^{87}Y States Populated by Single-Proton Stripping*

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The ^{86}Sr(^{3}He,d)^{87}Y reaction was studied at a laboratory beam energy of 20 MeV to test the possible closed-subshell behavior of the 38-proton configuration. The ground-state Q value is found to be 0.346 ± 0.015 MeV. Several new states were identified below an excitation energy of 3.45 MeV. A distorted-wave analysis was used to assign l values for the stripping transitions. Spectroscopic strengths agree well with sum-rule expectations for the $2p$, $1f$, and $1g_{7/2}$ configurations. Comparison with the results of Picard and Bassani for the ^{88}Sr(^{3}He,d)^{89}Y reaction indicates qualitative agreement with shell-model expectations for $2p$ and $1g$ strengths but significant deviations for $1f$ strength and for low-lying $2d$ strength.

I. INTRODUCTION

Nuclides in the $A \approx 90$ region have been extensively studied with single-nucleon-transfer reactions, and the results are well described by the simple shell model.[1-3] In particular, the nuclei ^{88}Sr and ^{90}Zr have both been characterized[1,2] as having a closed neutron shell at $N = 50$ and closed proton subshells – the former with a $2p_{3/2}$-$1f_{5/2}$ configuration closed at $Z = 38$ and the latter with a $2p_{1/2}$ subshell closed at $Z = 40$. Although neither of these nuclei exhibits all the properties of a doubly magic nucleus, both show an approximate doubly magic character in terms of the single-particle strength seen in pickup and stripping reactions.

The presence of holes in the $N = 50$ core, however, is expected to induce mixed proton configurations and to destroy some of the simplicity seen in reactions involving ^{88}Sr or ^{90}Zr. Thus, the level structure and distribution of single-particle strength in ^{87}Y, as populated in the ^{86}Sr(^{3}He,d) reaction, is of interest in order to test the hole structure of the 38-proton configuration in the presence of two neutron holes in the $N = 50$ shell.

Previous studies[4,5] of ^{87}Y have revealed only two levels – the ground state and the 0.379-MeV state, tentatively assigned $J^{\pi} = \frac{1}{2}^-$ and $\frac{9}{2}^+$, respectively, on the basis of β-decay studies. We report a study of the ^{86}Sr(^{3}He,d)^{87}Y reaction at a ^{3}He energy comparable to that used by Picard and Bassani[2] in their study of the ^{88}Sr(^{3}He,d)^{89}Y reaction. A distorted-wave Born-approximation (DWBA) analysis consistent with theirs has also been performed. Thus a direct comparison of proton strength is expected to be meaningful even in the absence of a sophisticated shell-model calculation.

Nuclei in the $A \approx 86$ region (e.g., ^{85}Rb) have also been discussed in terms of the Nilsson model,[6] but characteristic deformations appear quite small. Since the main interest in this work is to test the shell-model properties of the 38-proton configuration by a direct comparison between stripping on ^{86}Sr and stripping on ^{88}Sr, the Nilsson model will not be considered further.

II. EXPERIMENTAL PROCEDURE

The 20-MeV ^{3}He beam from the Argonne FN tandem was used to bombard an evaporated enriched target of ^{86}Sr(NO$_3$)$_2$ (97.6% ^{86}Sr, 0.7 ^{87}Sr, 1.7% ^{88}Sr),[7] about 180 μg/cm^2 thick. Emerging deuterons were detected in emulsions placed in the focal plane of an Enge split-pole spectrograph. The developed plates were scanned with the Argonne automatic plate scanner.[8] The over-all deuteron energy spread was about 25 (full width at half maximum) keV (FWHM). This can be mainly ascribed to effects of target thickness. A Si surface-barrier monitor detector (placed at 90° in the scattering chamber) measured the elastically scattered ^{3}He and indicated no appreciable target deterioration throughout the experiment.

An accurate beam energy (19.883 MeV) was determined (for use in Q-value calculations) by measuring the position of elastically scattered ^{3}He groups in the focal plane of the spectrograph. ^{3}He elastic scattering (at an energy of 9.9 MeV, where the scattering is assumed to follow the Rutherford formula) was also used to measure the target thickness.

III. DATA

A ^{86}Sr(^{3}He,d)^{87}Y spectrum obtained at a lab angle of 20° is shown in Fig. 1. Five strong peaks are seen in the spectrum below an excitation energy of 1.2 MeV. More strong states appear at excitation energies above 2.90 MeV. Several weak states appear in the ~1.8-MeV energy gap between strong excited states. Yields to the states labeled in Fig. 1 were extracted for each spectrum by using the least-squares peak-fitting program AUTOFIT.[9] The statistical error in the counts for strong

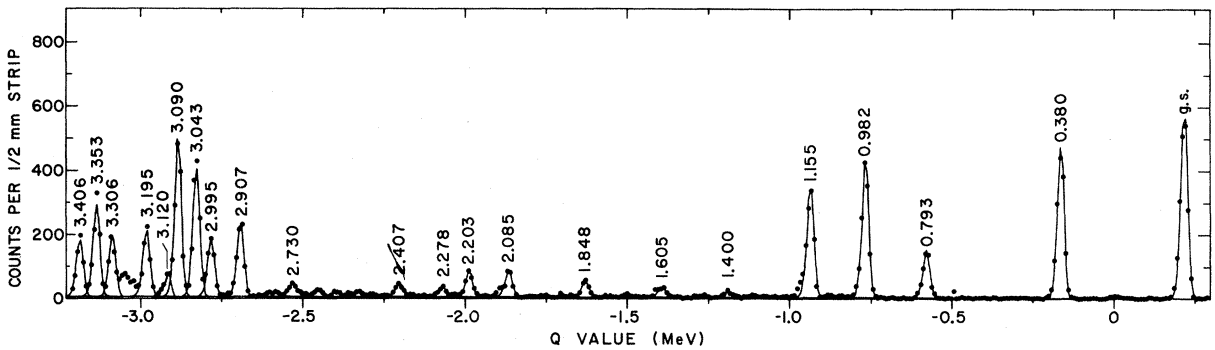

FIG. 1. ^{86}Sr(^{3}He, d)^{87}Y spectrum at $\theta_L = 20°$ and an incident ^{3}He energy of 20 MeV.

states was generally 3% or less. Absolute cross sections to an accuracy of better than 20% were derived from the target thickness measurement described above. The resultant angular distributions are shown for the low-lying states in Fig. 2, for the weak states in Fig. 3, and for the high-lying strong states in Fig. 4. The indicated errors are statistical only.

The ground-state Q value was found to be 0.346 ± 0.015 MeV. The previously reported value was 0.446 ± 0.200 MeV.

IV. DWBA ANALYSIS

Zero-range local distorted-wave calculations were made with the code JULIE[10] on an IBM 7094 computer. Calculations were made with the potential sets $H1$ and $D1$ used by Picard and Bassani[2] for the ^{88}Sr(^{3}He, d)^{89}Y reaction at 18 MeV and sets $H2$ and $D2$ used by Cates, Ball, and Newman[1] for the ^{90}Zr(^{3}He, d)^{91}Nb reaction at 25 MeV, both of which are listed in Table I. $H1$ is the deep (V = 170 MeV) ^{3}He potential used by Picard and Bassani. It was originally suggested by Bassel.[11] $H2$ is a very similar ^{3}He potential.[12] Both $D1$ and $D2$ are deuteron potentials taken from the work of Perey and Perey.[13] The two sets yielded virtually identical shapes for the angular distributions.

Computed angular distributions are compared with the data in Figs. 2–4. The fits are excellent for l = 1, 2, 3, and 4. There is little ambiguity in distinguishing l = 2 from l = 3 angular distributions. The first maximum for an l = 2 distribution is shifted 5° from that for l = 3, and the data fall into two distinct classes with no ambiguous cases. The distinction between l = 1 and l = 4 angular distributions is even more sure; their first maxima occur at 10 and 28°, respectively. A tentative l = 0 state is seen at 3.195 MeV. This assignment is based on a 3° shift in the second maximum of this angular distribution (as compared with the first maximum of an l = 2 distribution) as well as the suggestion of a strong forward peak (indicated by one point).

The weak states at 1.400 and 3.120 MeV are not fitted for any l value and are assumed to be populated by nondirect processes. As can be seen in the spectrum shown in Fig. 1, the 3.120-MeV state is only partly resolved from the much more strongly populated 3.090-MeV state. However, the AUTOFIT program allowed reliable separation of the peaks. There is some indication that there may be another weak state obscured by the low-energy tail of the 1.155-MeV state. It has not been possible to resolve this doubtful state.

Finite-range nonlocal (FRNL) DWBA calcula-

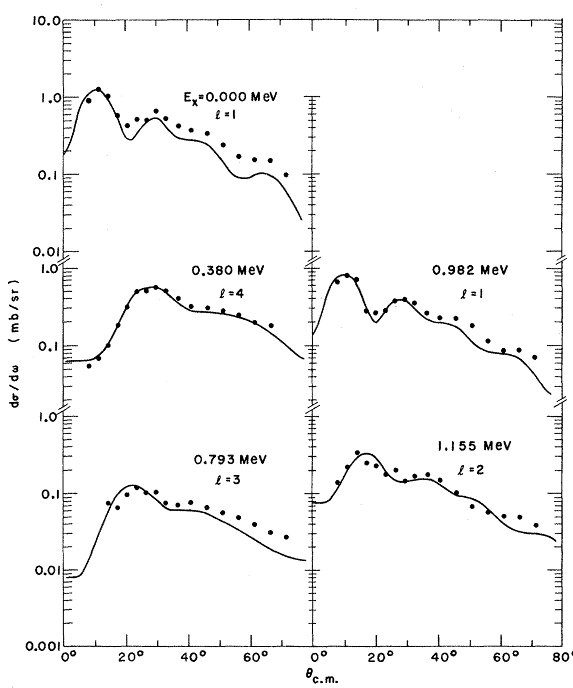

FIG. 2. Angular distributions for the ^{86}Sr(^{3}He, d)^{87}Y reaction. The curves were obtained from DWBA calculations.

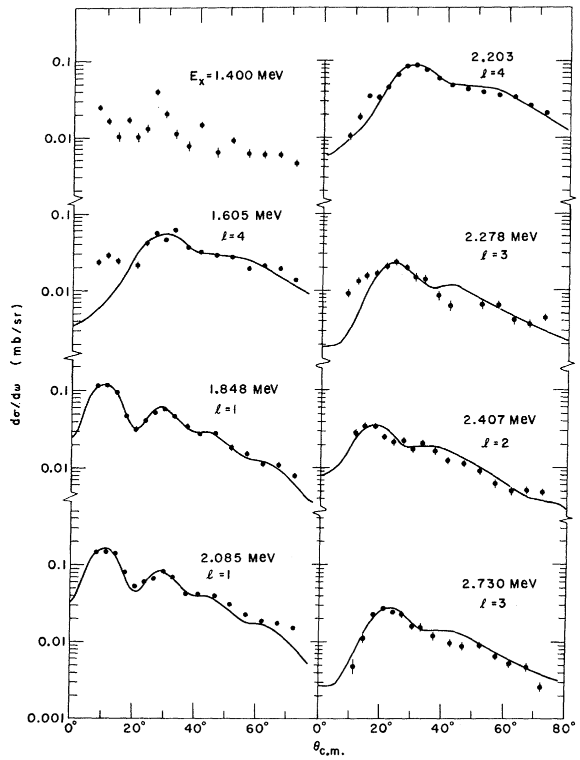

FIG. 3. Angular distributions for the ^{86}Sr(^{3}He, d)^{87}Y reaction. The curves were obtained from DWBA calculations.

tions were also performed[14] for the parameter set $H1$ and $D1$ listed in Table I. The shapes of the angular distributions were virtually identical to those predicted in the zero-range local (ZRL) case, and so were the predicted relative spectroscopic factors. Absolute cross sections for the FRNL cases were 1.38 times those for the corresponding ZRL cases.

V. SPECTROSCOPIC INFORMATION

Table II lists the spectroscopic information obtained from this experiment. Spectroscopic factors were extracted by use of the relation

$$[d\sigma(\theta)/d\omega]_{\text{exp}} = N(2J_F + 1)C^2 S \sigma_{\text{DWBA}}(\theta),$$

where $N = 4.42$. Column 1 in Table II lists excitation energies for all the observed states (with an uncertainty of ±4 keV). Column 2 lists the empirical l value and the assumed shell-model orbital of the transferred particle. Column 3 gives absolute spectroscopic factors as determined from the measured absolute cross sections and the (FRNL) DWBA. The sum rule states[15] that

proton holes in target

$$= \frac{2T}{2T+1} \sum g S(T_<) + \frac{1}{2T+1} \sum g S(T_>),$$

where $g = 2J_F + 1$, T is the target isospin, $T_>$ and

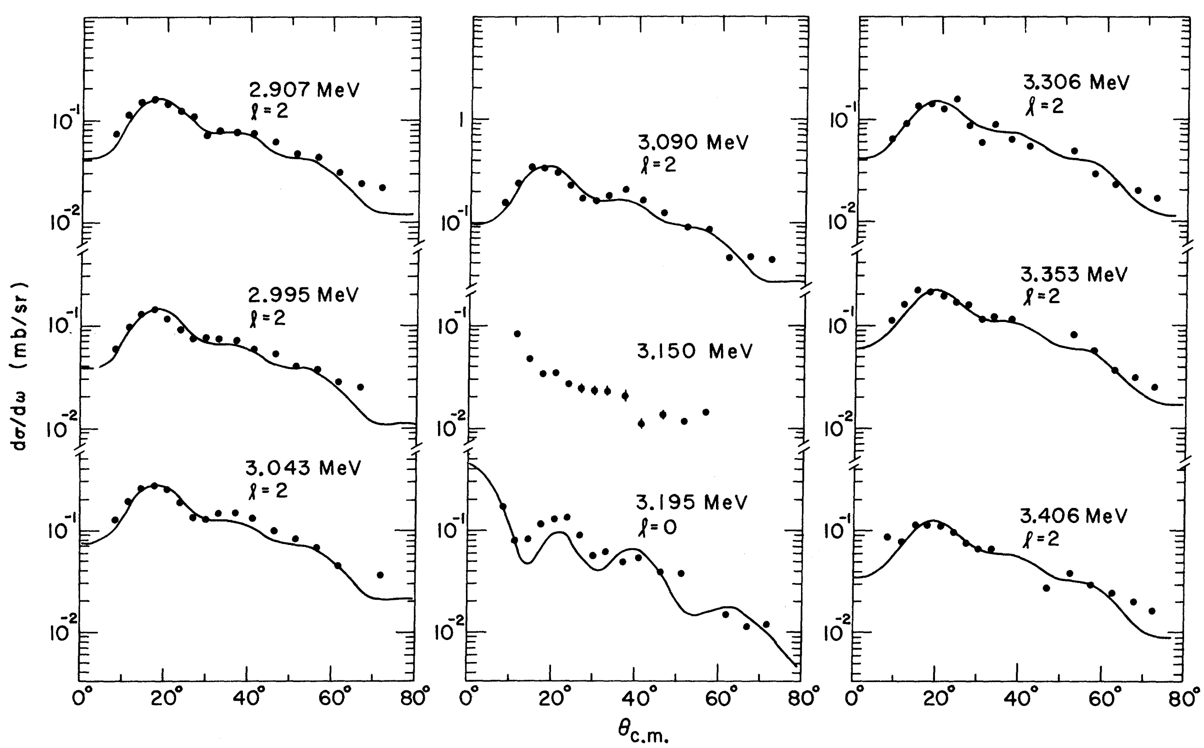

FIG. 4. Angular distributions for the ^{86}Sr(^{3}He, d)^{87}Y reaction. The curves were obtained from DWBA calculations.

TABLE I. Potential parameters used in the ^{86}Sr(^{3}He,d)^{87}Y reaction.

Potential set	V (MeV)	W (MeV)	r_0 (F)	r_c (F)	a (F)	V_{so} (MeV)	r' (F)	a' (F)	W' (MeV)
$H1$	170	20	1.14	1.40	0.75	···	1.60	0.80	···
$H2$	172	17	1.14	1.40	0.72	···	1.55	0.80	···
$D1$	98	···	1.10	1.30	0.85	6.0	1.40	0.70	72
$D2$	68.8	···	1.033	1.30	0.986	5.34	1.415	0.716	44

Bound-state parameters: $r_0=1.20$, $r_c=1.20$, $a=0.65$, $\lambda=25$.

$T_<$ are upper and lower residual-state isospins, respectively, and J_F is the final-state spin. For the ^{86}Sr target, $T=5$ and

$$\sum gS(T_>)=2 \text{ (i.e., two neutron holes in } N=50 \text{ core)},$$

so for states below the 50-proton shell closure (i.e., for $2p$, $1f$, and $1g$ states),

$$\frac{2T}{2T+1}\sum gS(T_<)=12-\tfrac{2}{11}=11.82\,.$$

The sum of absolute $2p$, $1f$, and $1g$ strengths from Table II is 11.88, in good agreement with sum-rule expectations. (Agreement to better than 10% is considered fortuitous.)

As discussed above, both potential sets in the ZRL approximation give relative spectroscopic factors identical to those listed in column 3. The consistency of the relative spectroscopic factors derived from the various calculations discussed above indicates that there is no noticeable sensitivity to the parameter set selected and that confidence can be placed in the spectroscopic information. It is important to note that these spectroscopic factors have been obtained from the same distorted-wave parameters as were used to study the ^{88}Sr(^{3}He,d)^{89}Y reaction.[2]

Figure 5 compares spectroscopic strengths from the ^{86}Sr(^{3}He,d)^{87}Y reaction with those derived from the ^{88}Sr(^{3}He,d)^{89}Y reaction. It is evident from these strengths that the 38-proton configuration is affected by introducing two neutron holes in the otherwise closed neutron shell. The $2p_{1/2}$ strength to the ground state of ^{87}Y is only 1.15 (slightly more than half the strength of this subshell) whereas the same state in ^{89}Y has a strength of 1.8. Similarly, the $1g_{9/2}$ state at 0.380 MeV accounts for 7.19 particles, and additional states at 1.605 and 2.203 MeV have 0.53 and 0.79 particles. In ^{89}Y the total $g_{9/2}$ strength is similar (8.8) but is concentrated in one state at 0.896 MeV.

The ^{89}Y results show 0.44 $2p_{3/2}$ particles at 1.490 MeV and 0.55 $1f_{5/2}$ particles at 1.725 MeV. In ^{87}Y an $l=1$ state at 0.982 MeV accounts for 0.54 $2p_{3/2}$ particles and an $l=3$ state at 0.793 MeV accounts for 1.15 particles. It is difficult to assess the importance of the crossing of the $p_{3/2}$ and $f_{5/2}$ levels since they are known to be nearly degenerate and to cross in this region (e.g., the ground state of ^{87}Rb is $J^\pi=\frac{3}{2}^-$ and the ground state of ^{85}Rb is $J^\pi=\frac{5}{2}^-$). However, the strong enhancement of $f_{5/2}$ strength in ^{87}Y over that in ^{89}Y is probably significant and will be discussed below. The $l=1$

TABLE II. Energies and spectroscopic factors for states observed in the ^{86}Sr(^{3}He,d)^{87}Sr reaction.

E_x (MeV)	(nlj)	$(2J_F+1)C^2S$
0.000	$2p_{1/2}$	1.15
0.380	$1g_{9/2}$	7.19
0.793	$1f_{5/2}$	1.15
0.982	$2p_{3/2}$	0.54
	$(2p_{1/2})$ [a]	(0.60)
1.155	$2d_{5/2}$	0.32
1.605	$1g_{9/2}$	0.53
1.848	$2p_{3/2}$	0.07
	$(2p_{1/2})$	(0.07)
2.085	$2p_{3/2}$	0.09
	$(2p_{1/2})$	(0.10)
2.203	$1g_{9/2}$	0.79
2.278	$1f_{5/2}$	0.14
2.407	$2d_{5/2}$	0.03
2.730	$1f_{5/2}$	0.16
2.907	$2d_{5/2}$	0.12
2.995	$2d_{5/2}$	0.11
3.043	$2d_{5/2}$	0.20
3.090	$2d_{5/2}$	0.25
3.195	$3s_{1/2}$	0.04
3.306	$2d_{5/2}$	0.11
3.353	$2d_{5/2}$	0.16
3.406	$2d_{5/2}$	0.09

[a] Parentheses indicate a less likely alternative.

strength to the 0.982-MeV level in ^{87}Y is similar to that of the corresponding ^{89}Y state if indeed both have $J^\pi = \frac{3}{2}^-$. Since there is so little $2p_{1/2}$ strength in the ^{87}Y ground-state transition, the 0.982-MeV level could well have $J^\pi = \frac{1}{2}^-$. It is, however, tempting to view this state as having $J^\pi = \frac{3}{2}^-$ in analogy with other nuclei in the region. Also, Morton[16] (despite severe resolution limitations) sees $l = 2$ strength with no detectable $l = 0$ admixture in populating this state through the ^{89}Y(p,t)-^{87}Y reaction. This is further evidence for a $J^\pi = \frac{3}{2}^-$ assignment. [Similarly, no $l = 0$ component is seen in the (p, t) reaction to the higher $2p$ states in ^{87}Y, so $\frac{3}{2}^-$ assignments are also favored for these.]

The remaining strong low-lying state in ^{87}Y has no stripping counterpart in ^{89}Y. This is a most significant difference between the two nuclei since this $l = 2$ state at 1.155 MeV has 0.32 particles (probably $2d_{5/2}$). However, a $J^\pi = \frac{5}{2}^+$ state has been reported[17] at 2.22 MeV in ^{89}Y although no state is observed at this energy in the (^{3}He, d) reaction. The experiment of Picard and Bassani was somewhat insensitive to weak states. They only weakly observe the known $1f_{5/2}$ state at 1.735 MeV in ^{89}Y, to which they assign a particle strength of 0.55. From their sensitivity to this $l = 3$ strength and from DWBA predictions of single-particle cross sections for $l = 2$ and 3, an upper limit on the strength of the unobserved $l = 2$ state at 2.22 MeV in ^{89}Y may be estimated to be ≤ 0.03 $d_{5/2}$ particles. This is significantly less strength than is seen in the case of ^{87}Y.

The states that appear between 1.2 and 2.9 MeV in ^{87}Y are seen to be predominantly $2p$, $1f$, and $1g$ states. No such states are seen in the corresponding region in ^{89}Y. When the strengths of these states are summed with those of the four strong states, the distribution of p, f, and g proton strengths is similar to that seen in ^{89}Y. The ^{86}Sr ground state has ~20% fewer $2p$ holes than the ^{88}Sr ground state and a very similar number of $1g$ holes. The major change is a threefold increase in $1f$ holes for ^{86}Sr and ^{88}Sr. These results are listed in Table III and discussed in Sec. VI.

The many $l = 2$ states in the region above 2.91 MeV are similar to those in ^{89}Y, although they are at lower excitation energy and appear to have lower individual strengths. In neither case does any one or a group of several states dominate the $2d$ strength. It is not meaningful to sum the $2d$ strengths since the present experiment has been limited to excitation energies below ~3.5 MeV. Considerable $2d_{5/2}$ strength is expected to lie above this excitation energy.

TABLE III. Summed proton stripping strengths.

Shell	^{86}Sr(^{3}He, d)^{87}Y	^{88}Sr(^{3}He, d)^{89}Y (Ref. 2)
$2p$	1.86	2.32
$1f$	1.45	0.55
$1g$	8.51	8.8

No $l = 0$ strength is reported for ^{89}Y, while a tentative $l = 0$ state is seen at 3.20 MeV in ^{87}Y. This state is very weak and no appreciable fraction (i.e., only ~2%) of the strength of the $3s_{1/2}$ subshell is present. The ^{89}Y angular distributions do not include angles less than ~25° so it is not possible to rule out the possibility of some weak $l = 0$ strength in the corresponding region of ^{89}Y.

VI. DISCUSSION

The main effect expected from introducing holes into the neutron core is a splitting of the states, the summed spectroscopic strengths being roughly preserved. The results for the $g_{9/2}$ states agree well with this qualitative expectation (as can be seen from Fig. 5 and Table III). The results for the $2p$ states are more difficult to interpret. If the assignment of the 0.982-MeV state in ^{87}Y were $J^\pi = \frac{1}{2}^-$, then the results shown in Fig. 5 might be interpreted as follows. Coupling of the $2p_{1/2}$ proton to a mixed configuration of neutron-hole pairs leaves 1.15 particles in the ground state (as compared with 1.8 particles in the ^{89}Y ground state). Most of the remaining strength (0.60 particles) is then concentrated in the 0.932-MeV state of ^{87}Y. Similarly, the 1.490-MeV state of ^{89}Y (0.44 parti-

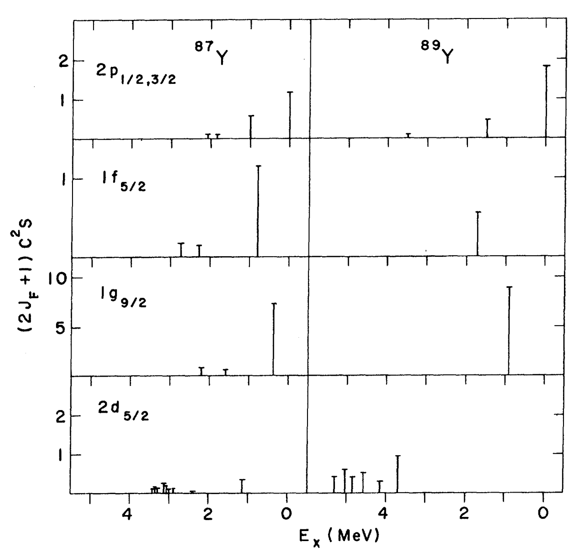

FIG. 5. Comparison of spectroscopic strengths and excitation energies for states in ^{87}Y and ^{89}Y populated in the (^{3}He, d) reaction on ^{86}Sr and ^{88}Sr. The ^{89}Y results are from Picard and Bassani (Ref. 2).

cles) is split into the 1.848-MeV state (0.07 particles) and 2.085-MeV state (0.09 particles) of ^{87}Y with possible smaller fragments undetected at higher excitations. However, as discussed in Sec. V, there is evidence that all the $l = 1$ levels of ^{87}Y except the ground state have $J^\pi = \frac{3}{2}^-$. If this is the case, significant $p_{1/2}$ strength has disappeared; i.e., there are more $p_{3/2}$ holes in the ground state of ^{86}Sr than in that of ^{88}Sr, and the $2p_{1/2}$ subshell (which in ^{88}Sr is nearly empty) is almost half full in ^{86}Sr. This latter case is not easily explained. Thus, theoretical considerations favor a $J^\pi = \frac{1}{2}^-$ assignment for the 0.982-MeV state of ^{87}Y, while the experimental results of Morton[16] favor a $\frac{3}{2}^-$ assignment.

As mentioned above, the results for the $1f_{5/2}$ states are most difficult to explain. In ^{89}Y there is one $l = 3$ state at 1.735 MeV. Picard and Bassani[2] estimate the strength of this state as 0.55 $1f_{5/2}$ particles but warn that this value is very uncertain due to experimental limitations. While these authors give no quantitative error estimate, it seems unlikely that the strength of this state could be low by a factor of 3. Thus it appears most probable that the $1f_{5/2}$ subshell is significantly less full in ^{86}Sr than in ^{88}Sr. In the absence of a full shell-model calculation, there is no simple explanation for this effect. Although the $2p_{3/2}$ and $1f_{5/2}$ single-proton states are nearly degenerate in this mass region, it seems unlikely that interaction between the $2p_{3/2}$ and $1f_{5/2}$ subshells can account for the apparent depletion of the $1f_{5/2}$ strength; no comparable effect has been found for the $1f_{5/2}$ single-neutron state in the Ni isotopes,[18] where the underlying $2p_{3/2}$ and $1f_{5/2}$ orbitals are also nearly degenerate. Further clarification of the strength of the 1.735-MeV level in ^{89}Y would be of great interest.

The $l = 2$ level at 1.155 MeV in ^{87}Y probably represents the most significant deviation of the 38-proton configuration from semiclosed-subshell behavior. No explanation is known for the surprising (0.32 particle) amount of strength drawn from the next major shell.

VII. CONCLUSION

The closed-subshell property of the 38-proton configuration in the presence of the 50-neutron closed shell has been seen to change significantly when two neutron holes are present. Considerable shell-model simplicity remains, however, for the $1g_{9/2}$ configuration and possibly for the $2p_{1/2,3/2}$ configurations. An increased $1f_{5/2}$ strength and the depression of a strong $2d_{5/2}$ state represent the principal evidence for significant departure from closed-subshell behavior.

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Energy-Dependent Beta-Gamma Circular Polarization and Nuclear Matrix Elements of Rb^{86}†

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The energy-dependent β-γ circular polarization was measured in order to set better limits on the matrix-element ratio $\Lambda = \int \vec{\alpha} / (\int i \vec{r}/\rho)$ for the 696-keV first-forbidden β transition of Rb^{86}. The results show that Λ is consistent with the Fujita-Eichler relation, so the contributions of the off-diagonal matrix elements of the Coulomb Hamiltonian to Λ are small for this transition. The nuclear matrix elements are in agreement with previous results, but the limits of error have been reduced.

I. INTRODUCTION

The determination of nuclear matrix elements for first-forbidden β transitions provides a sensitive test for nuclear models. Since there can be a relatively large number of matrix elements which contribute to the β transition, more can be learned about the details of nuclear structure from first-forbidden transitions than from allowed transitions. Even though the large number of quantities which can be measured makes the investigation interesting, the fact that there are a large number of unknowns also makes it difficult to extract the matrix elements from the experimental data.

A valuable aid to simplify the extraction of the matrix elements has been proposed by Fujita[1] and Eichler.[2] The conserved-vector-current theory of β decay can be used to predict the ratio Λ of two of the vector matrix elements.

$$\Lambda \equiv \int \vec{\alpha} \Big/ \left(\int i \vec{r}/\rho \right) .$$

If Λ can be used to remove one unknown from the problem, it is much easier to determine the matrix elements. The Fujita-Eichler method for predicting Λ is attractive because it does not depend on any details of nuclear structure. The assumption is made that the off-diagonal matrix elements of the Coulomb Hamiltonian H_C are so small that they can be neglected. The validity of this assumption was questioned by Damgaard and Winther.[3] They used nuclear-model calculations for Tl^{207} and Pb^{209} to propose that even if the off-diagonal matrix elements of H_C are small, it is possible that the vector matrix-element ratio Λ will depend on details of nuclear structure.

The Fujita-Eichler expression for Λ is

$$\Lambda \equiv \frac{\int \vec{\alpha}}{\int i \vec{r}/\rho} = \pm 2.4 \frac{\alpha Z}{2} + (W_0 \mp 2.5)\rho \quad \text{for } \beta^{\mp} \text{ decay} . \tag{1}$$

Z is the charge of the daughter, $\alpha = 1/137$, and natural units ($m_e = \hbar = c = 1$) are used for the nuclear radius ρ and the end-point energy W_0. When the off-diagonal matrix elements of H_C are included the following correction term[4] must be added to Λ:

$$\sum_{f' \neq f} \langle f | H_C | f' \rangle \langle f' | i \vec{r} | i \rangle - \sum_{i' \neq i} \langle f | i \vec{r} | i' \rangle \langle i' | H_C | i \rangle . \tag{2}$$

The final state $|f\rangle$ and the initial state $|i\rangle$ of the