Nuclear fission viewed as a diffusion process: Case of very large friction

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Using a Fokker-Planck equation, we consider induced nuclear fission for large values of the friction constant. Then, the equation simplifies to the Smoluchowski equation which we solve using a method of van Kampen. We find that the fission rate obeys a scaling law. From our numerical results and those obtained previously, we estimate the transient time τ for fission. For small nuclear temperature, τ is defined as the time delay between the onset of a nuclear reaction and the attainment of the quasistationary probability flow over the fission barrier. For nuclear temperatures large compared to the fission barrier, the entire fission process is governed by transients, and τ essentially yields the fission lifetime. We speculate on the significance of our results for recent experimental observations.

I. INTRODUCTION

Recent experimental data[1] indicate that the number of light particles evaporated in a heavy-ion induced reaction prior to fission considerably exceeds the expectations based on the statistical model. One possible explanation for this observation is based on a diffusion model for the fission process.[2,3] It has long been known[4] that the quasistationary diffusive probability flow over the fission barrier yields a fission rate which is in essential agreement with the statistical model.[5] However, there exists a time delay τ between the beginning of the diffusion process (here, the onset of the nuclear reaction), and the attainment of stationarity of the probability flow. For times t with $0 \leq t \leq \tau$, very little probability escapes over the fission barrier, and nucleon evaporation competes favorably with nuclear fission. If τ is of the order of a nucleon evaporation time or bigger, the number of light particles evaporated prior to fission may obviously considerably exceed the prediction of the statistical model.

This approach, which essentially postulates that the equilibration time for the nucleonic degrees of freedom is considerably shorter than τ, has been quantitatively implemented in Ref. 3. It was found that at excitation energies of 80 or 100 MeV, and for values of the friction constant which appear to be generally acceptable, the time delay τ is of the order of a neutron evaporation time. This substantiates the picture described above.

The investigations and results of Ref. 3 are limited by a technical restriction. Both the analytical and the numerical techniques apply only when the friction constant β [which has the dimension $(\text{time})^{-1}$] is not much in excess of $2\omega_0$. Here, ω_0 is the frequency of a harmonic oscillator potential osculating the fission potential at the nuclear ground state (the first minimum). [The physical significance of the value $2\omega_0$ is that for $\beta < 2\omega_0$ (or $\beta > 2\omega_0$), the motion in the first minimum is underdamped (or overdamped, respectively)]. With a typical value of 1 MeV for $\hbar\omega_0$, this restricts the friction constant to $\beta \leq 5\times 10^{21}$ sec^{-1}.

The data of Ref. 1, particularly the most recent of these, would seem to require values of τ that are substantially bigger than those obtained in Ref. 3. In the framework of a diffusion model, such values of τ require β to exceed the limit just mentioned. This fact prompted us to carry out the present investigations. By applying a method of van Kampen[6] suitable for the overdamped case, $\beta > 2\omega_0$, we open up a domain of β values which partly overlaps that investigated in Ref. 3 and which considerably extends the range of β values accessible in our work.

Van Kampen's method consists in constructing an analytic solution to the Smoluchowski equation. In Sec. II, we recall which approximations are encountered in replacing the original Fokker-Planck equation by the Smoluchowski equation. We present van Kampen's solution, and discuss scaling relations which are implied by this method. The application of these scaling rules to the results of Ref. 3 alone is almost sufficient to extend the results of Ref. 3 to larger values of β. The numerical calculations of Sec. III substantiate and corroborate the analytical results. Moreover, and more importantly, they allow us to examine situations where the first minimum is very shallow or altogether nonexistent. (This is the case in heavy-ion reactions with large angular momentum.) A summary of our work, and a semiquantitative evaluation of the dependence of τ on β, on the nuclear temperature T, and on the height of the fission barrier, which also includes the results of Ref. 3, is given in Sec. IV.

II. THE SMOLUCHOWSKI EQUATION: SCALING RULES

We begin with the same Fokker-Planck equation as in Ref. 3. It reads

$$\frac{\partial}{\partial t}P(x,p;t)+p\frac{\partial}{\partial x}P(x,p;t)-K(x)\frac{\partial}{\partial p}P(x,p;t) =\beta\frac{\partial}{\partial p}[pP(x,p;t)]+\epsilon\frac{\partial^2}{\partial p^2}P(x,p;t) . \qquad (2.1)$$

Here, x is the fission variable, $\tilde{p}$ its conjugate momentum; $p=\tilde{p}/\mu$ the velocity with μ the reduced mass; $K(x)=+\mu^{-1}(dV/dx)$, where $V(x)$ is the fission potential; and $\epsilon=\beta T/\mu$.

Chandrasekhar[7] has reviewed the transition from the Fokker-Planck equation (2.1) to the Smoluchowski equation which describes dissipation in the x variable only, and which assumes that β is so large that equilibration in momentum space is very rapid. The Smoluchowski equation thus applies to an overdamped situation. It reads

$$\frac{\partial}{\partial t}P(x,t)=\beta^{-1}\frac{\partial}{\partial x}[K(x)P(x,t)]+\beta^{-2}\epsilon\frac{\partial^2}{\partial x^2}P(x,t) . \tag{2.2}$$

The quantity $P(x,t)$ is obtained by integrating $P(x,p;t)$ over all p space. The Smoluchowski equation follows from Eq. (2.1) if the conditions

$$\beta t \gg 1 \text{ and } \beta \gg (T/\mu)^{1/2}(\Delta x)^{-1} \tag{2.3}$$

are valid. The first of these implies that momentum equilibration is attained, while the second states that the diffusive length scale $(T/\mu)^{1/2}\beta^{-1}$ must be small compared to the distance Δx over which either $V(x)$ or $P(x,p;t)$ changes appreciably. With typical values $T\cong 3$ MeV and $\mu\cong 62m_p$ (with m_p the proton mass), and using $\Delta x\cong 1$ fm, the last condition (2.3) yields $\beta \gg 2\times 10^{21}$ $\sec^{-1}$. This condition actually follows from the requirement[7] that a Gaussian be small compared to unity and is effectively fulfilled with good accuracy already for $\beta \geq 5\times 10^{21}$ $\sec^{-1}$ or so. This shows that the domain of validity of Eq. (2.2) smoothly joins the domain $\beta \lesssim 5\times 10^{21}$ $\sec^{-1}$ explored in Ref. 3. The first condition (2.3) obviously gives a lower limit for the time t at which Eq. (2.1) gives meaningful results. It follows from the requirement[7] that $\exp-(\beta t)$ be small compared to unity and is thus effectively fulfilled with sufficient accuracy for $\beta t \gtrsim 3$.

Van Kampen's method[6] of solving Eq. (2.2) is based on the assumption that $V(x)\to +\infty$ for $|x|\to\infty$. The potential of a fissioning nucleus actually tends to zero for $x\to +\infty$. Without affecting the dynamics of the fission process we may, however, add a steep barrier to $V(x)$ for large x and thus fulfull van Kampen's assumption. Fission is then modeled as a diffusion process out of the shallow valley into the deep valley of an asymmetric bistable potential $V(x)$.

We use the substitution

$$P(x,t)=\exp[-V(x)/(2T)]\phi(x,t) .$$

Then, the Smoluchowski equation takes the form

$$\mu\beta\frac{\partial}{\partial t}\phi(x,t)=[\tfrac{1}{2}V''-\tfrac{1}{4}T^{-1}(V')^2]\phi(x,t)+T\frac{\partial^2}{\partial x^2}\phi(x,t) . \tag{2.4}$$

This suggests using the expansion

$$\phi(x,t)=\sum_n c_n\phi_n(x)\exp(-\lambda_n t) , \tag{2.5}$$

where the ϕ_n are the stationary orthonormal eigenfunctions of the problem

$$[E_n-U(x)]\phi_n(x)+\frac{d^2}{dx^2}\phi_n(x)=0 , \tag{2.6}$$

subject to the proper boundary conditions at $x=\pm\infty$. Here,

$$\mu\beta\lambda_n=T(E_n-\mathscr{C}) \tag{2.7}$$

and

$$TU(x)=\tfrac{1}{4}T^{-1}(V')^2-\tfrac{1}{2}V''+T\mathscr{C} , \tag{2.8}$$

where $\mathscr{C}$ is a constant.

Instead of solving Eq. (2.2), van Kampen proposes solving Eq. (2.6) for some suitable potential $U(x)$. This is straightforward because Eq. (2.6) has the form of a Schrödinger equation. The fission potential $V(x)$ is then to be found from Eq. (2.8). This procedure has the great advantage that $U(x)$ can be chosen in such a way that Eq. (2.6) has simple analytical solutions $\phi_n(x),E_n$. It has the disadvantage that the fission potential $V(x)$ determined by solving Eq. (2.8) may not exactly correspond to the actual physical situation.

To solve Eq. (2.8), we set

$$z(x)=\exp[-V(x)/(2T)] .$$

Then, Eq. (2.8) takes the form

$$\frac{d^2}{dx^2}z(x)+[\mathscr{C}-U(x)]z(x)=0 . \tag{2.9}$$

This is formally the same as Eq. (2.6). Moreover, we must have $z(x)\geq 0$ for all x by definition. This is only possible if $z(x)=\phi_0(x)$ and $\mathscr{C}=E_0$, the *lowest* eigenvalue. Collecting all these results, we have

$$P(x,t)=\phi_0(x)\sum_{n=0}^{\infty}C_n\phi_n(x)\exp(-\lambda_n t) . \tag{2.10}$$

The ϕ_n are the orthonormal eigenfunctions of the boundary-value problem (2.6). The λ_n are related to the associated eigenvalues E_n by

$$\lambda_n=(E_n-E_0)T/(\beta\mu);\quad n=0,1,\ldots, . \tag{2.11}$$

For arbitrary $U(x)$, the fission potential is given by

$$V(x)=-2T\ln\phi_0(x) . \tag{2.12}$$

The initial condition $P(x,t)=\delta(x-x_1)$ is realized if we set

$$C_n=\phi_n(x_1)/\phi_0(x_1) . \tag{2.13}$$

Equations (2.6) and (2.10)–(2.13) completely define the solutions in terms of a given auxiliary potential $U(x)$. For $U(x)$ fixed, these solutions have the following *scaling properties:*

(i) For fixed $U(x)$, the solutions $\phi_n(x)$ are completely independent of T and β.

(ii) For fixed $U(x)$ [and, therefore, for fixed $\phi_0(x)$], the fission potential $V(x)$ is linear in T [see Eq. (2.12)]. As a corollary, it follows that for fixed $U(x)$, the quantity $\exp(-E_f/T)$ (where E_f is the height of the fission barrier) is independent of T. This quantity is of central importance for the fission rate.

(iii) The parameters λ_n and, hence, the fission rate, depend on T and β only via the combination $T/(\beta\mu)$. The fission rate scales with this factor. This last fact is also seen directly when one rewrites Eq. (2.2) with the help of

Eq. (2.12) in the form

$$\frac{\partial}{\partial t}P(x,t)=-[2T/(\beta\mu)]\frac{\partial}{\partial x}[P(x,t)\phi_0'(x)/\phi_0(x)]+[T/(\beta\mu)]\frac{\partial^2}{\partial x^2}P(x,t) . \quad (2.14)$$

With $t'=tT/(\beta\mu)$, Eq. (2.14) depends only on t' and x, not on t, T, and β, separately.

The scaling properties (i)—(iii) apply to a situation where the auxiliary potential $U(x)$ or, equivalently, the function $\phi_0(x)$, is held fixed. This is the case for the numerical calculations presented in Sec. III.

Unfortunately, these observations cannot be applied directly to the results of Ref. 3. Indeed, in these calculations, the physical fission potential $V(x)$ [and *not* the auxiliary quantities $\phi_0(x)$ or $U(x)$] was held fixed, and β and T were changed. Since the relation (2.12) connecting $\phi_0(x)$ and $V(x)$ involves the temperature T, there is no simple way of telling how the results of a calculation will change with T, if $V(x)$ is held fixed.

The Smoluchowski equation (2.2) does, however, have a simple scaling property as β is changed. This is seen immediately from Eq. (2.2) if one recalls that $\epsilon=\beta T/\mu$: The time scales with β, and the rate, therefore, with β^{-1}. To apply this result, we recall that in Ref. 3, the "time-dependent fission rate" $\lambda_f(t)$ was defined as follows [x_0 is the coordinate of the saddle point]:

$$\lambda_f(t)=-\frac{d}{dt}\int_{-\infty}^{x_0}dx\int_{-\infty}^{\infty}dp\,P(x,p;t)\Big/\int_{-\infty}^{x_0}dx\int_{-\infty}^{\infty}dp\,P(x,p;t) . \quad (2.15)$$

It follows that for two solutions of the Smoluchowski equation (2.2) associated with different values β_0 and β of the friction constant, but subject to identical initial conditions (independent of β) at time $t=0$, we have

$$\lambda_f(t,\beta)=(\beta_0/\beta)\lambda_f(\beta t/\beta_0,\beta_0) . \quad (2.16a)$$

We also recall that in Ref. 3, τ was defined as the time span between the onset ($t=0$) of a nuclear reaction, and the time at which λ_f attains (approximately) its quasistationary value. The scaling law (2.16a) then implies that

$$\tau(\beta)=(\beta/\beta_0)\tau(\beta_0) . \quad (2.16b)$$

We use Eqs. (2.16) to generalize the results of Ref. 3. There, we found that for fixed T, and for $\beta\lesssim 2\omega_1$, the quantity τ decreases with increasing β roughly like $\beta^{-1}\ln(10E_f/T)$, where E_f is the height of the fission barrier. We now know that for $\beta\gg 2\omega_1$, τ is proportional to β. Looking at Figs. 4 and 5 of Ref. 3 (corresponding to temperatures of 1 and 4 MeV), we find that for $\beta=5$, the values of τ are approximately given by 8 and 2, respectively, in units 10^{-21} sec. This suggests that for $\beta\gg 2\omega_1$, the delay time τ is roughly inversely proportional to T. We emphasize, however, that this statement is not based on a scaling law and only supported by two data points. Nevertheless, we have fitted the τ values obtained in Ref. 3 with a formula consisting of a term proportional to $\beta^{-1}\ln(10E_f/T)$ and a second term proportional to β/T. The result of the fit is

$$\tau(\beta,T)=1.4\beta^{-1}\ln(E_f 10/T)+1.4(\beta/T)10^{-42}\ \text{MeV sec}^2 . \quad (2.17)$$

This formula gives a good fit to the six values of τ which one may deduce from Figs. 4 and 5 of Ref. 3 for $\beta=0.5$, 1.0, and 5.0×10^{21} sec^{-1}, as well as for the totally different case of $T=1$ MeV of Fig. 6. We surmise that Eq. (2.17) gives a reasonable approximation to the delay time in all physical situations where $T\lesssim E_f$. It does not work in cases where $E_f<T$, where the fission process is altogether characterized by transients. This regime, which is of interest experimentally in heavy-ion induced reactions with sufficiently high angular momentum, must be explored by numerical calculations to which we now turn.

III. NUMERICAL RESULTS

For the numerical calculations, the auxiliary potential was chosen as follows:

$$U(x)=\begin{cases}+\infty & x\le 0,\\ 0 & 0<x\le a,\\ U_0>0 & a<x\le b,\\ 0 & b<x\le c,\\ +\infty & x>c.\end{cases} \quad (3.1)$$

The solutions of the associated eigenvalue problem (2.6) are trigonometric functions or exponentials, and the eigenvalues are solutions of a simple transcendental equation. All this is elementary quantum mechanics; the formulas are not given here. It is of interest, however, to discuss the qualitative features of the eigenfunction ϕ_0 belonging to the lowest eigenvalue since ϕ_0 determines via Eq. (2.12) the properties of the fission potential $V(x)$. We have

$$\phi_0(x)=\begin{cases}\alpha\sin k_0x, & 0\le x\le a,\\ \beta_1\exp(-k_1x)+\beta_2\exp(+k_1x), & a\le x\le b,\\ \gamma\sin k_0(x-c), & b\le x\le c,\end{cases} \quad (3.2)$$

where $k_0^2+k_1^2=U_0$. We wish to choose the parameters a, b, c, and U_0 in such a way that $V(x)$ has a deep minimum somewhere in the interval $b\le x\le c$, a shallow minimum somewhere in $0\le x\le a$, and a maximum somewhere in $b\le x\le c$. In view of Eq. (2.12), this requires $\phi_0(x)$ to have a very large maximum in $b\le x\le c$, a significantly less pronounced maximum in $0\le x\le a$, and a minimum in $a\le x\le b$. It is easy to see that these requirements can only be met by choosing U_0 quite high. More precisely, and since U_0 has the dimension (length)$^{-2}$, we must choose $U_0^{-1/2}$ small in comparison with the characteristic lengths a, $b-a$, $c-b$ of the problem. The latter quantities are typically several fm in the nuclear context, and we have therefore chosen $U_0^{-1/2}=0.45$ fm for our numerical work. The disadvantage of this choice, necessitated by the requirements mentioned above, is that the curvature of ϕ_0 in the interval $a\le x\le b$ is much larger than in the other two intervals. This means that the curvature of the fission

TABLE I. Parameter values and results for the numerical calculations.

Case No.	1	2	3	4	5	6	7
a (fm)	4.54	4.87	5.15	5.74	6.28	6.35	6.80
x_0 (fm)	4.58	4.92	5.21	5.84	6.46	6.83	7.20
E_f/T	0.125	0.250	0.400	0.875	1.625	1.750	3.250
E_f (MeV)/T (MeV)	0.5/4	0.5/2	1.0/2.5	3.5/4	6.5/4	3.5/2	6.5/4
R' (fm^{-2})	0.255	0.225	0.194	0.121	0.057	0.050	0.011
$(R')^{-1}$ (fm^2)	3.92	4.44	5.16	8.30	17.6	19.9	89.3
λ_0' (fm^{-2})	0.217	0.170	0.136	0.079	0.040	0.036	0.010

potential near the saddle point is much bigger than in the first minimum. This (unphysical) feature of our fission potential $V(x)$ is the price we have to pay for using van Kampen's method and the simple potential (3.1).

Table I contains a list of parameter values for which the calculations have been carried out. The values of b and c were fixed throughout and given by $b=18$ fm and $c=25$ fm. For this choice, the position x_1 of the first minimum of $V(x)$, and the position x_2 of the second (deep) minimum of $V(x)$, are virtually independent of the choice of the remaining free parameter a, and are given by $x_1=3.73$ fm and $x_2=21.27$ fm. In contradistinction, the position x_0 of the saddle point [the maximum of $V(x)$], depends on a and is given in the second row of Table I. According to Eq. (2.12), the curvature $V''(x)$ of the fission potential at the extremal points x_0, x_1, and x_2 is given by $-2T\phi_0''/\phi_0$. The values of ϕ_0''/ϕ_0 are again nearly independent of the choice of a; they are trivially independent of β, μ, and T; and they are given by $-\phi_0''/\phi_0\,|_{x_0}=4.64\ \mathrm{fm}^{-2}$ and $\phi_0''/\phi_0\,|_{x_1}=k_0^2=0.18\ \mathrm{fm}^{-2}$. Multiplying the second of these values by $2T=4$ MeV and equating it with $\mu\omega_1^2$, we obtain, with $\mu=62m_\mathrm{p}$ for ω_1, the harmonic-oscillator frequency at the first minimum, the value $\hbar\omega_1\cong 0.67$ MeV, a typical number. The frequency ω_0 at the saddle point is roughly 25 times larger than this value. This large figure displays an unphysical aspect of our simple model.

The near independence on a of x_1, x_2, k_0^2, and of $\phi_0''/\phi_0\,|_{x_0}$ find a simple explanation in the fact that the lowest eigenvalue of Eq. (2.6) is essentially determined by the fact that there is a large repulsive potential at $x=b$, and free motion in $b\le x\le c$.

According to Eq. (2.12), the height E_f of the fission barrier is given by

$$E_f/T=2\ln[\phi_0(x_1)/\phi_0(x_0)]\ . \tag{3.3}$$

The ratio E_f/T is obviously independent of β, μ, and T, and is listed in the third row of Table I. Values of E_f, corresponding to typical temperatures, are listed in the fourth row, and they show why the particular values of a given in the first row were chosen. We see that we cover a range of values including cases where $E_f \ll T$ and those where E_f/T is reasonably large.

In view of the scaling properties discussed in Sec. II, it is convenient to express all times t in units of the universal time scale $t'=tT/(\beta\mu)$, and all rates λ in units of $\lambda'=\lambda\beta\mu/T$. Equations (2.15) and (2.5) show that $\lambda'(t')$ is a universal function independent of β, μ, and T, and competely specified by the initial condition and the parameters a, b, c, and U_0. We prefer to present the results of our calculations in this way, using for t' the unit fm^2 and for λ' the unit fm^{-2}.

Because of the significance of Kramers's rate expression for fission theory, we compare our results with his formula.[4] For sufficiently large β and for the model here considered, it reads

$$R'=R\beta\mu/T$$
$$=(2\pi)^{-1}\exp[-E_f/T]2[E_0(U_0-E_0)]^{1/2}\ . \tag{3.4}$$

It is satisfactory that R' is independent of β, μ, and T, so that R has the correct scaling behavior. Values of R' and of $(R')^{-1}$ calculated from Eq. (3.4) are listed in the fifth and sixth rows, respectively, of Table I. In the seventh row, we list the stationary rate λ' obtained from our numerical work and our figures.

The numerical calculations were done using the lowest 90 eigenvalues and eigenfunctions in expansion (2.10), and employing the condition (2.13) with x_1 the position of the

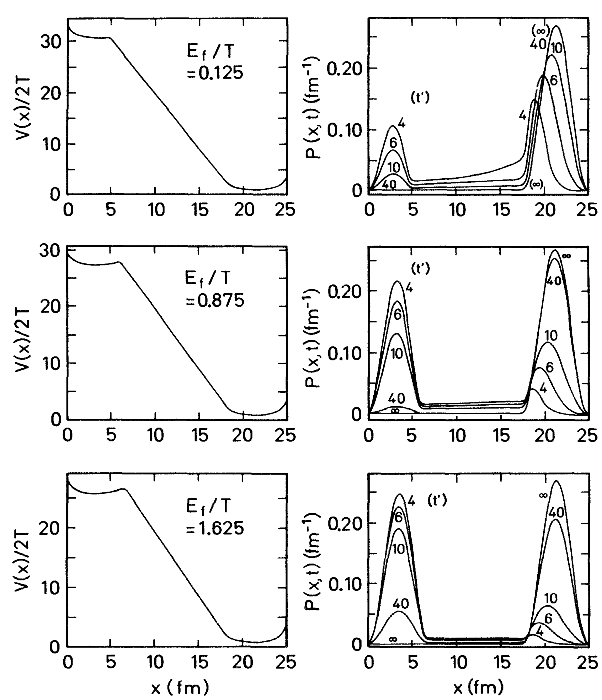

FIG. 1. Left side: The fission potential $V(x)$ in units $2T$ with T the temperature (in MeV) for various choices of a or equivalently, of E_f/T, as indicated. Right side: The corresponding probability density $P(x,t')$ in units (fm^{-1}) plotted versus x (in fm) for various times t'.

first minimum. This expansion is applicable for sufficiently large values of t' subject to the condition that all terms with $n>90$ have been damped out in expansion (2.10), i.e., subject to the condition that $\exp[-(E_n-E_0)t'] \ll 1$ for $n>90$. In our figures, we show results only for such t' values.

Figure 1 shows the fission potential $V(x)$ divided by $2T$ vs x (in fm) for three of the seven cases listed in Table I, corresponding to values of E_f/t as given in our figures. On the right-hand side, we show $P(x,t')$ for various times t' vs x (in fm). We see that for $E_f/T=0.125$ our method does not allow for the study of transients. All we can say is that the fission process is essentially finished at $t' \cong 10$ or so. The other cases show the passage of probability from one valley to the other very clearly. To interpret the figures in physical terms, it may be useful to recall that for $T=4$ MeV, $\mu=62m_{\rm p}$, and $\beta=10^{22}$/sec, a time t of 10^{-21} sec corresponds to a time t' of 0.6 $\rm fm^2$.

Figure 2 shows the function $\lambda'_f(t')$ defined as in Eq. (2.15), but in scaled units versus t' for the seven cases listed in Table I. As the ratio E_f/T decreases, we observe a characteristic change in the pattern. For $T \ll E_f$ ($E_f/T=3.25$), $\lambda'_f(t')$ reaches a stationary value after a transient time of $\tau' \cong 2$ $\rm fm^2$. After that time, a quasistationary current flows over the fission barrier, with a decay rate of $\lambda'_f \cong 0.01$ $\rm fm^{-2}$. A plot similar to the one given in Fig. 1 shows that it takes a long time—$t' \cong 100$ or so—before most of the probability has escaped over the barrier: The transient is a small correction to an otherwise quasistationary process. This picture is completely consistent with the results of Ref. 3. As the ratio E_f/T decreases, $\lambda'_f(t')$ develops the following pattern: Starting at zero for $t'=0$, it overshoots its asymptotic value which is then approached from above. Such features, too, were seen already in Ref. 3 but are driven here to the extreme, cf. the topmost curves in Fig. 2. Our method does, in fact, not allow us to calculate the behavior of these curves reliably for $t' \le 2$ $\rm fm^2$ or so. Therefore, we show in Fig. 3 the quantity

$$P_s(t')=\int_{-\infty}^{x_0} dx \int_{-\infty}^{\infty} dp\, P(x,p;t') ,$$

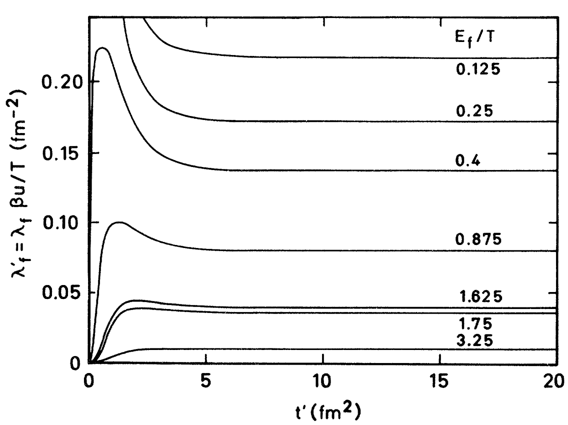

FIG. 2. The scaled rate $\lambda'_f(t')$ versus t' for various choices of a or, equivalently, E_f/T, as indicated.

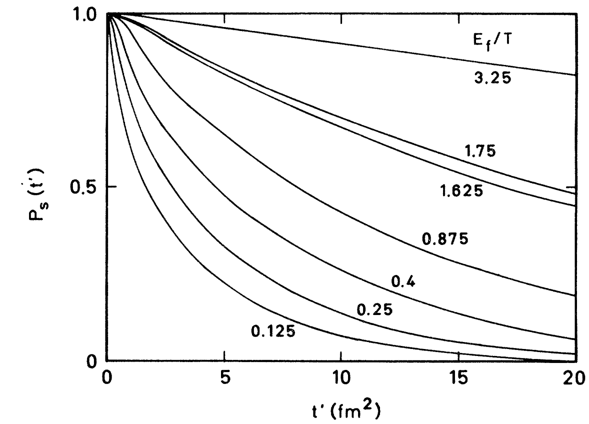

FIG. 3. The probability $P_s(t')$ of finding the system to the left of the saddle point versus t' for various choices of a or, equivalently, of E_f/T, as indicated.

which has a simple behavior and can therefore be displayed without detailed knowledge of the behavior at small t'. (In addition, it is also numerically more stable than λ_f for small t'.) We use also the fact that $P_s(0)=1$. We see that for small values of E_f/T, the behavior of the system is altogether of a transient type: Most of the probability passes the barrier in a single swoop, and only a small remainder follows the quasistationary pattern and an exponential decay law. For these processes, the delay time τ' gives, at the same time, the scale for the lifetime of the entire process. For the physical interpretation of the rates given in Fig. 2, we recall that a rate $\lambda=10^{21}$ $\rm sec^{-1}$ corresponds to a rate $\lambda'=1.7$ $\rm fm^{-2}$ if we choose $\beta=10^{22}$ $\rm sec^{-1}$, $\mu=62m_{\rm p}$, and $T=4$ MeV.

Comparing the quasistationary rates λ'_0 taken from Fig. 2 and listed in the seventh row of Table I with the Kramers's rates R' listed in the fifth row, we see that there is close agreement for large values of E_f/T, and that this agreement gets worse as E_f/T decreases. This is not surprising, since Kramers's formula is based on the approximation that $T \ll E_f$. It is astonishing, in fact, that the two rate values do not differ by more than 20 percent even for $E_f/T=0.125$. We bear in mind, however, that for such small values of E_f/T, the asymptotic (quasistationary) rates are physically void of information since the process is completely governed by transients.

We define the delay time τ' as the minimum of the following two values: (a) the time when λ'_f has approached its asymptotic value with sufficient accuracy (5 percent, say), and (b) the time when $P_s(t')$ is reduced to $1/e$ of its original value unity. This definition is intended to include the cases where the entire fission process is a transient phenomenon. A glance at Figs. 2 and 3 shows that with this definition, $\tau' \cong 3$ $\rm fm^2$ for all values of E_f/T displayed there. This is in semiquantitative agreement with the fit formula (2.17) which, in the limit of very large β and for $\mu=62m_{\rm p}$, yields $\tau'=2.16$ $\rm fm^2$.

This shows that τ' has a very different physical significance depending on whether $E_f/T \ll 1$ or $E_f/T \gg 1$. In the first case, τ' signifies the total duration time of the fission process which is altogether of a transient nature. In the second case, τ' is the delay time until the onset of fis-

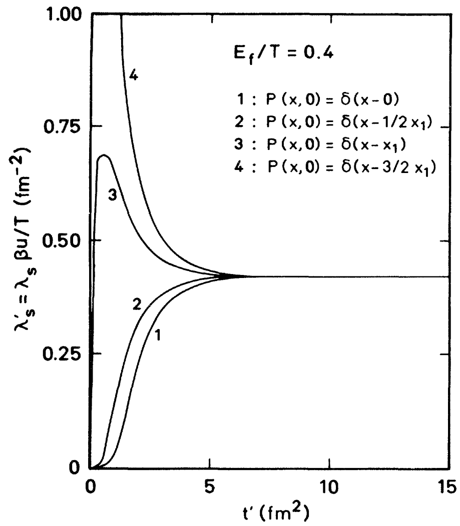

FIG. 4. The scaled rate $\lambda'_f(t')$ versus t' for various initial conditions as explained in the text.

sion. Which of the two interpretations applies can be seen by comparing τ' with $(\lambda'_0)^{-1}$, or with the more easily accessible estimate $(R')^{-1}$. Whenever $(R')^{-1}$ is not at least a factor 3 bigger than τ', we do not deal with a transient phenomenon followed by a long quasistationary process of physical significance.

Figure 4 illustrates the fact that the transient behavior is strongly dependent on which starting configuration is chosen. The calculations were performed for four initial conditions, all given by a delta function, located either at $x=0$ (case 1), at $\frac{1}{2}x_1$ (case 2), at x_1 (case 3), or at $\frac{3}{2}x_1$ (case 4). The curves show $\lambda'_f(t')$ for the four cases, calculated for a choice $E_f/T=0.4$ (case 3 of Table I). The physical interpretation of this pattern can be given in terms of the flow of a drop of honey. If the drop starts far to the left of the fission barrier as a sphere of very small radius, viscous dissipation has smeared it out long before it reaches the saddle point, and the current over the saddle rises slowly towards its quasistationary value, which at the same time signifies the existence of a nontransient (secular) decay process. If, on the other hand, the drop starts with the same shape but close to the barrier, a good portion of it will flow over the barrier in one swoop.

IV. SUMMARY AND CONCLUSIONS

Using the Smoluchowski equation (2.2) and the method of van Kampen tailored to its solution, we have shown that for large values of the friction constant β, the fission rate is governed by scaling laws. There are two time scales which determine the nature of the fission process, a delay time τ approximately given by Eq. (2.17), and the inverse of the quasistationary decay rate. We have seen that the latter can be reasonably well approximated by Kramers's formula even when the fission barrier is much lower than the nuclear temperature T. Both τ and Kramers's rate R obey the scaling laws for large β, and thus allow a discussion of the fission process independent of the actual values of β, of the mass parameter μ, and of T. We have seen from numerical examples that the significance of τ changes: As long as $\tau \ll R^{-1}$, we deal with a transient phenomenon which finally (at time $\sim\tau$) approaches the quasistationary pattern governed by Kramers's rate. Here, all the considerations of Ref. 3 apply, and virtually no fission takes place for times $t \leq \tau$, this leading to an enhancement of light-particle evaporation. With decreasing values of E_f/T, R^{-1} becomes smaller. Whenever $\tau \gtrsim 3R^{-1}$, we enter a new regime. Here, fission is altogether a transient phenomenon of duration time τ, and R loses all physical significance. We have seen that for values of β in excess of 5×10^{21} sec^{-1}, τ is numerically well approximated (to within 50 percent or so) by the expression 2.16 fm$^2\beta\mu/T$.

This finding suggests the following speculative comment on the results reported in Ref. 1. In heavy-ion induced reactions, the angular momentum is large, and the effective fission barrier may be substantially smaller than in neutron-induced fission. At the same time, we deal with nuclear temperatures of several MeV. Using the results of Sec. III, we argue that in this domain where $E_f < T$, fission is altogether a transient process with a characteristic time scale τ as given above. Using the values $\mu=62m_p$, $T=4$ MeV, and $\beta=10^{22}$ sec^{-1} to convert τ to an actual time, we find $\tau\cong3.5\times10^{-21}$ sec, or an effective fission width $\Gamma_{eff}=\hbar/\tau\cong200$ keV. (A further increase of β would decrease Γ_{eff} proportionally.) On the other hand, neutron decay widths at excitation energies around 100 MeV are of the order of an MeV or bigger. This suggests that for values of β which are not altogether unreasonable, a fairly substantial nucleon evaporation prior to fission may be expected. The quantitative analysis of this question, as well as of the consequences to be drawn from the findings of Ref. 3, is being investigated by Hassani and Grangé.

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