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Angular Distribution of Internal Bremsstrahlung in Orbital-Electron Capture from Polarized Nuclei

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The angular distribution of the internal bremsstrahlung accompanying allowed electron-capture transitions is calculated using the theory of Glauber and Martin. It is shown that when Coulomb effects as well as relativistic and screening corrections are taken into account, the asymmetry coefficients associated with the radiative capture of K- and L-shell electrons acquire a dependence on photon energy which is quite pronounced at energies below about $Z\alpha mc^2$. This is quite contrary to the results of previous calculations, which have predicted these coefficients to be energy independent. The present results are found to give much better agreement with recent experimental observations than that obtained with the previous theory.

I. INTRODUCTION

Internal bremsstrahlung emitted during orbital electron capture has been the subject of many investigations, both theoretical and experimental. Although such radiation is of low intensity, its study is valuable in providing us with information on the properties of nuclear states and as a means of testing basic features of the theory of weak interactions.

Most of the work which has been done is concerned with the determination of the spectral distribution of the radiation. For allowed K-capture transitions, this distribution was first calculated by Morrison and Schiff[1] neglecting completely the influence of the nuclear Coulomb field. After early experiments[2,3] revealed a large excess of photons at low energies, Glauber and Martin[4] carried out much more extensive calculations for capture from both K and L shells, which took complete account of the nuclear field and included both relativistic and screening effects. Their results, applicable to moderately light nuclei, have been found to be in good agreement with experiment.[5]

With the discovery of parity nonconservation in weak interactions, interest in internal bremsstrahlung shifted to studies of the polarization and angular distribution of the radiation. Cutkosky[6] was the first to show that a two-component neutrino theory predicts that the radiation will be circularly polarized. While Cutkosky's results included nuclear Coulomb field effects only to lowest order of $Z\alpha$, more exact calculations on the polarization of the radiation were reported shortly thereafter.[7]

The work of Cutkosky also implies the existence of an anisotropy in the angular distribution of the radiation emitted from oriented nuclei. Studies of this angular asymmetry are of considerable interest, since they provide us with information such as the nuclear spin change in electron capture and the relative magnitude of the nuclear matrix elements operative in electron capture. In particular, the ratio of the Gamow-Teller and Fermi nuclear matrix elements can be determined for $J_i = J_f \neq 0$ transitions. The internal-bremsstrahlung angular-distribution function has been calculated by Timashev and Kaminskii[8] and by Koh, Miyatake, and Watanabe[9] neglecting all relativistic effects and the intermediate-state Coulomb effects for the electron. Both papers report the interesting result that for allowed K-capture transitions the angular distribution of the radiation is independent of the energy of the photon.

Recently the first experimental results of an electron-capture angular-distribution study have been reported by Brewer and Shirley[10] who have examined the internal bremsstrahlung accompanying the decay of ^{119}Sb. Their results do not agree very well with the theoretical predictions of Koh, Miyatake, and Watanabe and of Timashev and Kaminskii, particularly at low photon energies. On the contrary, their results suggest an appreciable energy dependence in the angular distribution at low energies. Although it has been pointed out that the inclusion of relativistic and Coulomb corrections might lead to an energy dependence in the angular distribution,[9] it has been generally believed that any such dependence will be only slight.

With the expectation that relativistic and Coulomb effects do lead to an appreciable energy dependence in the angular distribution of the radiation, we reexamine in the present paper the comprehensive theory of Glauber and Martin with the intention of extending it to the calculation of the angular-distribution function for radiative electron capture. In order to make the present paper coherent and reasonably self-contained, it is necessary that we briefly review the extensive work of Glauber and Martin on this process. We provide only an outline of their analysis and refer the reader to their original papers[4] for further details and discussion. In Sec. II, the matrix element for radiative electron capture is briefly discussed, while in Sec. III the transition rates and angular distribution functions for 1s-, 2s-, and p-state radiative capture are calculated. Section IV is devoted to a discussion of the results and their comparison with the recent experimental observations of Brewer and Shirley.

II. MATRIX ELEMENT FOR INTERNAL BREMMSTRAHLUNG

The lowest-order Feynman diagram describing internal bremsstrahlung (IB) is shown in Fig. 1 in which the electron lines describe electron propagation in the presence of the Coulomb field of a nucleus of charge Ze. For a β interaction consisting of the standard $V-\lambda A$ coupling, the corresponding matrix element is given by[11]

$$M = ieC_V\left(\frac{\pi}{kV}\right)^{1/2}\int d\vec{r}_N\int d\vec{r}\,\bar{\phi}_f^N(\vec{r}_N)\Gamma_\mu\phi_i^N(\vec{r}_N)\bar{\phi}_\nu(\vec{r}_N) \times\Lambda_\mu G_E(\vec{r}_N,\vec{r})\vec{\gamma}\cdot\vec{e}^*\phi_i(\vec{r})e^{-i\vec{k}\cdot\vec{r}}, \quad (1)$$

where $\Gamma_\mu=\gamma_\mu(1+\lambda\gamma_5)$ and $\Lambda_\mu=\gamma_\mu(1+\gamma_5)$. C_V is the vector coupling constant of the β interaction, and $\lambda=|C_A/C_V|\approx 1.18\pm 0.02$. The emitted photon has momentum $\vec{k}$, and its polarization state is described by the unit vector $\vec{e}$. ϕ_i^N and ϕ_f^N represent the initial and final states of the nucleus,[12] while E_ν, E_i and ϕ_ν, ϕ_i are the energies and wave functions of the emitted neutrino and the initial electron, respectively. In the intermediate state the electron has an energy $E=E_i-k$, and its propagation is described by the Dirac-Coulomb Green's function $G_E(\vec{r}_N,\vec{r})$.

Glauber and Martin have studied this matrix element extensively for the case of unpolarized nuclei in two well-known papers.[4] Their approach is based on the introduction of the second-order Dirac-Coulomb Green's function $g_E(\vec{r}_N,\vec{r})$, which is related to the first-order Green's function by

$$G_E(\vec{r}_N,\vec{r})=g_E(\vec{r}_N,\vec{r})\left[\vec{\gamma}\cdot\overleftarrow{\nabla}+\gamma_4\left(E+\frac{Z\alpha}{r}\right)+1\right], \quad (2)$$

and satisfies the inhomogeneous second-order equation,

$$g_E(\vec{r}_N,\vec{r})\left[\nabla^2+\left(E+\frac{Z\alpha}{r}\right)^2-1-iZ\alpha\vec{\alpha}\cdot\overleftarrow{\nabla}\left(\frac{1}{r}\right)\right] = -\delta(\vec{r}_N-\vec{r}). \quad (3)$$

With the introduction of (2), the matrix element

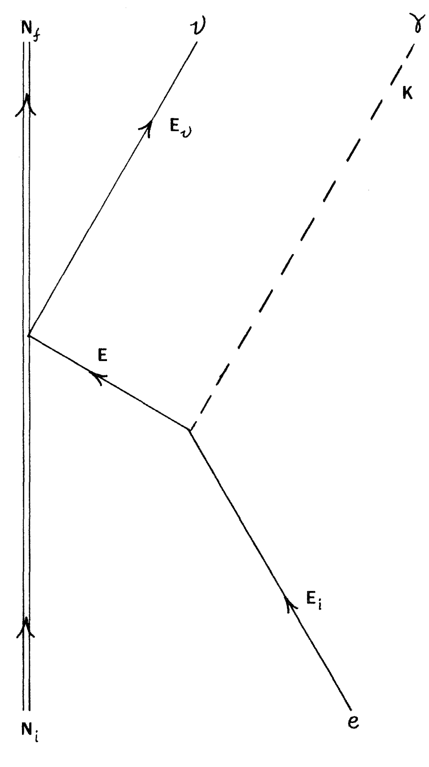

FIG. 1. Feynman diagram for radiative electron capture.

(1) lends itself to considerable simplification after which, for allowed transitions, it may be written in the following form

$$M = ieC_V\left(\frac{\pi}{kV}\right)^{1/2}\bar{\phi}_\nu(0)\mathcal{B}\int d\vec{r}\langle g_E(\vec{r}_N,\vec{r})\rangle_\Omega e^{-i\vec{k}\cdot\vec{r}}$$

$$\times(-2\vec{e}^*\cdot\vec{\nabla}+\vec{\Sigma}\cdot\vec{e}^*\times\vec{k}+ik\vec{\alpha}\cdot\vec{e}^*)\phi_i(\vec{r}), \qquad (4)$$

where $\mathcal{B}=B_\mu\Lambda_\mu$ with $B_j=i\lambda(\phi_f^N,\sigma_j\phi_i^N)$, $B_4=(\phi_f^N,\phi_i^N)$. Since $g_E(\vec{r}_N,\vec{r})$ is weakly singular at $\vec{r}_N=0$, it is necessary to take account of the fact that the β interaction takes place over a finite nuclear volume. This is most easily done by averaging g_E with respect to the variable $\vec{r}_N$ over the nuclear volume Ω. We have denoted this averaged Green's function by $\langle g_E(\vec{r}_N,\vec{r})\rangle_\Omega$.

To go further, one must specify which orbital electron undergoes the radiative-capture process. As is well known from the work of Glauber and Martin, only radiative capture from s states is significant at higher photon energies, with $1s$-state capture making the predominant contribution to the IB spectrum. However, at energies below $k\approx Z\alpha$, the photon spectrum associated with radiative capture from p states becomes very intense and it is $2p$-state capture which largely determines the low-energy portion of the photon spectrum. Consequently, to determine the angular distribution of the radiation accurately, we shall take into consideration radiative capture from both $1s$ and $2s$ states as well as from $2p$ and $3p$ states. Contributions to the photon spectrum due to capture from other orbital states are then expected to be negligible.[13]

Relativistic effects will be most important for the capture of $1s$ electrons, since they have the greatest probability of being found in the neighborhood of the nucleus. To a lesser extent, this will also be true for the capture of $2s$ electrons. Thus, for $1s$-state capture it is important to maintain an exact relativistic treatment, while for $2s$-state capture it is sufficient to include relativistic effects to a relative accuracy of order $Z\alpha$. This approximation is further strengthened by the fact that the $2s$-state spectrum is much less intense than that of the $1s$ state. In the case of p-state capture, relativistic effects are quite unimportant and a nonrelativistic treatment is sufficient. Indeed, in this case it is screening which contributes the most important corrections to the nonrelativistic results. (Screening effects are discussed at the end of Sec. III.) On the basis of these considerations, we now evaluate the transition rate and the IB angular-distribution function for radiative capture from s and p states.

III. TRANSITION RATE AND IB ANGULAR-DISTRIBUTION FUNCTION

A. 1s-State Radiative Capture

In order to compute the matrix element for radiative capture from a $1s$ state it is only necessary to insert the appropriate relativistic forms for ϕ_{1s} and $\langle g\rangle_\Omega$ into (4) and carry out the indicated integration. The form of ϕ_{1s} is well known, while the averaged second-order Green's function $\langle g_E(\vec{r}_N,\vec{r})\rangle_\Omega$ has been constructed in MG from the solution of (3). Furthermore, these authors have already carried out the required integration and shown that after considerable algebraic reduction it leads to the following result for the radiative K-capture matrix element:

$$M_{1s}=\frac{eC_V}{2k}\left(\frac{\pi}{kV}\right)^{1/2}\langle\phi_{1s}\rangle_\Omega\bar{\phi}_\nu(0)\mathcal{B}$$

$$\times[iA_{1s}(k)\vec{\Sigma}\cdot\vec{e}^*\times\vec{k}-kB_{1s}(k)\vec{\alpha}\cdot\vec{e}^*]\begin{pmatrix}\chi_{1s}\\0\end{pmatrix}. \qquad (5)$$

Here, χ_{1s} is a Pauli spinor describing the spin state of the $1s$ electron. The quantities A_{1s} and B_{1s} refer to certain integrals whose definitions may be found in the Appendix [Eqs. (A1)]. At this point we note only that they are functions of Z and k and leave a discussion of their detailed evaluation for the Appendix.

The transition rate for radiative K capture is related to M_{1s} by

$$dw_{1s}=2\pi\int\sum|M_{1s}|^2\delta(Q-k-E_\nu)\frac{d\vec{k}V}{(2\pi)^3}\frac{d\vec{P}_\nu V}{(2\pi)^3}, \qquad (6)$$

where Q is the energy difference between the parent and (excited) daughter atom. After the integration over all possible neutrino momenta and the summations over all possible spin states of the neutrino, photon, and $1s$ electron have been carried out, it may be written in the form

$$\frac{dw_{1s}}{d\vec{k}}=\frac{\alpha C_V^2}{(2\pi)^3k^2}\langle\phi_{1s}\rangle_\Omega^2(Q-k)^2$$

$$\times[k(A_{1s}^2+B_{1s}^2)(\vec{B}\cdot\vec{B}^*+B_4B_4^*)+2A_{1s}B_{1s}$$

$$\times i\vec{B}\times\vec{B}^*\cdot\vec{k}+4A_{1s}B_{1s}\,\mathrm{Re}(iB_4\vec{B}^*\cdot\vec{k})]. \qquad (7)$$

Since we are interested in situations where the initial nucleus is polarized, it only remains to sum the transition rate over all angular momentum magnetic substates of the final nucleus. To this end we introduce reduced matrix elements $\langle f\|1\|i\rangle$ and $\langle f\|\vec{\sigma}\|i\rangle$ through the Wigner-Eckart theorem, according to which

$$B^{(\mu)}\equiv\langle\alpha'J'M'|\sigma^{(\mu)}|\alpha JM\rangle=C(J1J';M\mu M')\langle f\|\vec{\sigma}\|i\rangle,$$

with $\mu = 1, 0, -1$ denoting the spherical components of $\vec{B}$ and

$$B_4 = \langle \alpha' J' M' | \alpha J M \rangle = C(J0J'; M0M') \langle f \| 1 \| i \rangle .$$

J, M and J', M' denote the angular momentum quantum numbers of the initial and final nuclear states, respectively, while α, α' represent all additional quantum numbers associated with these states. Using the techniques of Wigner algebra, we find after a short calculation:

$$\sum_M \vec{B} \cdot \vec{B}^* = \lambda^2 \left(\frac{2J'+1}{2J+1}\right)^{1/2} (-1)^{J-J'} |\langle f \| \vec{\sigma} \| i \rangle|^2 , \quad (8a)$$

$$\sum_{M'} B_4 B_4{}^* = |\langle f \| 1 \| i \rangle|^2 , \quad (8b)$$

$$\sum_{M'} \vec{B} \times \vec{B}^* \cdot \vec{k} = -i \vec{\mathcal{P}}_M \cdot \vec{k} N_J \sum_{M'} \vec{B} \cdot \vec{B}^* , \quad (8c)$$

$$\sum_{M'} B_4 \vec{B}^* \cdot \vec{k} = \frac{-i\lambda J}{[J(J+1)]^{1/2}} \vec{\mathcal{P}}_M \cdot \vec{k} \langle f \| 1 \| i \rangle \langle f \| \vec{\sigma} \| i \rangle^* , \quad (8d)$$

where $\vec{\mathcal{P}}_M \equiv \langle \alpha J M | \vec{J} | \alpha J M \rangle / J$ is the polarization vector of the initial nuclear state and

$$N_J = -J/(J+1) \quad \text{if} \quad J' = J+1 ,$$
$$= 1/(J+1) \quad \text{if} \quad J' = J ,$$
$$= 1 \quad \text{if} \quad J' = J-1 .$$

Using these results we can write the transition rate for radiative K capture in the final form

$$\frac{dw_{1s}}{d\vec{k}} = w_K \frac{I_{1s}}{4\pi k^2} W_{1s}(\theta) , \quad (9)$$

where w_K is the transition rate for ordinary K capture and is given by

$$w_K = \frac{C_V{}^2}{\pi} \langle \phi_{1s} \rangle_\Omega{}^2 Q^2 \sum_{M'} (\vec{B} \cdot \vec{B}^* + B_4 B_4{}^*) . \quad (10)$$

Here, I_{1s} is the relative intensity of the $1s$-state IB spectrum and is given by

$$I_{1s} = \frac{\alpha}{2\pi} (A_{1s}{}^2 + B_{1s}{}^2) k \left(\frac{Q-k}{Q}\right)^2 , \quad (11)$$

and $W_{1s}(\theta)$ is the $1s$-state IB angular-distribution function and is given by

$$W_{1s}(\theta) = 1 + \alpha_{1s} a_K \mathcal{P}_M \cos\theta , \quad (12)$$

where $\mathcal{P}_M \equiv |\vec{\mathcal{P}}_M|$ and θ is the angle between $\vec{k}$ and the direction of nuclear polarization. For a pure Fermi transition $a_K = 0$, while for a pure Gamow-Teller transition $a_K = N_J$. Otherwise,

$$a_K = \left[\frac{\lambda^2}{(J+1)} |R|^2 + \frac{\lambda J (R + R^*)}{[J(J+1)]^{1/2}}\right] (1 + \lambda^2 |R|^2)^{-1} ,$$

where $R \equiv \langle f \| \vec{\sigma} \| i \rangle / \langle f \| 1 \| i \rangle$. The function $\alpha_{1s}(k)$ is defined by

$$\alpha_{1s}(k) = 2A_{1s} B_{1s} / (A_{1s}{}^2 + B_{1s}{}^2) \quad (13)$$

and describes the energy dependence of the asymmetry coefficient associated with radiative capture from $1s$ states. Using the results of the Appendix, we have evaluated A_{1s}, B_{1s}, and α_{1s} for ^{119}Sb which corresponds to $Z = 51$. The results are shown in Fig. 2.

B. 2s-State Radiative Capture

As has been stated, in calculating the matrix element for radiative capture from $2s$ states it is sufficient to retain a relative accuracy of order $Z\alpha$. Normally, a nonrelativistic approximation is adequate for this purpose. However, for the low-energy portion of s-state spectra, the factor of $Z\alpha$ is partially compensated by an increased probability of radiation.[14] As a result, it is necessary to work to the next order in $Z\alpha$ and omit only those resulting terms which are clearly of order $Z\alpha$. It

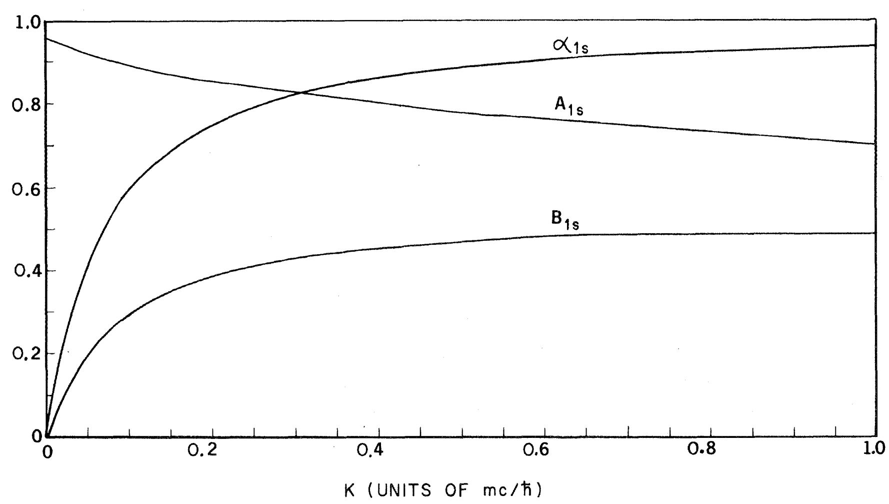

FIG. 2. A plot of the $1s$-state-capture asymmetry function $\alpha_{1s}(k)$ for ^{119}Sb ($Z = 51$). The associated amplitude functions $A_{1s}(k)$ and $B_{1s}(k)$ are also shown.

has been shown in GM that this may be conveniently accomplished by first introducing the Foldy-Wouthuysen transformation

$$g_E(\vec{r}_N, \vec{r}) = e^{-i\vec{\alpha}\cdot\vec{\nabla}_N/2E} g'_E(\vec{r}_N, \vec{r})\, e^{i\vec{\alpha}\cdot\vec{\nabla}/2E}, \tag{14a}$$

$$\phi_{2s}(\vec{r}) = e^{i\beta\vec{\alpha}\cdot\vec{\nabla}/2}\phi'_{2s}(\vec{r}). \tag{14b}$$

After neglecting terms of order $(Z\alpha)^2$, we find that $\phi'_{2s}(\vec{r})$ satisfies the ordinary nonrelativistic Coulomb-Schrödinger equation, while the nonrelativistic Green's function $g'_E(\vec{r}_N, \vec{r})$ satisfies

$$g'_E(\vec{r}_N, \vec{r})(\nabla^2 + E^2 - 1 + 2EZ\alpha/r) = -\delta(\vec{r}_N - \vec{r}). \tag{15}$$

The solution of this equation, valid for small $\vec{r}_N$, has been constructed in GM, and with it these authors have evaluated the $2s$-state-capture matrix element to a relative accuracy of order $Z\alpha$. The final result of their calculation is

$$M_{2s} = \frac{ieC_V}{2k}\left(\frac{\pi}{kV}\right)^{1/2}\bar{\phi}_\nu(0)\mathcal{B} \times[\vec{\Sigma}\cdot\vec{e}^*\times\vec{k} + ikB_{2s}(k)\vec{\alpha}\cdot\vec{e}^*]\phi'_{2s}(0), \tag{16}$$

with the function $B_{2s}(k)$ defined by (A3). The transition rate is then determined as before, with the final result

$$\frac{dw_{2s}}{d\vec{k}} = w_K\frac{I_{2s}}{4\pi k^2}W_{2s}(\theta), \tag{17}$$

where the relative intensity of the $2s$-state IB spectrum is given by

$$I_{2s} = \frac{\alpha k}{8\pi}R_{2s}\left(\frac{E_{2s} - E_{1s} + Q - k}{Q}\right)^2, \tag{18}$$

with the relativistic correction factor R_{2s} defined by

$$R_{2s} = \tfrac{1}{2}(1 + B_{2s}{}^2). \tag{19}$$

The $2s$-state IB angular-distribution function has the form

$$W_{2s}(\theta) = 1 + \alpha_{2s}a_K\mathcal{P}_M\cos\theta, \tag{20}$$

with the function $\alpha_{2s}(k)$ defined by

$$\alpha_{2s}(k) = B_{2s}/R_{2s} \tag{21}$$

describing the energy dependence of the asymmetry coefficient associated with $2s$-state radiative capture. Using the results of the Appendix, we have also evaluated B_{2s}, R_{2s}, and α_{2s} for ^{119}Sb obtaining the results shown in Fig. 3.

C. *p*-State Radiative Capture

In the calculation of radiative capture from p states, it is sufficient to work in the nonrelativistic limit. Furthermore, as has been shown in GM, there is appreciable p-state radiation only at low photon energies where $k \lesssim Z\alpha$. Retaining a relative accuracy of order $Z\alpha$, we may, under these circumstances, neglect the retardation factor in the matrix element for p-state capture, after which the contributions from the $\vec{\Sigma}\cdot\vec{e}^*\times\vec{k}$ and $ik\vec{\alpha}\cdot\vec{e}^*$ terms vanish by orthogonality and the matrix element reduces to

$$M_{np} = -2ieC_V\left(\frac{\pi}{kV}\right)^{1/2}\bar{\phi}_\nu(0)\mathcal{B}\int d\vec{r}\, g'_E(0, r)\vec{e}^*\cdot\vec{\nabla}\phi'_{np}(\vec{r}), \tag{22}$$

from which it is clear that the IB associated with p-state capture is distributed isotropically. Using the spherically symmetric solution of (15), Glauber and Martin have evaluated the p-state-radiative-capture matrix element to a relative accuracy of order $Z\alpha$ for both $2p$ and $3p$ states. Once again we quote only their final result

$$M_{np} = -ieC_V\left(\frac{2Z\alpha}{kV}\right)^{1/2}\bar{\phi}_\nu(0)\mathcal{B}\begin{pmatrix}\chi_{np}\\0\end{pmatrix}Q_{np}, \tag{23}$$

where the integrals Q_{np} depend only on Z and k and are defined by (A6) of the Appendix. Using this result, we find the transition rate to have the final form

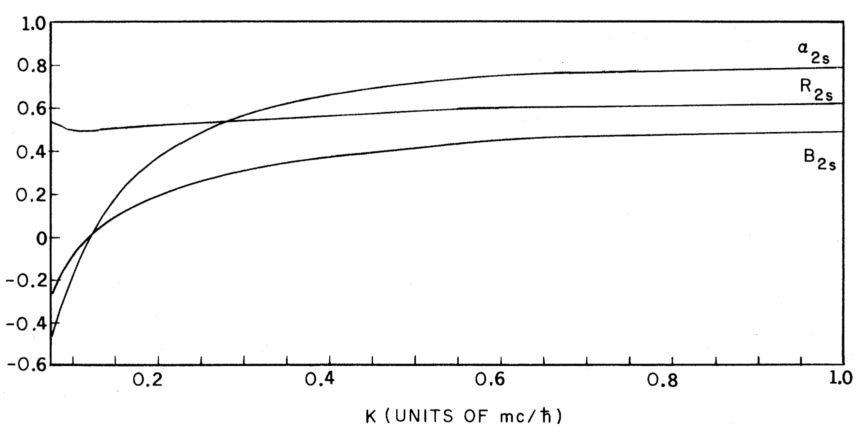

FIG. 3. A plot of the $2s$-state-capture asymmetry function $\alpha_{2s}(k)$ for ^{119}Sb. The associated amplitude function $B_{2s}(k)$ and the relativistic correction factor $R_{2s}(k)$ are also shown.

$$dw_{np}/d\vec{k} = w_K I_{np}/4\pi k^2, \tag{24}$$

where the relative intensity of the np-state IB spectrum is given by

$$I_{np} = \frac{4k}{\pi Z^2 \alpha} Q_{np}{}^2 \left(\frac{E_{np} - E_{1s} + Q - k}{Q}\right)^2. \tag{25}$$

The previous calculations have, of course, ignored the influence of screening on the radiative-capture transition rates. These effects are expected to be particularly important for p-state capture. To take account of screening effects in a simple way, we shall multiply the unscreened transition rates by the factors

$$S_i = |\phi_i^{sc}(0)|^2 / |\phi_i(0)|^2, \tag{26}$$

where ϕ_i^{sc} and ϕ_i are the screened and unscreened wave functions for the initial electron state. The justification for this procedure is given in MG, where graphs of $S_i(Z)$ for $i = 1s, 2s, 2p, 3p$ may also be found.

IV. RESULTS AND DISCUSSION

The preceding sections have been devoted to the detailed determination of the angular-distribution functions for s-state and p-state radiative capture from polarized nuclei. Whereas the p-state radiation is distributed isotropically, the s-state radiation shows a marked angular asymmetry in its distribution. In the case of radiative capture from 1s states, relativistic and Coulomb effects have been included in both initial and intermediate electron states exactly, while in our treatment of 2s-state radiative capture we have included such effects everywhere to a relative accuracy of order $Z\alpha$. These results show that for ns-state radiative capture the angular-distribution function has the form

$$W_{ns}(\theta) = 1 + \alpha_{ns}(k) a_K \mathcal{P}_M \cos\theta, \tag{27}$$

with the energy-dependent asymmetry functions $\alpha_{1s}(k)$ and $\alpha_{2s}(k)$ defined by (13) and (21), respectively.

Although the evaluation of these asymmetry functions must be carried out numerically in general, in the low- and high-energy limits it is possible to evaluate them analytically. In the case of $\alpha_{1s}(k)$ we need only note the following results of MG: in the neighborhood of $k = 0$, $A_{1s} = \frac{1}{3}(2\lambda + 1)(1 - k)$ and $B_{1s} = 0 + \mathcal{O}(k/Z\alpha)$, while at high energies ($k \sim 1$), A_{1s} and B_{1s} differ from unity by terms of order $Z\alpha$, the actual expressions being given by Eqs. (4.5) of MG. Using these results, one easily finds that $\alpha_{1s} \to 0$ as $k/Z\alpha$ as $k \to 0$, while in the high-energy region

$$\alpha_{1s}(k) = 1 - (Z\alpha)^2[\mu + 2(1 - k)\tan^{-1}(k/\mu)]^2/2k^4.$$

Indeed, one expects that, in general, $\alpha_{ns} = 1 - \mathcal{O}(Z\alpha)^2$ in the high-energy limit, since the only effect of changing the radial wave function of the capturing state will be a change in the coefficient of the $(Z\alpha)^2$ term. In particular, we expect $\alpha_{2s}(k)$ to be of this form at high energies. However, since our calculations for 2s-state radiative capture have retained only a relative accuracy of order $Z\alpha$, the results are not adequate for a determination of the coefficient of the $(Z\alpha)^2$ term. However, the low-energy limit of $\alpha_{2s}(k)$ can be easily obtained, since it has already been shown in GM that $B_{2s}(0) = -\frac{3}{2}$. From this result it immediately follows that $\alpha_{2s}(0) = -\frac{12}{13}$. In fact, one may state quite generally that $\alpha_{ns}(0) \neq 0$ for $n \geq 2$.

It is clear from Figs. 2 and 3 that, while at higher photon energies the asymmetry coefficients are approximately energy independent, for photon energies below $k \approx Z\alpha$ these asymmetry coefficients show a marked dependence on photon energy. This is in sharp contrast to what has been previously believed.

Previous calculations of the asymmetry coefficients for allowed K-capture transitions have been reported by Timashev and Kaminskii[8] and by Koh, Miyatake, and Watanabe.[9] The former authors give no details of their work and state only that it was carried out "according to the usual method of the Born approximation in the Coulomb field of the nucleus." The calculation of the latter authors is similar to that of the present paper but differs from it in that nonrelativistic forms are used throughout to describe the electron's motion, and all Coulomb effects are neglected in the intermediate electron state. Presumably, this is what Timashev and Kaminskii also have done. In any event, both groups find that the angular distribution of the IB accompanying K capture is described by a distribution function of the form (27), but with $\alpha_{1s} = 1$. Indeed, it is carefully pointed out in each of the papers that the respective calculations show the asymmetry coefficient to be independent of photon energy, although Koh, Miyatake, and Watanabe do remark that a slight energy dependence may appear because of relativistic and Coulomb corrections.

Whereas the IB energy spectrum has been measured for a number of allowed transitions, where it has shown rather satisfactory agreement with the Glauber and Martin theory both with respect to partial as well as total rates, it was only recently that Brewer and Shirley[10] reported the first observations on the IB angular distribution. These investigators measured the forward-backward asymmetry of the IB accompanying electron capture in polarized ^{119}Sb nuclei. Assuming an angular-distribution function of the form $W(\theta) = 1 + A(k)Q_1 \mathbf{P}\cos\theta$, where $A(k)$ is the over-all asymmetry coefficient, Q_1 is a correction for the finite solid angle of their detector, and $\mathcal{P}$ is the temperature-dependent average nuclear polarization, they were able to determine $W(0)$ as a function of photon energy, obtaining the results shown in Fig. 3 of their paper. However, as these authors point out, their results are subject to correction for effects due to the scattering of photons by the polarizing magnet. These effects tend to attenuate the anisotropy and can be accounted for through the introduction of an attenuation coefficient $\beta(k)$ in the angular-distribution function: $W(\theta)=1+\beta(k)A(k)Q_1\mathcal{P}\cos\theta$. Brewer[15] has made estimates for $\beta(k)$ assuming Klein-Nishina scattering from the known geometry of his experiment, and with them he has obtained the lower dashed curve shown in Fig. 3 of Ref. 10. To appreciate the importance of the attenuation coefficient, this curve should be compared with the upper dashed curve in the same figure which assumes no attenuation. As expected, scattering becomes more important as the photon energy decreases. Indeed, $\beta(k)$ is only about 0.7 at the lowest photon energies observed.

Since the attenuation coefficient depends on the experimental arrangement, it would seem more appropriate, for the purpose of comparing theory with experiment, to examine $A(k)$ rather than $W(0)$ as Brewer and Shirley have done. The estimated values for $\beta(k)$ as well as the experimental values for $A(k)$, corrected for scattering, may be deduced from Fig. 3 of Ref. 10. In Fig. 4 we have plotted the experimental values for $A(k)$ so obtained.

Since the experiment of Brewer and Shirley does not differentiate between the various initial states from which the electron may be captured, their results refer to the over-all angular-distribution function obtained by summing the partial rates over all initial states. Assuming that only capture from 1s, 2s, 2p, and 3p states is appreciable and including screening effects, we obtain

$$\frac{dw}{d\vec{k}}=\frac{dw_{1s}}{d\vec{k}}+\frac{dw_{2s}}{d\vec{k}}+\frac{dw_{2p}}{d\vec{k}}+\frac{dw_{3p}}{d\vec{k}}=\frac{w_K I W(\theta)}{4\pi k^2}, \qquad (28)$$

where $I=I_{1s}S_{1s}+I_{2s}S_{2s}+I_{2p}S_{2p}+I_{3p}S_{3p}$ is the total relative intensity of the IB, and the over-all angular-distribution function is given by

$$W(\theta)=1+A(k)\mathcal{P}_M\cos\theta, \qquad (29)$$

with the over-all asymmetry coefficient $A(k)$ defined by

$$A(k)=a_K(I_{1s}S_{1s}\alpha_{1s}+I_{2s}S_{2s}\alpha_{2s})/I. \qquad (30)$$

The electron-capture decay of ^{119}Sb is a transition for which $J'=J-1$, in which case $a_K=1$. Using the result of the Appendix and screening factors taken from Fig. 3 of MG we have numerically evaluated $A(k)$, as defined by (30), for this case obtaining the solid curve shown in Fig. 4. For comparison we have also plotted the over-all asymmetry coefficient predicted by the simpler theory in which α_{1s} and α_{2s} are taken to have the value 1. Under these circumstances, one needs only to know the s-state intensity and the over-all intensity of the IB spectrum to determine $A(k)$. These intensities have been measured for ^{119}Sb by Olsen, Mann, and Lindner,[16] who report results which agree well with the Glauber and Martin theory, i.e., with the intensities used in the present paper. Using the results

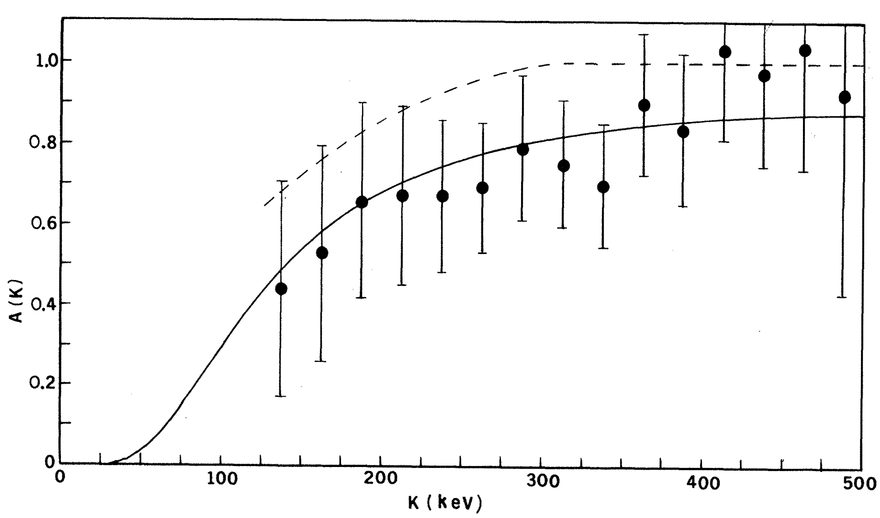

FIG. 4. Over-all asymmetry coefficient $A(k)$ for ^{119}Sb. The experimental points represent the observations of Brewer and Shirley. The solid curve is the result predicted by the present theory, while the dashed curve is the result predicted by the simpler theory for which $\alpha_{1s}=\alpha_{2s}=1$.

of these workers, one obtains[17] for $A(k)$ the dashed curve shown in Fig. 4.

At this point it is worth pointing out what the theoretical curves would look like for K capture alone. In this case, (30) reduces to $A(k) = a_K\alpha_{1s}$ so that the dashed line in Fig. 4 would become the horizontal line $A(k) = 1$, while the solid curve would become $A(k) = \alpha_{1s}(k)$ with $\alpha_{1s}(k)$ as shown in Fig. 2. Thus the curvature of the dashed line is due solely to the inclusion of higher-shell contributions, whereas the solid curve still goes to zero at $k = 0$ for K capture alone.

From this we may conclude that relativistic and Coulomb effects are indeed important not only for the determination of the intensity of the IB spectrum, as Glauber and Martin have shown, but also for the determination of the angular distribution of the IB, particularly at low photon energies. Relativistic and Coulomb corrections enter into this determination in two ways, first through their influence on the s-state intensities, and secondly through their influence on the asymmetry parameters α_{ns} associated with radiative capture from ns states. From the present work it is clear that an accurate determination of the over-all asymmetry coefficient requires that both modes of influence be taken into account.

Detailed experimental studies on the angular distribution of the IB accompanying electron capture are difficult to carry out, and therefore it is not surprising that a first determination of the angular asymmetry involves appreciable experimental errors. Although the results of Fig. 4 indicate that the experimental results of Brewer and Shirley appear to be in much better agreement with the present theory than with the simpler theory which assumes $\alpha_{ns} = 1$, the rather appreciable experimental errors make the results less than conclusive. Indeed, the practical distinction between the two theories lies not in their k dependence but in their magnitudes; the difference of ~15% being of the same order as the experimental uncertainties.

In view of this, further experimental work on allowed transitions would be of value in providing a more rigorous test of the present theory. In particular, anisotropy studies involving the detection of photons of the continuous spectrum in coincidence with characteristic K-shell x rays would make it possible to determine α_{1s} itself. It is hoped that the present calculations will lend encouragement to such studies.

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APPENDIX

In this Appendix we discuss the evaluation of the various integrals, depending parametrically on Z and k, which appear in the transition rates and IB angular-distribution functions. First we consider the functions $A_{1s}(k)$ and $B_{1s}(k)$ which occur in the rate for $1s$-state capture. They were first introduced in MG and are defined by

$$A_{1s}(k) = \frac{(\lambda+1)k}{\Gamma(2\lambda+1)\mu}\int_0^\infty dr\int_0^\infty ds\left\{j_0(kr)\left[1+\frac{a^2}{3(\lambda+1)^2}\right] - \frac{j_1(kr)}{kr}\,\frac{2a^2}{(\lambda+1)^2} - \frac{j_2(kr)2a^2}{3(\lambda+1)^2}\right\}$$
$$\times s^{-\eta+\lambda-1}(1+s)^{\eta+\lambda-1}(2\mu r)^{2\lambda}e^{-\mu r(2s+1)}e^{-ar} \tag{A1a}$$

and

$$B_{1s}(k) = \frac{(\lambda+1)k}{\Gamma(2\lambda+1)\mu}\int_0^\infty dr\int_0^\infty ds\left\{j_0(kr)\left[1-\frac{4a}{3(\lambda+1)}\left(\frac{\lambda}{kr}-\frac{a}{k}\right)+\frac{a^2}{3(\lambda+1)^2}\right]\right.$$
$$\left.+j_2(kr)\left[\frac{4a}{3(\lambda+1)}\left(\frac{a}{k}+\frac{(3-2\lambda)}{2kr}\right)-\frac{2a^2}{3(\lambda+1)^2}\right]\right\}s^{-\eta+\lambda-1}(1+s)^{\eta+\lambda-1}(2\mu r)^{2\lambda}e^{-\mu r(1+2s)}e^{-ar}, \tag{A1b}$$

in which the following general definitions have been used: $a = Z\alpha$, $\lambda = (1-a^2)^{1/2}$, $\mu = (1-E^2)^{1/2}$, $\eta = aE/\mu$. For $1s$-state capture in particular, $E = E_{1s} - k$ with $E_{1s} = \lambda$. To simplify these expressions we first carry out the r integration using elementary methods and introduce the change of variable $x = s/(1+s)$ into the remaining integrals. After algebraic reduction, we obtain the following forms:

$$A_{1s} = 2C\int_0^1 dx\, x^{-\eta+\lambda-1}f_A(x), \tag{A2a}$$

$$B_{1s}=\frac{C}{k(1-\lambda)}\int_0^1 dx\,x^{-\eta+\lambda-1}f_B(x)\,, \tag{A2b}$$

in the writing of which we have made use of the definitions

$$C=-\frac{(2\mu)^{2\lambda-1}}{\lambda(2\lambda-1)k^2},\quad \Sigma=k^2+(a+\mu)^2,$$

$$\epsilon=2\frac{\mu^2-a^2-k^2}{\Sigma},\quad \delta=\frac{k^2+(a-\mu)^2}{\Sigma},$$

$$S=\Sigma(1+\epsilon x+\delta x^2),\quad \sigma=a+\frac{\mu(1+x)}{(1-x)},$$

$$\theta=\tan^{-1}\frac{k}{\sigma},$$

in terms of which the functions appearing in the integrands are defined by

$$f_A(x)=[2k\lambda\sigma\cos(2\lambda\theta)-\sigma^2\sin(2\lambda\theta)]/S^\lambda,$$

$$\begin{aligned}f_B(x)=\big(&2k\lambda\sigma[a\sigma-2a^2+(1-\lambda)k]\cos(2\lambda\theta)\\&+\{k^2(2\lambda-1)[k(\lambda-1)-2a^2+a\sigma]\\&+\sigma^2[2a^2-k(1-\lambda)-a\sigma]\}\sin(2\lambda\theta)\big)/S^\lambda.\end{aligned}$$

Unfortunately, the remaining integration in (A2) cannot be carried out analytically. However, inspection shows that f_A and f_B are very slowly varying functions of x over the entire range of integration for all values of the photon energy of interest. Thus, after an integration by parts to remove the weak singularity in the integrand at $x=0$, A_{1s} and B_{1s} have forms which may be easily evaluated numerically with the aid of any high-speed computer.

The function $B_{2s}(k)$, which appears in the rate for $2s$-state capture, is defined by

$$B_{2s}(k)=1+\frac{2}{3\phi_{2s}'(0)}\int d\vec{r}\,g_E^{(1)}(0,r)r\frac{d}{dr}\phi_{2s}'(r)\,, \tag{A3}$$

where, following GM, we have introduced the p-wave part of the nonrelativistic Green's function $g_E^{(1)}(0,r)$, which may be defined by the representation

$$g_E^{(1)}(0,r)=\frac{\mu^3}{\pi}e^{-\mu r}\int_0^\infty e^{-2\mu rs}s^{1-\eta}(1+s)^{1+\eta}ds \tag{A4}$$

for $\eta<2$. Here, μ and η are defined as before, except that now $E=E_{2S}-k$ with $E_{2s}=1-a^2/8$. With the introduction of (A4) and the standard nonrelativistic hydrogenic form for ϕ_{2s}', the $\vec{r}$ integration in (A3) is easily completed. After transforming the remaining integration variable as before, we arrive at the following result:

$$B_{2s}(k)=1-\frac{8a^2\mu^3}{\phi_{2s}'(0)}\left(\frac{a}{2\pi}\right)^{1/2}\frac{1}{(\mu+\frac{1}{2}a)^4}\int_0^1 dx\frac{x^{1-\eta}(\Delta+x)}{(1+\Delta x)^5}, \tag{A5}$$

where $\Delta=(\mu-\frac{1}{2}a)/(\mu+\frac{1}{2}a)$. Once again, the remaining integral is easily evaluated numerically by computer.

The functions $Q_{np}(k)$, which help determine the intensities of the IB spectrum for p-state capture, are defined by

$$Q_{np}(k)=\left(\frac{2\pi}{a}\right)^{1/2}\int d\vec{r}\,g_{E_{np}-k}'(0,r)\vec{e}^*\cdot\vec{\nabla}\phi_{np}'(\vec{r})\,. \tag{A6}$$

The nonrelativistic Green's function $g_E'(0,r)$ is the spherically symmetric solution of (15) and has the well-known representation

$$g_E'(0,r)=\frac{\mu}{2\pi}e^{-\mu r}\int_0^\infty e^{-2\mu rs}(1+s)^\eta s^{-\eta}ds\,. \tag{A7}$$

With this form and the standard nonrelativistic form for ϕ_{np}', the $\vec{r}$ integration in (A6) is easily completed. In particular, for the $2p$ and $3p$ states we obtain, after transforming the remaining integration variable,

$$Q_{2p}(k)=\frac{\eta_2^2}{(1+\eta_2/2)^4}\int_0^1 dx\frac{x^{-\eta_2}(1-x^2)}{(1+\lambda_2x)^4}, \tag{A8}$$

with $\eta_2=a/[2k+\frac{1}{4}a^2]^{1/2}$ and $\lambda_2=(2-\eta_2)/(2+\eta_2)$, and

$$Q_{3p}(k)=\frac{16\eta_3^2}{27(1+\frac{1}{3}\eta_3)^4}\int_0^1 dx\frac{x^{-\eta_3}(1-x^2)(x+\lambda_3)}{(1+\lambda_3x)^5}, \tag{A9}$$

with $\eta_3=a/[2k+\frac{1}{9}a^2]^{1/2}$ and $\lambda_3=(3-\eta_3)/(3+\eta_3)$. A final integration by parts to remove the weak singularity at $x=0$ renders the remaining integrals suitable for computer evaluation.

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[1]P. Morrison and L. I. Schiff, Phys. Rev. 58, 24 (1940).

[2]L. Madansky and F. Rasetti, Phys. Rev. 94, 407 (1954).

[3]B. Saraf, Bull. Am. Phys. Soc. 29, 24 (1954).

[4]R. J. Glauber and P. C. Martin, Phys. Rev. 104, 158 (1956); P. C. Martin and R. J. Glauber, *ibid.* 109, 1307 (1958). These papers are hereafter referred to as GM and MG, respectively.

[5]A recent review of the subject is given by J. Zylicz, in *Proceedings of the Conference on Electron Capture and Higher Order Processes in Nuclear Decays, Debrecen, Hungary, 1968* (Eötvös Lóránd Physical Society, Budapest, 1968), Vol. 2, p. 123. See also the article by B. G. Pettersson, in *Alpha-, Beta-, and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, The Netherlands, 1965), p. 1575.

[6]R. E. Cutkosky, Phys. Rev. 107, 330 (1957).

[7]See Martin and Glauber, Ref. 4, Sec. 6.

[8]S. F. Timashev and V. A. Kaminskii, Zh. Eksperim. i Teor. Fiz. 38, 284 (1960) [transl.: Soviet Phys. – JETP

11, 206 (1960)].

[9]Y. Koh, O. Miyatake, and Y. Watanabe, Nucl. Phys. 32, 246 (1962).

[10]W. D. Brewer and D. A. Shirley, Phys. Rev. Letters 20, 885 (1968).

[11]We employ units in which $m = c = \hbar = 1$ (m is the electron mass), and $e^2 = \alpha = \frac{1}{137}$. In terms of the standard Dirac matrices $\vec{\alpha}$ and $\vec{\beta}$, the γ matrices are defined as $\vec{\gamma} = -i\beta\vec{\alpha}$, $\gamma_4 = \beta$, $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$. Also, $\vec{\Sigma} = -\gamma_5\vec{\alpha}$ and the Pauli adjoint is defined by $\overline{\phi} = \phi^+\gamma_4$. Our normalization volume is denoted by V.

[12]In the nuclear wave functions, the coordinates of all the nucleons except the one undergoing the capture process have been suppressed. It is to be understood that an integration over this coordinate implies an integration over the coordinates of the other nucleons as well. A summation over all the protons in the initial nucleus is also implied.

[13]It is assumed that $1s$-state capture is energetically allowed, so that our approximation amounts to including the two most important s-state contributions as well as the two most important p-state contributions. On this basis we neglect the very small contribution from $3s$-state capture. However, should $1s$-state capture be energetically forbidden, inclusion of this latter contribution would then be in order.

[14]See Glauber and Martin, Ref. 4, Sec. 9.

[15]W. D. Brewer, University of California Radiation Laboratory Report No. UCRL-19533, 1969 (unpublished).

[16]J. L. Olsen, L. G. Mann, and M. Lindner, Phys. Rev. 106, 985 (1957).

[17]W. D. Brewer, private communication.

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Proton-Proton Elastic Scattering from 9.6 to 13.6 MeV*

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Accurate differential-cross-section data for proton-proton elastic scattering are presented at 9.690, 9.918, and 13.600 MeV. These data agree with current energy-dependent phase-shift analyses and resolve discrepancies between previous sets of data and between those data and the phenomenological analysis.

I. INTRODUCTION

The importance of nucleon-nucleon scattering information to the understanding of the strong nuclear force can hardly be overestimated. Not only is it the most accessible process concerning the strong interaction between simple noncomposite nuclear particles, but it is greatly benefited by the large energy region free from strongly interacting inelastic channels. A detailed theoretical description of the scattering does not now exist, although there has been progress in the one-boson-exchange model.[1] A unique and complete phenomenological description is very desirable as a basis for a theoretical description and for the correlation of experiments, and, to a given accuracy, should eliminate the necessity for performing further experiments in the region of interest. In proton-proton scattering several energy-dependent phase-shift analyses[2,3] that give a good fit to all data up to 400 MeV have been published. Some of the remaining difficulty has been in the region below 10 MeV, where involved electromagnetic corrections[4] are necessary, partly because of the accuracy of the data. There has also been disagreement between sets of experimental data and difficulty in fitting some of the data without serious problems in the phenomenological method.

In particular, near 10 MeV the cross-section data of Johnston and Young[5] (Minnesota) at 9.69 MeV disagree markedly with the data at 9.918 MeV of Slobodrian *et al.*[6] (Berkeley) in both shape and absolute magnitude. Sher, Signell, and Heller[4] show that the central phase parameter, $9\Delta_C = \delta_{10} + 3\delta_{11} + 5\delta_{12}$ where the δ_{LJ}'s are the P-wave phases, extracted from the Berkeley and Minnesota data disagree by several standard deviations. MacGregor, Arndt, and Wright,[7] using an energy-dependent phase-shift analysis, have shown the Berkeley 9.918-MeV data to be inconsistent with the Berkeley data at 6.141 and 8.097 MeV when combined with other data at nearby energies. In addition, Holdeman, Signell, and Sher[8] (HSS) found the $^1S_0$ phase and the Δ_C parameter extracted from the Berkeley data incompatible with a reasonable phenomenological prediction, and that in order to make a fit, serious readjustment of fits to a number of well-accepted data at other energies would be necessary (see Figs. 2 and 3 of Ref. 8).

To help resolve these inconsistencies, we measured accurate differential cross sections at 9.690 and 9.918 MeV. These results were reported in an earlier Letter[9] and analyzed by HSS. They found that the angular shape of our data agreed with their