Nuclear molecular barrier resonances in the scattering of ^{28}Si on ^{28}Si studied by coupled channel calculations

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(Received 3 October 1983)

For the scattering of ^{28}Si on ^{28}Si coupled channel calculations of the elastic scattering and inelastic single excitation of the first 2^+ state of ^{28}Si are carried out. The real coupling potentials are calculated in the framework of an adiabatic model. The resulting cross sections reveal structures in agreement with the observed ones and support their interpretation as nuclear molecular resonances.

I. INTRODUCTION

The experimental differential cross sections for the elastic and inelastic scattering of ^{28}Si on ^{28}Si, measured by Betts *et al.*[1] in the energy range $E_{\text{c.m.}}=45$–75 MeV, show similar gross and intermediate structures as those already found in lighter systems, for example, in ^{12}C on ^{12}C (Ref. 2), ^{16}O on ^{16}O (Ref. 3), and ^{12}C on ^{16}O (Ref. 4). Betts *et al.*[5] assigned the angular momenta $l=34$–$42\hbar$ to the maxima of the gross structures observed in the elastic excitation function of ^{28}Si on ^{28}Si. These angular momenta are among the highest nuclear spins measured so far in nuclear physics.

Correlations of the gross and intermediate structures between the elastic and inelastic cross sections of lighter scattering systems have been successfully explained by the double resonance mechanism[6,7] acting between nuclear molecular resonances or, more simplified, by the band crossing model.[8] In this paper, we will study the gross and intermediate structures in the elastic and inelastic cross sections of ^{28}Si on ^{28}Si in the adiabatic coupling model of Könnecke *et al.*[9] We show that the structures in the cross sections can be explained with the excitation of resonances on top of the potential barriers which are the relevant molecular resonances in heavier scattering systems. The excitation of quasibound resonances inside the potential pockets, established already in the lighter systems, can be studied when inelastic excitations of higher lying states of ^{28}Si are involved.

II. THE ADIABATIC COUPLING MODEL

This model serves to calculate the potential energy as a function of the relative distance of the nuclei and of multipole coordinates describing the deformations of the nuclear surface. If we restrict the multipoles to the quadrupoles only, described by the coordinates $\alpha_{2m}^{(1)}$ and $\alpha_{2m}^{(2)}$, the real potential energy surface can be generally expanded for equal nuclei up to quadratic terms as follows:

$$V=V^0(r)+V_2^{(1)}(r)\sum_{i=1}^{2}[\alpha_2^{(i)}\otimes Y_2]^{[0]} + \sum_{\Lambda=0,2,4}\left\{V_\Lambda^{(2)}(r)\sum_{i=1}^{2}[[\alpha_2^{(i)}\otimes\alpha_2^{(i)}]^{[\Lambda]}\otimes Y_\Lambda]^{[0]} + V_\Lambda^{(1,2)}(r)[[\alpha_2^{(1)}\otimes\alpha_2^{(2)}]^{[\Lambda]}\otimes Y_\Lambda]^{[0]}\right\}. \quad (1)$$

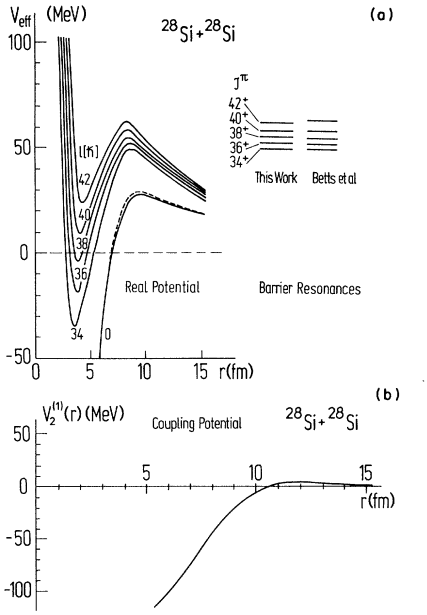

FIG. 1. (a) The real potential for the system ^{28}Si+^{28}Si. The sum of the real and centrifugal potentials is shown for angular momenta between 34 and $42\hbar$. The dashed curve represents the calculation of $V^0(r)$ taken from Seiwert (Ref. 11), who used the folding model with an M3Y-two-body interaction. On the right-hand side the energies of the corresponding potential resonances near the barriers are indicated and compared with those given by Betts *et al.* (Ref. 5). (b) The radial potential for the coupling term linear in the quadrupole coordinates [see Eq. (1)].

In the adiabatic coupling model the potentials V^0, $V_2^{(1)}$, $V_\Lambda^{(2)}$, and $V_\Lambda^{(1,2)}$ are determined by the Coulomb interaction and an effective nuclear Yukawa two-body interaction acting on the density, which is homogeneously distributed over the volume of the composite system for all relative distances (no compression effects as in the usual folding models due to overlapping nuclear densities; for details of the adiabatic coupling model see Ref. 9). The parameters of the potentials are the strength V_0 and range μ of the Yukawa interaction and the radius R_0 of the homogeneous density distributions of the separated ^{28}Si nuclei. The range μ is set $\mu^{-1}=1.4$ fm^{-1}. The parameters V_0 and R_0 are determined by the condition that the energies of the potential resonances, calculated with $V^0(r)$ of Eq. (1) and lying slightly above the corresponding barriers, agree with those for $l=34$–$42\hbar$ obtained by Betts *et al.*[5] from the maxima of the experimental elastic gross structures. We found the parameters $V_0=-438$ MeV fm and $R_0=4.67$ fm.

Figure 1(a) shows the sum of the potential $V^0(r)$ and the centrifugal potential $l(l+1)\hbar^2/2\mu r^2$ for $l=34$–42 and the energies of the barrier resonances (calculated without an absorptive potential) in comparison to those found by Betts *et al.*[5] The radius parameter R_0 of the homogeneous density distribution of the separated nuclei leads to a barrier at a relative distance of about 8 fm for angular momenta in the range of 34–$42\hbar$. The large value $R_0=4.67$ fm corresponds to a radius where the charge density of ^{28}Si, represented by Helm's distribution, has already dropped to 5% of its maximum value,[10] and reflects the softness of the Si nuclei if they touch each other. The same position and height of the potential barrier have been obtained by Seiwert,[11] who calculated $V^0(r)$ by the usual folding model using an M3Y-two-body interaction. This calculation is shown by the dashed curve in Fig. 1(a).

After having fixed the parameters for V_0 and R_0 we can determine all the radial coupling potentials in Eq. (1). In this paper we only use the coupling potential proportional to $V_2^{(1)}(r)$ [see Fig. 1(b)], which linearly couples the ground state configuration of ^{28}Si+^{28}Si to the single excitation of the first excited 2^+ state of ^{28}Si.

III. THE ABSORPTIVE POTENTIAL

The effects of the channels which are not explicitly treated are described by an absorptive potential. For its calculation we apply the statistical model of Ref. 12. In this model the imaginary potential is set proportional to the transition probability out of the elastic or inelastic channels into precompound channels, multiplied by the level density which depends on the excitation energy $E^*=E-V^0(r)$ and quantum number I of the total angular momentum.

$$W(r,E^*,I)\propto |\langle\psi_{\text{el or inel}}|V_{\text{int}}|\psi_{\text{precomp}}\rangle|^2\rho(E^*,I)\ . \quad (2)$$

We assume that the radial dependence of the square of the matrix element is proportional to the number of nucleons $N(r)$ in the overlap region of the collision partners.[7] For the calculation of $N(r)$ we approximate the density of the nucleons of ^{28}Si by the following function [$\theta(x)$=step function]:

$$\rho_{\text{model}}=\rho_0\{\theta(r_0-r)+\theta(r-r_0)\exp[-\lambda(r-r_0)^2]\}\ . \quad (3)$$

The parameters ρ_0, λ, and r_0 are determined by fitting the model density to the measured charge density of ^{28}Si.[10] Finally we obtain an imaginary potential with two free parameters, α and β:

$$W(r,E^*,I)=\alpha N(r)\frac{2I+1}{\sigma^3 E^{*5/4}}\exp\left[2\sqrt{aE^*}-\frac{(I+\frac{1}{2})^2}{2\sigma^2}\right], \quad (4)$$

where

$$\sigma^2=\frac{\theta}{\hbar^2}\sqrt{E^*/\beta}\ , \quad (5)$$

with θ as the moment of inertia of the system.

The spin cutoff parameter σ and the level density constant a are taken the same as in the work of Fink *et al.*[7] Optical model calculations with this imaginary potential have shown that the value of β has to be taken in such a way that the contour lines of constant imaginary potentials in the (E,I) plane (at a fixed relative distance $r=9$ fm, i.e., in the surface interaction zone) have the same slope dE/dI as the band formed by the barrier resonances (for details see Ref. 13). The strength parameter α of the imaginary potential is then fixed by fitting the order of magnitude of the calculated elastic excitation function to the experimental one.

IV. RESULTS

Figure 2 shows the elastic 90° differential cross section calculated with the optical potential consisting of the real part $V^0(r)$ and the imaginary potential (4). For comparison we have drawn the experimental data of Betts *et al.*[1] (dashed line). The theoretical curve in Fig. 2 clearly reveals the gross structures observed in the experimental cross section. These structures arise from the barrier resonances shown in Fig. 1(a), as an analysis of the partial wave functions has shown. In the case of no absorptive potential the resonating partial waves oscillate rapidly with small amplitudes inside the potential pockets (6–12

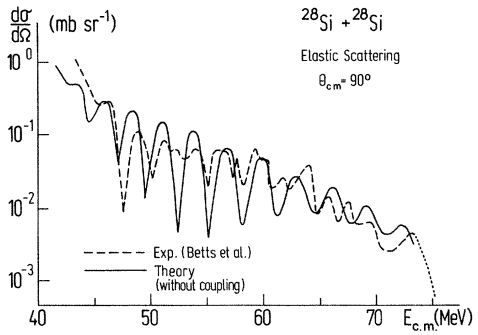

FIG. 2. The differential cross section for the elastic scattering of ^{28}Si on ^{28}Si at $\theta_{\text{c.m.}}=90°$ as a function of the incident energy $E_{\text{c.m.}}$. The theoretical curve (solid line) represents an optical model calculation without coupling of inelastic channels. The experimental data of Betts *et al.* (Ref. 1) are shown by the dashed curve.

nodes) and are strongly peaked in the barrier region (8—9 fm). The imaginary potential damps the small oscillations out, but leaves the surface peaks unaffected. Therefore, the $^{28}Si+^{28}Si$ system forms molecular configurations at larger relative distances than one is used for lighter systems like $^{12}C+^{12}C$ and $^{16}O+^{16}O$. In the latter systems the fitted potentials are flatter, and the positions of the resonance states are found at smaller relative distances than the positions of the barriers.

In the next step we coupled the single excitation of the first 2^+ state of ^{28}Si to the elastic channel. The reduced matrix element $\langle 2^+||\alpha_2||0^+\rangle$, needed for the evaluation of the linear coupling matrix elements, has been determined by fitting the order of magnitude of the calculated inelastic single 2^+ cross section to the experimental one. We found a value of -0.25. With the effective radius $R_0=4.67$ fm used for the calculation of the real potentials this yields a $B(E2,0^+\rightarrow 2^+)$ value of 332 e^2 fm^4 [experimental value 331 e^2 fm^4 (Ref. 14)]. The same imaginary potential is used in all the channels. Compared with the calculation shown in Fig. 2, the imaginary potential has been slightly changed by taking a smaller value for β.

Figure 3 shows the resulting elastic and inelastic cross sections for $\theta_{c.m.}=90°$ in comparison with the experimental ones of Betts *et al.*[1] We recognize that the coupling of the single 2^+ excitation of ^{28}Si affects the elastic cross section in the energy range at about 60 MeV. The finer structures in the theoretical cross sections arise due to the coupling of the barrier resonances in the considered channels. The large peak-to-valley ratios of the calculated elastic scattering cross section near 50—55 MeV may get smoothed out if more reaction channels are coupled to the considered ones. If we had taken ^{28}Si states with higher excitation energies and spins into account in the coupled channel calculations, then quasibound resonances, which lay more inside the potential pockets and have a smaller number of nodes than the virtual barrier resonances, could also be excited.

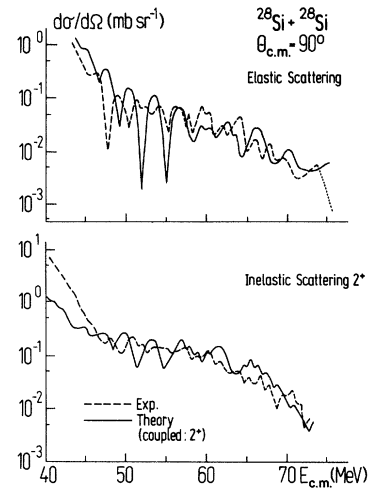

FIG. 3. The 90° differential cross sections (a) for the elastic scattering of $^{28}Si+^{28}Si$ and (b) for the inelastic single excitation of the first 2^+ state in ^{28}Si: $^{28}Si[^{28}Si, ^{28}Si(2^+)]^{28}Si$. The theoretical curves (solid lines) are calculated by coupling the elastic channel with the channels for the single excitation of the first 2^+ state. The experimental data of Betts *et al.* (Ref. 1) are shown by dashed curves.

V. CONCLUSIONS

Our calculations clearly show that the structures in the cross sections of ^{28}Si on ^{28}Si can be traced back to the excitation of resonance states in the relative motion of the nuclei. The corresponding molecular configurations of the considered system are formed at large internuclear distances in the barrier region of the real potential. That is a novel effect compared with the molecular configurations found in lighter systems, where the constituent nuclei have a larger overlap of their surfaces.

We suppose that also in systems heavier than $^{28}Si+^{28}Si$ molecular configurations can exist with internuclear distances lying in the barrier region. As shown in Ref. 15, a barrier and a strong absorption behind this barrier are the only prerequisites for the appearance of barrier resonances. These conditions can be fulfilled also in heavier systems, like, e.g., U + U or U + Cu, as the line structure, observed in the measurements of the positrons emerging from these overcritical systems, reveals.[16,17]

Thus, it is likely that nuclear molecules are a rather general phenomenon appearing in many binary nuclear systems. Therefore, the ^{28}Si-^{28}Si results furnish an important step in proving this conjecture.

ACKNOWLEDGMENTS

We are grateful to Dr. Rainer Könnecke for his help and discussions during the early stage of this investigation. This work was supported by BMFT and GSI (Darmstadt).

[1]R. R. Betts, S. B. DiCenzo, and J. F. Petersen, Phys. Rev. Lett. 43, 253 (1979).
[2]D. A. Bromley, J. A. Kuehner, and E. Almqvist, Phys. Rev. Lett. 4, 365 (1960).
[3]J. V. Maher, M. W. Sachs, R. H. Siemssen, A. Weidinger, and D. A. Bromley, Phys. Rev. 188, 1665 (1969).
[4]R. Stokstad *et al.*, Phys. Rev. Lett. 28, 1523 (1972); R. E. Malmin *et al.*, *ibid.* 28, 1590 (1972).
[5]R. R. Betts, S. B. DiCenzo, and J. F. Petersen, Phys. Lett. 100B, 117 (1981).
[6]W. Scheid, W. Greiner, and R. Lemmer, Phys. Rev. Lett. 25, 176 (1970).
[7]H. J. Fink, W. Scheid, and W. Greiner, Nucl. Phys. A188, 259 (1972).
[8]Y. Abe, in *Proceedings of the Second International Conference on Clustering Phenomena in Nuclei, College Park, Maryland, 1975,* edited by D. A. Goldberg, J. B. Marion, and S. J. Wallace (National Technical Information Service, Springfield, Virginia, 1975), p. 500.
[9]R. Könnecke, J. Guirguis, W. Greiner, and W. Scheid, J. Phys.

G 8, 535 (1982).
[10]R. H. Helm, Phys. Rev. 104, 1466 (1956).
[11]M. Seiwert (private communication).
[12]G. Helling, W. Scheid, and W. Greiner, Phys. Lett. 36B, 64 (1971).
[13]R. Könnecke, W. Greiner, and W. Scheid, J. Phys. G 8, L11 (1982).
[14]G. C. Ball *et al.*, Nucl. Phys. A349, 271 (1980).
[15]W. A. Friedman and C. J. Goebel, Ann. Phys. (N.Y.) 104, 145 (1977).
[16]M. Clemente, E. Berdermann, P. Kienle, H. Tsertos, W. Wagner, F. Bosch, C. Kozhuharov, and W. Koenig, in *Proceedings of the International Conference on Nuclear Physics, Florence, 1983,* edited by R. A. Ricci (Tipografia Compositori, Bologna, 1983), Vol. I, p. 693.
[17]H. Bokemeyer, H. Folger, H. Grein, T. Cowan, J. S. Greenberg, J. Schweppe, K. Bethge, A. Gruppe, K. E. Stiebing, D. Schwalm, P. Vincent, and N. Trautmann, see Ref. 16, p. 694.