Figure 9 displays a comparison of extended R-matrix theory results (inversion and weak coupling) calculated with different choices for the channel radii $\{a_\alpha\}$ and matching radii $\{\bar{a}_\alpha\}$. The dependence on the choice is not very severe in this 10-level calculation. For the extended R-matrix theory (diagonalization) the dependence is somewhat stronger. All the other extended R-matrix-theory and L-operator-theory results use $\{a_\alpha = \bar{a}_\alpha = 7\ \mathrm{F}\}$. The X-matrix theory results do not depend on the channel radii or matching radii.

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Measurement of the Magnitude and Sign of the Nuclear Quadrupole Interaction via the Spin Precession of an Angular Correlation*

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Perturbed angular-correlation techniques are proposed for measuring the spin precessions of a polarized nucleus interacting with an axially symmetric electric field gradient. A specific method is suggested which makes use of Coulomb excitation to polarize and excite a nuclear state even as it recoils the nucleus into a suitable crystalline environment. Calculations indicate that this spectroscopic technique should make practical the measurements of the sign and magnitude of quadrupole moments of subnanosecond nuclear states.

The possibility of measuring the spin precession of a nonspherical nuclear state under the influence of a crystalline electric field gradient has received only passing mention in the literature.[1] It is the purpose of this paper to stress the usefulness of such measurements and to detail one experimental procedure for the observation of spin precession. In principle, the proposed technique of perturbed angular correlation (PAC) following nuclear polarization via the Coulomb-excitation interaction should make practical the determination of the sign and magnitude of quadrupole interactions of excited states which live much less than a nanosecond.

The importance of measurements of quadrupole moments of short-lived states has increased markedly in recent years with the discovery that a wide class of first 2^+ states of so-called spherical nuclei have substantial static quadrupole moments.[2,3] All of these measurements use Coulomb-excitation reorientation techniques wherein the higher-order matrix element $\langle I|Q|I\rangle$ is extracted from measurements of the Coulomb-excitation cross section. Sophisticated methods have been found, yet many of the details of the experiment and analysis remain subject to controversy.[4] It would be valuable to check the results from the reorientation technique by spectroscopic methods which involve the measurement of the interaction of the quadrupole moment of short-lived states with a crystalline electric field gradient. Present methods use the Mössbauer effect, which is impractical for the higher-energy 2^+ states, and the perturbations of angular correlations, which are time-integrated functions of the square of the angular parameter $\omega_Q\tau$. Unfortunately, $\omega_Q\tau$ is generally much smaller than 1 for subnanosecond states, so that methods which depend upon $(\omega_Q\tau)^2$ become prohibitively difficult for most of the states of interest. To carry out measurements on such short-lived states, one should either make use of abnormally high values of $\partial E/\partial Z$, such as might be encountered in certain ionized atoms or in special crystalline environments, or use a method which depends on $(\omega_Q\tau)$ in first order. It is the latter which is discussed here.

Consider the action of an axially symmetric electric field gradient on an intrinsic quadrupole moment according to the semiclassical vector model (Fig. 1). For a given product of $Q(\partial E_Z'/\partial Z)$ the precession direction of the nuclear spin depends upon the sign of the M substate involved, in contrast to the magnetic case in which the Larmor precession is independent of the sign. Thus an aligned nucleus – the usual symmetry in a directional angular correlation – yields a superposition of equal and opposite precessions, which produces effects proportional to $(\omega_Q\tau)^2$. The polarization of a nucleus, on the other hand, results in a definite rotation sense and the γ-ray angular distribution generally has terms proportional to $\omega_Q\tau$.

It has long been recognized that the sign of the quadrupole interaction can be determined either from the measurement of the circular polarization of the γ rays involved in the angular correlation or from the measurement of the anisotropy of the correlation from nuclei polarized in β decay, though in neither case has the analogy to the magnetic precession been considered.[5] Moreover, though it has been known for some time that under suitable conditions the products of nuclear reactions will be polarized,[6] almost no consideration has been given to the consequences for PAC. Perhaps even more surprising is the little attention given to the fact that Coulomb excitation, one of

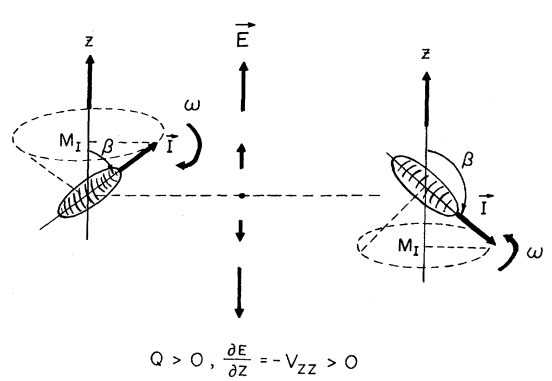

FIG. 1. Classical precession $\omega(\beta) = \frac{3}{4}(eQV_{ZZ}/I\hbar)\cos\beta$ of a nuclear angular momentum I and quadrupole moment Q ($Q>0$) around the symmetry axis of a field gradient $\partial E_Z/\partial Z = -V_{ZZ} > 0$ for β smaller and larger than 90°.

the most effective means of producing low-lying states of nuclei, can result in a substantial degree of nuclear polarization for particle scattering angles $\vartheta < 180°$ [see Fig. 3(a) for example]. In this case the degree of polarization and all angular distribution coefficients involved can be calculated.[7,8]

The method suggested here for the determination of the sign and magnitude of quadrupole interactions of short-lived states requires the determination of the particle–γ-ray angular precession following Coulomb excitation which excites and polarizes the nuclear states even as it impels the nucleus into a suitable single-crystal host. Only a few rather arbitrary geometries are discussed.

The perturbed particle-γ correlation for Coulomb excitation can be written in analogy to perturbed γ-γ correlation

$$W_{\vartheta,\,\varphi}(\Omega_\gamma, t) = (4\pi)^{1/2} \sum_{k_1k_2N_1N_2} \left(\frac{2k_1+1}{2k_2+1}\right)^{1/2} A_{k_1k_2} a_{k_1N_1}(\vartheta, \varphi) \times G_{k_1k_2}^{N_1N_2}(t) Y_{k_2N_2}(\Omega_\gamma)\,. \tag{1}$$

The coefficients $a_{k_1N_2}(\vartheta, \varphi)$ describe the Coulomb-excitation process. Formulas for $a_{k_1N_1}$ in first-order perturbation theory are given by Alder *et al.*[7] (Eq. II A67); for multiple Coulomb excitation (including the reorientation effect) the computer program of Winther and de Boer[8] can be used. In this present paper we consider only axially symmetric quadrupole interactions and choose the coordinate system for all quantities in Eq. (1) in such a way that the quadrupole interaction is diagonalized. Then the attenuation coefficients are

$$G_{k_1k_2}^{NN}(t) = \sum_{n\geq 0} S_{nN}^{k_1k_2} \begin{cases} \cos n\omega_0 t & \text{for } (k_1+k_2) \text{ even}, \\ (-i)\sin n\omega_0 t & \text{for } (k_1+k_2) \text{ odd}, \end{cases} \tag{2}$$

with

$$S_{nN}^{k_1k_2} = 2(1-\tfrac{1}{2}\delta_{n0})(2k_1+1)^{1/2}(2k_2+1)^{1/2} \times \begin{pmatrix} I & I & k_1 \\ m-N & -m & N \end{pmatrix} \begin{pmatrix} I & I & k_2 \\ m-N & -m & N \end{pmatrix} \quad \text{for } N\neq 0, \tag{3}$$

$$\omega_0 = 3\omega_Q, \quad m = \tfrac{1}{2}N + \tfrac{1}{2}(n/N) \quad \text{for integer } I, \tag{4a}$$

$$\omega_0 = 6\omega_Q, \quad m = \tfrac{1}{2}N + (n/N) \quad \text{for half-integer } I, \tag{4b}$$

$$S_{n0}^{k_1k_2} = \delta_{n0}\delta_{k_1k_2} \quad \text{for } N=0,$$

$$\omega_Q = \frac{eQV_{ZZ}}{4I(2I-1)\hbar}\,.$$

The definition of $S_{nN}^{k_1k_2}$ [Eq. (3)] which holds for all k_1, k_2 combinations is an extension of the usual one[9] which holds only when (k_1+k_2) is even. $S_{nN}^{k_1k_2} \neq 0$ only if $m(n,N)$ is integer or half-integer in Eqs.

(4a) and (4b), respectively.

The sum over k_1 in Eq. (1) extends over even and odd values; the odd values bringing the necessary $\sin n\omega_0 t$ into play [Eq. (2)]. $A_{k_1k_2}$ are the usual γ-γ angular-correlation coefficients.

To get maximum nuclear precession, according to the semiclassical picture, the $\hat{c}$ axis should be aligned parallel to the direction of the polarization, which is perpendicular to the scattering plane. In the following examples we restrict ourselves to this geometry and therefore choose the so-called "focal" coordinate system for Eqs. (1) and (2); see Fig. 2. The Z axis is perpendicular to the scattering plane, and the X axis halves the hyperbolic orbit of the projectile. In this coordinate system, the $a_{kN}^{(\text{focal})} \neq 0$ for even N only, and are real in first-order perturbation theory.[7] For $E2$ Coulomb excitation one gets

$$a_{kN}^{(\text{focal})}(\vartheta) = -\tfrac{16}{5}\pi(2k+1)^{-1/2}\begin{pmatrix} 2 & 2 & k \\ 1 & -1 & 0\end{pmatrix}^{-1}\{\tfrac{3}{2}[I_{2-2}(\vartheta,\xi)]^2+[I_{20}(\vartheta,\xi)]^2+\tfrac{3}{2}[I_{22}(\vartheta,\xi)]^2\}^{-1}$$
$$\times\sum_{\mu}\sum_{\mu'}(-1)^{\mu}\begin{pmatrix} 2 & 2 & k \\ \mu & -\mu' & N\end{pmatrix}Y_{2\mu}(\pi/2,0)Y_{2\mu'}(\pi/2,0)I_{2\mu}(\vartheta,\xi)I_{2\mu'}(\vartheta,\xi)\,. \qquad (5)$$

The $I_{2\mu}(\vartheta, \xi)$ are the classical orbital integrals tabulated by Alder *et al.*[7] as functions of ϑ and the adiabaticity parameter ξ. For this geometry and for nuclear spins of $\frac{3}{2}$ and 2, $a_{32}^{(\text{focal})}(\vartheta)$ is the only coefficient with odd k_1 which appears in Eq. (1). The dependence of this coefficient on ϑ and ξ is shown in Fig. 3(b).

To display the essentials of Eq. (1) as applied to specific cases it is convenient to introduce the abbreviations

$$g_{nN}^{e}(\theta) = (4\pi)^{1/2}2(1-\tfrac{1}{2}\delta_{N0})\sum_{k_1=0,2,4}\;\sum_{k_2=0,2,4}(2k_1+1)^{1/2}(2k_2+1)^{-1/2}A_{k_1k_2}a_{k_1N}^{(\text{focal})}(\vartheta)S_{nN}^{k_1k_2}Y_{k_2N}(\theta,0)\,, \qquad (6)$$

for the sum over even k_1 and the corresponding parameter $g_{nN}^{0}(\theta)$ for the single sum for $k_1 = 3$. The dependence of g_{nN} on the particle scattering angle is omitted for simplicity.

For $I=\frac{3}{2}$ the time-dependent angular distribution then takes the form

$$W_{\vartheta}(\theta,\Phi,t) = \{g_{00}^{e}(\theta)+g_{12}^{e}(\theta)[(1-p)\cos\omega_0 t\cos 2\Phi + p\cos 2(\Phi-\tfrac{1}{2}\omega_0 t)]\}e^{-t/\tau}\,,$$
$$p=p(\vartheta,\xi)=\frac{g_{12}^{0}(\theta)}{g_{12}^{e}(\theta)}=\frac{1}{4}\sqrt{5}\,\frac{A_{32}}{A_{22}}\,\frac{I_{22}(\vartheta,\xi)-I_{2-2}(\vartheta,\xi)}{I_{22}(\vartheta,\xi)+I_{2-2}(\vartheta,\xi)}\,. \qquad (7)$$

In the square brackets the first term corresponds to a breathing mode that disappears, e.g., at $\Phi = 45^{\circ}$, and the second to a rotation of the angular distribution. Thus, in this simple case it is possible to find geometries which result in a rotation of the χ angular correlation similar to that produced by magnetic interactions. The factor p is a measure of the polarization of the intermediate state, since the orbital integrals $I_{2\mu}(\vartheta, \xi)$ for $\mu = \pm 2$ are proportional to the excitation amplitudes for a quadrupole transition with components $\mu = M_i - M_f = \pm 2$ $[I_{2\pm 1}(\vartheta, \xi) = 0]$.

As to the dependence of the precession on θ, ϑ, ξ the decisive quantity is the product $p \times g_{12}^{e}(\theta) = g_{12}^{0}(\theta)$ which is proportional to $\sin^2\theta\, a_{32}^{(\text{focal})}(\vartheta, \xi)$. [Compare Fig. 3(b).]

The important case of $I = 2$ is more complicated than that of $I=\frac{3}{2}$. We only consider the 0-2-0 time-integrated angular distribution, a special example which nevertheless has wide applicability. The angular distribution, keeping the geometry of Fig. 2, can be written in the following instructive way.

$$W_{\vartheta}(\theta,\Phi) = g_{00}^{e}(\theta)+g_{04}^{e}(\theta)\cos 4\Phi + g_{42}^{e}(\theta)\frac{[1+(p\times 4\omega_0\tau)^2]^{1/2}}{1+(4\omega_0\tau)^2}\cos 2(\Phi-\delta)\,,$$
$$\delta = \tfrac{1}{2}\tan^{-1}(p\times 4\omega_0\tau)\,, \qquad (8)$$
$$p=p(\vartheta,\xi)=\frac{g_{42}^{0}(\theta)}{g_{42}^{e}(\theta)} = -\frac{I_{22}(\vartheta,\xi)-I_{2-2}(\vartheta,\xi)}{I_{22}(\vartheta,\xi)+I_{2-2}(\vartheta,\xi)}\,.$$

$p(\vartheta, \xi)$ is again a measure of the polarization of the nucleus since here the classical orbital integrals, $I_{2\mu}(\vartheta, \xi)$, are proportional to the excitation amplitudes of the substates $(M = -\mu)$. When $p(\vartheta, \xi) = 0$, Eq. (8) reduces to the usual case of a quadrupole interaction in a single crystal, and when $p(\vartheta, \xi) = 1$, the last term in Eq. (8) appears like a time-integrated Larmor precession. It should also be noted that the unperturbed nature of the $\cos 4\Phi$ term is a general result for a quadrupole interaction on $I=2$ when the $\hat{c}$ axis is parallel to the Z axis.

The coefficients $g_{42}^{e}(\theta)$ and $g_{42}^{0}(\theta)$, being proportional to $\sin^2\theta - \sin^4\theta$, are maximum for $\theta = 45°$ and disappear for $\theta = 90°$. The latter is consistent with the general rule that a pure quadrupole interaction will not perturb an angular distribution through an $I=2$ state if the radiations involve a $0-2$ γ radiation emitted perpendicular to the crystalline symmetry axis.[10] Thus for the geometry of Fig. 2, the γ rays should be detected on a cone of half angle $\theta = 45°$ for maximum perturbations. (Calculations for a more convenient geometry with γ counters in the particle scattering plane and arbitrary direction of the $\hat{c}$ axis are in progress.)

The perturbation effects are essentially proportional to the product $p \times g_{42}^{e}(\theta) = g_{42}^{0}(\theta)$ which in turn is proportional to $a_{32}^{(\mathrm{focal})}(\vartheta, \xi)$. The largest values of this decisive parameter, plotted in Fig. 3(b), are obtained at higher bombarding energies, E (i.e., low ξ values) and smaller scattering angles. On the other hand, the excitation probability $P_{0\to 2^+}$ per scattered particle decreases as ϑ and E decrease. The optimum values depend on the specific case under study.

As an illustration of such a case, we consider the recoil implantation of $^{184}W(2^+)$ by ^{16}O ions of 37 MeV ($\xi = 0.1$) into gadolinium single crystals. The unpolarized counterpart of this experiment was recently performed using backscattered particles ($\vartheta = 180°$) and $|\omega_Q\tau| = 0.25 \pm 0.04$ was measured.[11] In this case Eq. (8) yields for $\vartheta = 90°$

$$W_{90°}(45°, \Phi) = 1 - 0.15\cos 4\Phi + 0.10\cos 2(\Phi - \delta)$$

with $\delta = \pm 17°$. The sign of δ depends on the sign of ω_Q.

To see how, in this specific example, the effect varies with ϑ and E, the relative statistical error of $a_{32}^{(\mathrm{focal})}$, which is proportional to $(P_{0\to 2^+})^{-1/2} \times (a_{32}^{(\mathrm{focal})})^{-1}$ is plotted in Fig. 3(c). For this calculation the total particle counting rate and the ratio of true to accidental coincidences were kept constant. The lowest statistical error is obtained for the highest bombarding energy, 37 MeV, and $\vartheta \approx 90°$.

The example above is presently under experimental study to test specific techniques when all parameters including Q are reasonably well known. Even if these experiments are successful it is probable that each case will require further exploratory studies to find suitable crystalline environments. It is hoped that the importance of such studies will prove sufficient incentive for the

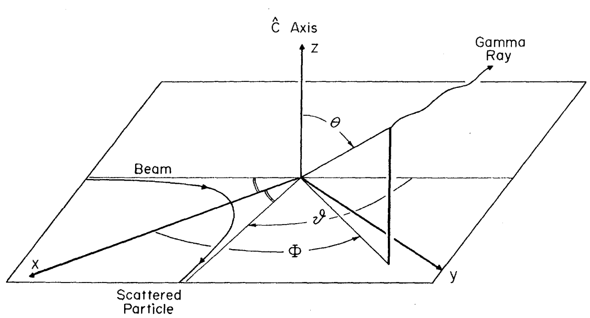

FIG. 2. Illustration of the scattering geometry, the crystalline symmetry axis ($\hat{c}$ axis), and the polar coordinates of the deexcitation γ ray in the center-of-mass system.

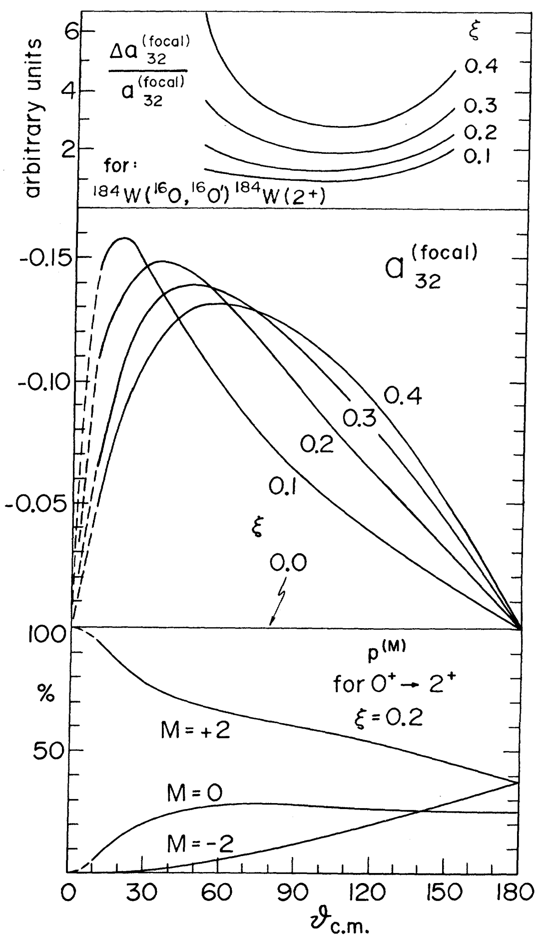

FIG. 3. The following quantities are calculated in first-order perturbation theory for $E2$ Coulomb excitation as functions of the scattering angle ϑ (center-of-mass system) and the adiabaticity parameter ξ: (a) (bottom) population probability $p^{(M)}$ for different sublevels M assuming the scattering geometry of Fig. 2 $[p^{(\pm 1)} = 0]$; (b) (middle) angular correlation coefficient a_{kN} for $k=3$, $N=2$ in the "focal" coordinate system; (c) (top) relative statistical error of $a_{32}^{(\mathrm{focal})}$ calculated for different bombarding energies $E_{^{16}O} = 37, 24, 18, 14$ MeV ($\xi = 0.1-0.4$) in the reaction $^{184}W(^{16}O, {}^{16}O')^{184}W(2^+)$.

development of the above and related techniques.

Finally, it should be noted that second-order perturbation theory of Coulomb excitation shows that reorientation experiments should yield a precession of the γ-angular distribution.[3,12] It is then of interest to compare the rotation expected in such experiments with that expected in the method proposed above. In the former case the effective field gradient is of the order of 10^{11} times larger than hyperfine fields in typical noncubic crystals. However, the collision times in Coulomb excitation are so short that the net rotation is about an order of magnitude smaller than expected spectroscopically for a nsec lifetime, as typified by the example above.

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Phenomenological Wave Function of ^{6}Li

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In the framework of the α-deuteron cluster model, the $J=1$ deformation is considered in the phenomenological wave function of ^{6}Li so as to produce the charge form factor and the quadrupole moment at the same time. In our model the deviation from the S state of the relative motion between the α particle and the deuteron compensates the quadrupole moment of the free deuteron, $Q_d^{\rm exp}=+0.278\pm0.008$ fm^2, to give that of ^{6}Li, $Q_{\rm Li}^{\rm exp}=-0.08\pm0.008$ fm^2. It is shown that the quadrupole contribution plays an important role in filling up the diffraction minimum around $q^2=8$ fm^{-2} in the elastic charge form factor.

1. INTRODUCTION

As is pointed out in the previous paper,[1] various problems of ^{6}Li still remain unsolved in spite of elaboration by many physicists. It may be necessary for the time being to reproduce theoretically the charge form factor, in the sense that high-energy electron elastic scattering by the ^{6}Li nucleus cannot be explained as simply as other p-shell nuclei with the simple harmonic-oscillator wave function. In particular, one of the most important problems is to explain the charge form factor in the region of large momentum transfer ($q^2\gtrsim3$) and also the diffraction minimum which is known to exist around $q^2=8$ fm^{-2}.[2] All theories[3-9] hitherto have reported that the parameter set adjusted to give a good fit for the charge form factor results in a quadrupole moment almost 10 times larger