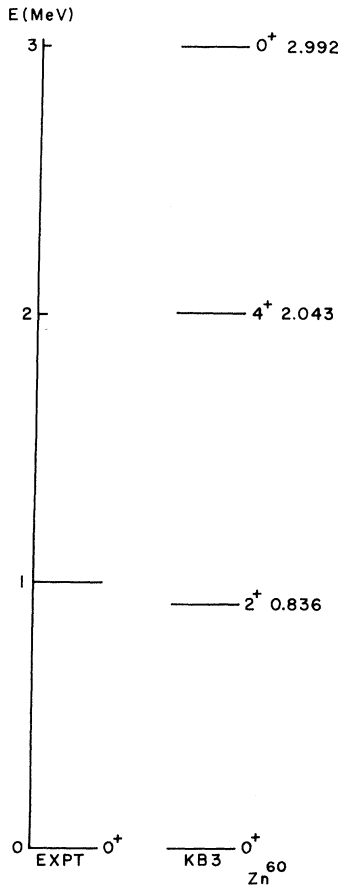

FIG. 2. Comparison of experimental (EXPT) and shell-model spectrum for Zn^{60}. KB3 denotes results of calculations with three orbitals and Kuo and Brown reaction matrix elements (Ref. 16).

4. SHELL-MODEL CALCULATIONS FOR Zn^{60}

As in the calculations for Cu^{60}, the shell-model Hamiltonian matrix between different allowed four-nucleon states for $T=0$ is set up. The single-particle orbitals included are $1p_{3/2}$, $0f_{5/2}$, and $1p_{1/2}$. The dimensions of various matrices are 21×21 for $J^\pi=0^+$, 31×31 for 1^+, 56×56 for 2^+, 45×45 for 3^+, 44×44 for 4^+, 24×24 for 5^+, 17×17 for 6^+, 5×5 for 7^+, and 3×3 for 8^+. The matrices are diagonalized and the energy spectrum obtained is shown in Fig. 2. The ground-state spin is found to be 0^+, in agreement with the observation. A level has been observed[20] at an excitation energy of 1.02 MeV, but its spin remains undetermined. Perhaps its spin is 2^+ and it coresponds to the calculated level at 0.936 MeV. The wave functions for the levels are listed in Table II.

It should also be pointed out that the detailed shell-model calculations reported in this paper will help us in testing both $T=0$ and $T=1$ matrix elements of Kuo and Brown.

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Analysis of Proton Scattering and Reactions with the Generalized Optical Model

J. E. Spencer,* E. R. Cosman, H. A. Enge, and A. K. Kerman
Physics Department and Laboratory for Nuclear Science,† Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 19 August 1970)

Isobaric analog resonances in proton scattering experiments are analyzed using a set of coupled optical equations which include both elastic and inelastic channels as well as other channels which can couple to these via the charge-exchange operator $(\vec{\tau}\cdot\vec{T})$. The method has been applied to the particular case of $Sr^{88}+p$, in which only the $p_0(0_1^+)$ and $p_1(2_1^+)$ proton channels are dominant and mixing of the $(j\times 0_1^+)$ and $(j'\times 2_1^+)$ configurations describes the primary fragmentation of the $2d_{5/2}$, $3s_{1/2}$, $2d_{3/2}$, and $1g_{7/2}$ neutron states in the parent nucleus Sr^{89}. Our purpose in this investigation was to see whether or not one could obtain a consistent match to both the level structure of the bound parent analog system and the analog resonance data in the various open channels with a general expression for the effective nucleon-nucleus interaction. Restricting as many parameters and potential terms as possible using independent sources of data, we obtained a detailed fit to the level structure of Sr^{89} and the resonant $Sr^{88}(p,p_0)$ and (p,p_1') excitation functions, the resonant $Sr^{88}(p,n)$ total cross-section data, and the elastic polarization functions for an incident proton energy from $E_p=4.8$ to 8.2 MeV. The predicted on-resonance $p'(2_1^+)$ angular distributions follow the main features of the data but not the finer details. It was found that the interference between the resonance and the direct inelastic background depends mostly on the nuclear rather than the Coulomb core excitation, and that direct inelastic charge-exchange does not affect the results. Our ability to fit all of the data in a consistent manner is interpreted as a confirmation of the main features of the generalized potential model assumed.

I. INTRODUCTION

Since the discovery of isobaric analog states in intermediate-mass nuclei,[1] a number of experiments using different types of reactions have been performed in order to understand how these states manifest themselves. They have been excited in direct (p,n) and (He^3,t) reactions[1,2] as states in the residual nucleus and also as intermediate states in particle-transfer reactions to particle unstable states. The intermediate structure observed in proton elastic scattering on intermediate-mass nuclei[3] was interpreted as resulting from the excitation of such states in the compound system.[3] Many subsequent studies have shown that these processes occur systematically through most of the Periodic Table, so that it is important to establish their various reaction mechanisms and attempt to