Phys. A97, 458 (1967); E. F. Gibson, B. W. Ridley, J. J. Kraushaar, M. E. Rickey, and R. H. Bassel, Phys. Rev. 155, 1194 (1967).

[12]B. M. Preedom, E. Newman, and J. C. Hiebert, Phys. Rev. 166, 1158 (1968); M. R. Cates, Ph. D. thesis, Texas A & M University, 1969 (unpublished).

[13]C. M. Perey and F. G. Perey, Phys. Rev. 152, 923 (1966).

[14]F. G. Perey, in *Proceedings of the Rutherford Jubilee International Conference, Manchester, England, 1961,* edited by J. B. Birks (Heywood and Company, Ltd., London, England, 1962), p. 125; P. J. A. Buttle and L. J. B. Goldfarb, Proc. Phys. Soc. (London) 83, 701 (1964); G. Bencze and J. Zimanyi, Phys. Letters 9, 246 (1964); F. G. Perey and D. Saxon, Phys. Letters 10, 107 (1964).

[15]M. H. MacFarlane and J. B. French, Rev. Mod. Phys. 32, 567 (1960); J. B. French and M. H. MacFarlane, Nucl. Phys. 26, 168 (1961).

[16]J. Morton, private communication; Rutherford Laboratory Progress Report, 1968 (unpublished).

[17]D. M. Van Patter, Bull. Am. Phys. Soc. 15, 573 (1970).

[18]S. Cohen, R. D. Lawson, M. H. MacFarlane, S. P. Pandya, and M. Soga, Phys. Rev. 160, 903 (1967), Table IX.

PHYSICAL REVIEW C VOLUME 3, NUMBER 3 MARCH 1971

Energy-Dependent Beta-Gamma Circular Polarization and Nuclear Matrix Elements of Rb^{86}†

J. J. Bosken, D. E. Ohlms,* and P. C. Simms
Department of Physics, Purdue University, Lafayette, Indiana 47907
(Received 20 July 1970)

The energy-dependent β-γ circular polarization was measured in order to set better limits on the matrix-element ratio $\Lambda = \int\vec{\alpha}/(\int i\vec{r}/\rho)$ for the 696-keV first-forbidden β transition of Rb^{86}. The results show that Λ is consistent with the Fujita-Eichler relation, so the contributions of the off-diagonal matrix elements of the Coulomb Hamiltonian to Λ are small for this transition. The nuclear matrix elements are in agreement with previous results, but the limits of error have been reduced.

I. INTRODUCTION

The determination of nuclear matrix elements for first-forbidden β transitions provides a sensitive test for nuclear models. Since there can be a relatively large number of matrix elements which contribute to the β transition, more can be learned about the details of nuclear structure from first-forbidden transitions than from allowed transitions. Even though the large number of quantities which can be measured makes the investigation interesting, the fact that there are a large number of unknowns also makes it difficult to extract the matrix elements from the experimental data.

A valuable aid to simplify the extraction of the matrix elements has been proposed by Fujita[1] and Eichler.[2] The conserved-vector-current theory of β decay can be used to predict the ratio Λ of two of the vector matrix elements.

$$\Lambda \equiv \int \vec{\alpha} \Big/ \left(\int i\vec{r}/\rho\right).$$

If Λ can be used to remove one unknown from the problem, it is much easier to determine the matrix elements. The Fujita-Eichler method for predicting Λ is attractive because it does not depend on any details of nuclear structure. The assumption is made that the off-diagonal matrix elements of the Coulomb Hamiltonian H_C are so small that they can be neglected. The validity of this assumption was questioned by Damgaard and Winther.[3] They used nuclear-model calculations for Tl^{207} and Pb^{209} to propose that even if the off-diagonal matrix elements of H_C are small, it is possible that the vector matrix-element ratio Λ will depend on details of nuclear structure.

The Fujita-Eichler expression for Λ is

$$\Lambda \equiv \frac{\int \vec{\alpha}}{\int i\vec{r}/\rho} = \pm 2.4\frac{\alpha Z}{2} + (W_0 \mp 2.5)\rho \quad \text{for } \beta^{\mp} \text{ decay}. \tag{1}$$

Z is the charge of the daughter, $\alpha = 1/137$, and natural units ($m_e = \hbar = c = 1$) are used for the nuclear radius ρ and the end-point energy W_0. When the off-diagonal matrix elements of H_C are included the following correction term[4] must be added to Λ:

$$\sum_{f' \neq f} \langle f | H_C | f' \rangle \langle f' | i\vec{r} | i \rangle - \sum_{i' \neq i} \langle f | i\vec{r} | i' \rangle \langle i' | H_C | i \rangle. \tag{2}$$

The final state $|f\rangle$ and the initial state $|i\rangle$ of the

β transition are members of the complete set of states $|f'\rangle$ and $|i'\rangle$, respectively. Damgaard and Winther proposed that the correction term be evaluated by using a realistic form for the Coulomb Hamiltonian. A more general result for Λ is then obtained.[4]

$$\Lambda = \pm\frac{\alpha Z}{2}(3-\lambda) + (W_0 \mp 2.5)\rho \quad \text{for } \beta^{\mp} \text{ decay}. \tag{3}$$

The parameter λ is the ratio of a higher-order to a first-order matrix element:

$$\lambda = \frac{\int \vec{r}(r/\rho)^2}{\int \vec{r}}.$$

When $\lambda = 0.6$ the Fujita-Eichler relation and the general formula give the same result for Λ.

It is evident from Eq. (3) that the theoretical value for Λ depends on the value for λ. The experimental limits which are set on Λ also depend upon the value of λ, because the higher-order matrix element $\int \vec{r}(r/\rho)^2$ contributes to the β transition.

We have shown[4,5] in a previous experiment that the Fujita-Eichler relationship for Λ is not valid for the 2.2-MeV β transition of La^{140} ($\lambda = 2.45$). Nevertheless, it is still interesting to measure Λ for other nuclei for two reasons. First, a measurement of Λ shows whether or not the off-diagonal matrix elements of the Coulomb Hamiltonian H_C are important for the transition. This is useful additional information for nuclear-structure studies, and it can also be used to investigate isobaric analog states.[4] Second, if it can be shown that the off-diagonal elements of H_C are not important for a nucleus, then the Fujita-Eichler relation can be useful in the analysis of similar nuclei.

Rb^{86} is an interesting nucleus to study because earlier experimental work[6] has shown that a measurement of the energy dependence of the β-γ circular polarization would greatly improve the experimental limits which could be set on Λ for Rb^{86}.

II. EXPERIMENTAL METHOD

The instrument that was used to measure the β-γ circular polarization P_γ has been described previously.[5] The electronic system has the following major features: it can measure the energy of β particles with reasonable accuracy at counting rates of 5×10^5 counts/sec, the detectors are stabilized by using feedback systems to control the high voltage for the photomultipliers, and strong sources can be used because true and accidental coincidences are recorded simultaneously. The circular polarization P_γ of the γ rays was measured by forward Compton scattering from magnitized iron. The stabilizing system was very effective. There was no appreciable change in the gain of the β detector when the β counting rate was changed from 10^4 to 5×10^5 counts/sec. The difference in β and γ single counting rates as a function of the magnetic field was typically 0.02%. The average deviation of the single counting rates was typically 0.1%.

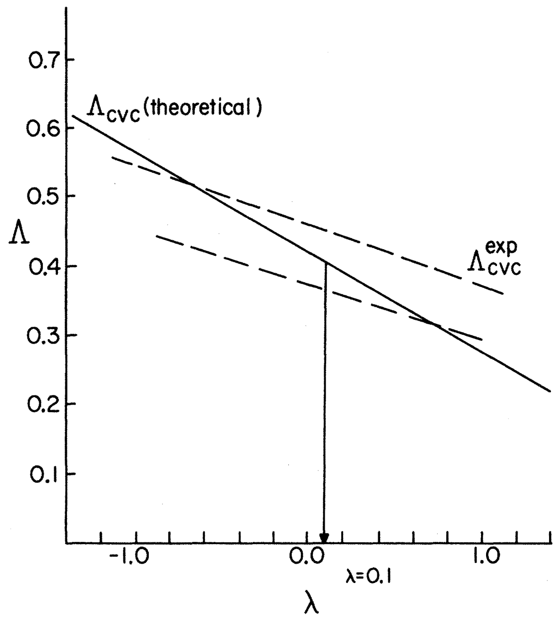

FIG. 1. Variation of the theoretical value and experimental limits of the matrix-element ratio $\Lambda = \int \vec{\alpha}/(\int i\vec{r}/\rho)$ as a function of the parameter λ.

The efficiency of the circular-polarization analyzing magnet was calculated with a computer program. The accuracy of the program was checked by measuring P_γ for Co^{60}. The calculated efficiency agreed to within 5% with the experimental value for Co^{60}. Since the degree of polarization of the electrons in the iron was only known to 5%, the agreement was considered to be good.

The sources were prepared by vacuum evaporating the Rb onto thin (0.6-mg/cm^2) aluminum leaf mounts. This procedure is not difficult for Rb, since evaporation occurs from RbCl at a temperature between 800 and 1000°C. The procedure has the advantage that it produces a source with uniform surface density (typically 1 mg/cm^2 in this experiment).

The β-γ angular correlation function for a first-forbidden β transition can be expressed in the following manner:

$$N(W,\theta,S) = A_0(W) + SA_1(W)P_1(\theta) + A_2(W)P_2(\theta) + SA_3(W)P_3(\theta), \tag{4}$$

where W is the total energy of the β particle, θ is the angle between the direction of emission of the β and γ rays, and the helicity factor S is +1 and −1

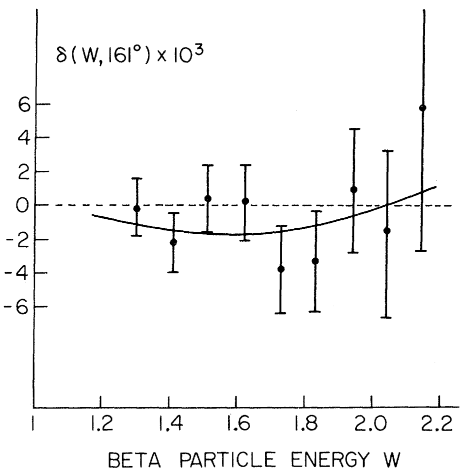

FIG. 2. Observed circular-polarization effect δ as a function of β-particle energy (in natural units). The solid line shows the theoretical values for δ which were calculated with a typical set of acceptable matrix elements.

for right-hand and left-hand circular polarization, respectively. The coefficients A_l are functions of the nuclear matrix elements, and P_l are Legendre polynomials. The β-γ circular polarization P_γ is defined as

$$P_\gamma = \frac{N(+1)-N(-1)}{N(+1)+N(-1)} = \frac{A_1P_1+A_3P_3}{A_0+A_2P_2}. \qquad (5)$$

The experimental difference in coincidence counting rate δ can be related to P_γ by the efficiency ϵ of the analyzing magnet,[7]

$$\delta(W) = 2\frac{C_+(W)-C_-(W)}{C_+(W)+C_-(W)}$$
$$= \frac{\epsilon_1 A_1(W)+\epsilon_3 A_3(W)}{A_0(W)+\epsilon_2 A_2(W)}. \qquad (6)$$

$C_\pm(W)$ is the β-γ true coincidence counting rate for the two directions of magnetic field in the analyzing magnet.

The parameters ϵ_j are obtained by a numerical integration over the geometry of the experiment. Solid-angle corrections, average values for P_l, and the circular-polarization efficiency are all combined in ϵ_j. The following values were calculated for the efficiency factors:

$$\epsilon_1 = -0.0512, \quad \epsilon_2 = 0.770, \quad \epsilon_3 = -0.0328.$$

In order to determine P_γ from δ, the relative size of A_1 to A_3 and A_0 to A_2 must be known. Even though these parameters are relatively well known from measurements of the angular dependence of P_γ[8,9] and the β-γ directional correlation,[9] the most accurate procedure in extracting the matrix elements is to compare the theoretical calculations directly with δ using Eq. (6).

The average angle between the directions of emission of the β and γ rays was 161°. At this angle, the factors $A_1P_1(\theta_{\beta\gamma})$ and $A_3P_3(\theta_{\beta\gamma})$ are approximately equal in magnitude and have opposite signs so that P_γ is very sensitive to the nuclear matrix elements.[6] Small changes in the matrix elements cause the sign of the circular polarization to change.

The computer program which was used to set limits on the matrix elements and the vector matrix-element ratio has been described previously.[4,6] The program uses exact electron wave functions, corrects for screening by atomic electrons, and considers the effect of higher-order matrix elements. Previous experimental results for the energy dependence of the β-γ directional correlation,[9] the shape correction factor,[10] the angular dependence of P_γ,[8,9] and the present measurement of the energy dependence of P_γ were used to determine the matrix elements.

III. RESULTS

Figure 1 shows plots of Λ(theoretical) and limits on Λ(experimental) as a function of λ. The theoretical value of Λ lies within the experimental limits as long as λ is within the following range:

$$-0.6 \le \lambda \le 0.8.$$

The experimental values of the circular-polarization effect δ are plotted in Fig. 2 and tabulated in Table I. In the calibration measurement, the average value for δ divided by v/c for Co^{60} was +0.0170 ± 0.0006. The solid curve in Fig. 2 shows the theoretical values of δ which were calculated with a typical set of acceptable matrix elements. It is clear from Eqs. (5) and (6) that additional uncertainty is introduced when P_γ is obtained from δ because the relative size of A_1 and A_3 are not known accurately. However, a good indication of the size and energy dependence of P_γ can be obtained from Fig. 3 where the theoretical value of $P_\gamma(\theta_{\beta\gamma} = 161°)$ is plotted for the same set of matrix elements used in Fig. 2. The matrix-element param-

TABLE I. Value of the circular-polarization-effect parameter δ as a function of the total energy W (in natural units) of the β particle at $\langle\theta_{\beta\gamma}\rangle = 161°$.

W	$10^3 \times \delta$
1.30	−0.2 ± 1.8
1.41	−2.2 ± 1.9
1.51	+0.4 ± 2.1
1.62	+0.2 ± 2.3
1.73	−3.8 ± 2.6
1.83	−3.3 ± 3.0
1.94	+0.9 ± 3.7
2.04	−1.7 ± 5.0
2.14	+5.8 ± 7.7

eters obtained with the new data and analysis are given in Table II. The limits of error are valid for all acceptable values of λ. The matrix-element parameters which were obtained from the previous analysis[6] are also given for comparison. The matrix elements which are associated with the parameters are also given in Table II. For x and u the matrix element is equal to the matrix-element parameter. For the other parameters the relationship to the matrix element is not exact because of higher-order matrix elements. The complete expressions for the matrix-element parameters are given in Ref. 4. The matrix elements are normalized such that their maximum physical size is $\sqrt{2}$.

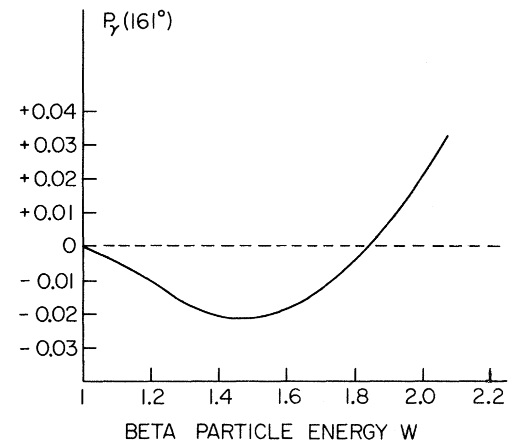

FIG. 3. Theoretical value of P_γ as a function of β-particle energy (in natural units) for an angle of 161° between the β and γ rays. The calculation used the same set of matrix elements which were used to calculate the theoretical values of δ shown in Fig. 2.

IV. DISCUSSION

The contributions of the off-diagonal matrix elements of the Coulomb Hamiltonian H_C cannot be as large in Rb^{86} as they are[4] in La^{140} ($\lambda = 2.45$). Furthermore, the results for the vector matrix-element ratio Λ are consistent with a zero contribution ($\lambda = 0.6$) – that is, the Fujita-Eichler relation may be valid for this transition. A calculation was performed to see how the value of λ would be affected if the present experimental result for δ were used and the limit of errors were reduced by a factor of 3. In that case, λ would be 0.5 ± 0.3. Therefore, it is likely that the contribution of the off-diagonal elements of H_C to Λ are small for this transition.

It is evident from Eq. (2) that the correction term in the expression for Λ will be small if the off-diagonal matrix elements of H_C ($\langle f|H_C|f'\rangle$, $\langle i'|H_C|i\rangle$) or the additional radial matrix elements ($\langle f'|i\vec{r}|i\rangle$, $\langle f|i\vec{r}|i'\rangle$) are small. The existence of relatively pure isobaric analog states is one of the strong indications that $\langle f|H_C|f'\rangle$ and $\langle i'|H_C|i\rangle$ are small. The explanation that has been offered[4] for the large correction in La^{140} is that the additional matrix elements $\langle f'|i\vec{r}|i\rangle$ and $\langle f|i\vec{r}|i'\rangle$ are large compared to the transition matrix element $\langle f|i\vec{r}|i\rangle$ which was shown to be reduced by an order of magnitude from its maximum physical size. However, that same possibility exists for Rb^{86}, because $\langle f|i\vec{r}/\rho|i\rangle$ (called $\int i\vec{r}/\rho$ in Table II) is also reduced by an order of magnitude from its maximum physical size ($\sqrt{2}$). Therefore there is no obvious reason why the correction term is large for La^{140} ($\lambda = 2.45$) and apparently small ($\lambda \approx 0.6$) for Rb^{86}.

Until a clear reason can be given for the fact that the simple Fujita-Eichler relation is a good approximation for Rb^{86}, it will not be possible to use the relation with confidence to simplify ma-

TABLE II. Results for matrix-element parameters $D = 0.149$, $d = 0.205$, $a = -0.032$, and $\rho = 0.0135$.

Matrix element	Matrix-element parameters	Result of present work	Previous results (Ref. 6)
$C_A\int\gamma_5 + DC_A\int i\vec{\sigma}\cdot\vec{r}/\rho$	$\approx\eta DV$	-0.0057 ± 0.0004	-0.006 ± 0.0015
$-C_A\int i\vec{\sigma}\cdot\vec{r}/\rho$	$\approx\eta w_0$	0 ± 1.0	0 ± 1.0
$C_V\int\vec{\alpha} - D\left(C_V\int i\vec{r}/\rho + C_A\int\vec{\sigma}\times\vec{r}/\rho\right)$	$\approx\eta DY$	-0.0063 ± 0.0007	-0.001 ± 0.001
$C_V\int\vec{\alpha}$	$\approx\eta D' y_0$	-0.054 ± 0.01	-0.05 ± 0.015
$C_V\int i\vec{r}/\rho$	$=\eta x$	-0.14 ± 0.03	-0.13 ± 0.05
$C_A\int\vec{\sigma}\times\vec{r}/\rho$	$=\eta u$	-0.22 ± 0.048	-0.23 ± 0.05
$-C_A\int iB_{ij}/\rho$	$\approx\eta z_0$	$+0.77 \pm 0.08$	$+0.78 \pm 0.15$

trix-element extractions in other nuclei. However, the general expression [Eq. (3)] can be useful in matrix-element extractions.

Given a value of $\int \vec{\alpha}$ and λ, the values of $\int i\,\vec{r}$ and $\int i\,\vec{r}(r/\rho)^2$ are fixed by Eq. (3) and the definition of λ. Since $\int \vec{\sigma}x\vec{r}$ and $\int i\,\vec{r}$ have the same radial dependence, it also is reasonable to use the following approximation:

$$\int \vec{\sigma}x\vec{r}\left(\frac{r}{\rho}\right)^2 \approx \lambda \int \vec{\sigma}x\vec{r}\,.$$

The matrix elements $\int i\,\vec{r}(r/\rho)^2$ and $\int \vec{\sigma}x\vec{r}(r/\rho)^2$ are the most important higher-order matrix elements because they enter the expression for the transition probability with relatively large coefficients. Therefore, the general expression for Λ can be used to include important high-order terms in the matrix-element extraction without making the extraction more difficult. $\int \vec{\alpha}$ and λ can be treated as unknown rather than $\int \vec{\alpha}$ and $\int i\,\vec{r}$.

A brief analysis of the matrix elements using the shell model was presented in our earlier work on this transition.[6] A detailed study of Rb^{86} has been made by S. Wahlborn.[11] The better limits of error obtained in this work would not alter the descriptions provided in either of these earlier papers. The present results confirm our earlier observation[6] that the experimental limits of error used by Wahlborn for the matrix-element parameter w are too restrictive. The limits placed on λ provide new nuclear-structure information. The higher-order matrix element $\int i\,\vec{r}(r/\rho)^2$ must be less than or equal in magnitude to $\int i\,\vec{r}$, and the two matrix elements may have equal or opposite sign.

The authors would like to thank Dr. H. A. Smith for his assistance in the matrix-element extraction.

†Work supported by U. S. Atomic Energy Commission under Contract No. AT(11-1)1746.

*Present address: IBM Space Guidance Center, Owega, New York.

[1]J. I. Fujita, Phys. Rev. 126, 202 (1962).
[2]J. Eichler, Z. Physik 171, 463 (1962).
[3]J. Damgaard and A. Winter, Phys. Letters 23, 345 (1966).
[4]H. A. Smith and P. C. Simms, Phys. Rev. C 1, 1809 (1970).
[5]D. Ohlms, J. Bosken, and P. C. Simms, Phys. Rev. C 1, 1804 (1970).
[6]P. C. Simms, Phys. Rev. 138, B748 (1965).
[7]R. M. Steffen and H. Fraunfelder, in *Alpha-, Beta-, and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, The Netherlands, 1966), p. 1465.
[8]F. Boehm and J. Rogers, Nucl. Phys. 45, 392 (1963).
[9]P. C. Simms, A. Namenson, T. H. Wei, and C. S. Wu, Phys. Rev. 138, B777 (1965).
[10]R. L. Robinson and L. M. Langer, Phys. Rev. 112, 481 (1958).
[11]S. Wahlborn, Nucl. Phys. 58, 209 (1964).

PHYSICAL REVIEW C VOLUME 3, NUMBER 3 MARCH 1971

Shell-Model Calculations for Cu^{60} and Zn^{60} with Reaction Matrix Elements*

R. P. Singh and M. L. Rustgi
Department of Physics, State University of New York, Buffalo, New York 14214
(Received 31 August 1970)

Employing Ni^{56} as core, and Kuo and Brown renormalized matrix elements for the Hamada-Johnston nucleon-nucleon potential as the residual interaction, the level structure of both Cu^{60} and Zn^{60} is studied within the framework of the shell model. The four active nucleons are allowed to populate the $1p_{3/2}$, $0f_{5/2}$, and $1p_{1/2}$ single-particle orbitals. A comparison with the observed level spectra is made, and good agreement with those levels whose spins and parity are definitely known is found. Other levels whose spins and parity assignments are not definite have corresponding theoretical levels which should help in determining their spins in the future measurements.

1. INTRODUCTION

Until a few years ago, nuclear structure calculations could be performed only by resorting to purely phenomenological models. This was partly due to the lack of the knowledge of the nucleon-nucleon interaction and partly due to the lack of development of theoretical techniques needed to carry out such calculations. However in recent years because of the availability of high-speed computers nuclear physics has entered its quantitative phase. Detailed shell-model calculations are feasible if