Multifragmentation and the partition of angular momentum: General statistical theory

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In the wake of the statistical theory for angular momentum in binary (deep inelastic) processes, a statistical theory for the distribution of angular momentum between the fragments has been developed for the case of multifragmentation (three or more fragments). From the generalized partition function, the average energy and angular momentum of each fragment are derived as well as the corresponding variances. The first moments in the two quantities suggest a "rigid rotation" limit analogous to the binary case. The components of the polarization tensor are calculated for each fragment. The role of thermally generated angular momentum versus that arising from rigid rotation is discussed. Comments are offered on the applicability of the theory to various reactions.

INTRODUCTION

In deep inelastic processes, where only two major fragments are observed in the exit channel, the fate of the entrance channel angular momentum has been studied in great detail both experimentally and theoretically.[1]

It has been found that, in the great majority of cases, especially at the largest Q values, the fragment spin is well described in terms of one vector aligned with the entrance channel angular momentum, arising from the limit of rigid rotation, plus a second vector with randomly distributed components along the three coordinates.[2] The second vector has the effect of introducing fluctuations both in the length and orientation of the resulting total fragment angular momentum. Experimental information on this subject has been obtained in various ways. Gamma-ray multiplicities have provided the sum of the moduli of the fragment angular momenta, sequential alpha and fission decay have provided information on the aligned component of an individual fragment spin, and finally, the angular distributions of sequentially emitted gamma rays or fission fragments have allowed for the measurement of the misaligned component of each fragment's angular momentum.

On the theoretical front, one is confronted with the dynamical results obtained from time-dependent Hartree-Fock (TDHF) theory[3,4] on one hand, or with those obtained from the excitation of high and low frequency collective modes on the other.[5,6] The effect of single particle transfer has also been studied either by itself[7] or by incorporating it into a diffusion equation which allows for statistical fluctuations.[8] The latter treatment falls into the category of time-dependent statistical theories which have been very successful in dealing with many aspects of deep inelastic reactions.[9,10] In contrast with the time-dependent statistical treatment which has the ambition either of knowing or of wanting to find the transfer mechanism, the equilibrium statistical model, brought forth by Moretto and Schmitt,[11] is completely independent of the reaction mechanism and thus can be calculated with a good degree of confidence. In this model, the normal modes of the dinuclear system that can bear angular momentum are identified (bending, twisting, wriggling, and tilting modes) and the partition function is calculated from the corresponding Hamiltonian.

The success of the statistical model in describing the misaligned component of the fragment angular momentum can be attributed to two possible causes. The first and more restrictive possibility implies that the angular-momentum-bearing modes are completely relaxed and thermalized. The second and milder possibility relies on the remarkable fact that the variances closely approach their full magnitude in a time comparable to or shorter than one relaxation time.[12] Consequently, if the first moment is zero by symmetry considerations (bending, twisting, tilting) or it is taken from experiment (wriggling), the equilibrium statistical approach may well suffice for a complete explanation of the experiment. The latter possibility is strongly favored by the success of the statistical model in the quasielastic region. Whatever the judgment may be on the predictive abilities of the statistical model, it is fair to say that, even in the most unflattering judgment, its role still must be considered significant in defining the background against which dynamical or otherwise nonequilibrium effects ought to be observed.

Prompted by the above considerations, we have felt that the time is right to describe the fate of the angular momentum in collisions resulting in a larger number of major fragments within the equilibrium statistical framework.

The production of three or more major fragments is expected to be a dominant mechanism in the region of 10 to 50 MeV/nucleon and higher. The evidence for multifragmentation in reactions induced by Ar or lighter fragments is still somewhat ambiguous due to the difficulty of deciding whether, for instance, an alpha particle is a primary or a secondary particle. On the other hand, this problem should be greatly alleviated by the use of very large targets and projectiles. Already evidence of tripartition is accumulating for Kr-induced reactions.[13–15] There is little doubt that the strong kinematic fix given by the detection of the three or more major fragments will provide the experimenter with a powerful tool to unscramble these complicated processes in the same way as deep inelastic processes have dramatically benefited from their binary nature.

In a reaction regime where several large fragments are found, it should be possible, if not easy, to determine either their average or their individual intrinsic angular momentum by means of more or less standard sequential decay measurements. A more ambitious scientist may even find that the measurement of the spin alignment of the fragments is not altogether impossible. It is to those people willing to stake their lives and reputations in the research of the unknown that this paper is dedicated, with the hope that it will provide them with insight and guidance.

THEORY

Let us consider a collision giving rise to n fragments. In the "expansion" phase, we assume statistical equilibrium, until beyond a critical shape, or mass distribution, the fragments decouple from each other and the equilibrium remains frozen in.

For simplicity, let us suppose that the critical shape is approximately spherical. Then, it is completely general to choose the z axis to coincide with the direction of the angular momentum. Also, for simplicity, let us assume that each fragment is spherical. The Hamiltonian of the system can be written as follows:

$$H=\sum H_i=\sum\left[\frac{I_x^2+I_y^2+I_z^2}{2\mathscr{I}}+\frac{l_z^2}{2mr^2}+\frac{1}{2m}(p_r^2+p_z^2)\right], \tag{1}$$

where the sum $\sum$ is to be carried over the fragments (the corresponding index is omitted for simplicity); I_x, I_y, and I_z are the intrinsic components of the angular momentum for a given fragment of moment of inertia $\mathscr{I}$; l_z is the z component of the orbital angular momentum of a fragment of mass m and distance r from the z axis; and p_r and p_z are the other two generalized momenta for the translational motion of a fragment in cylindrical coordinates. The choice of cylindrical coordinates for the relative motion has the advantage of nicely isolating the z component of the orbital angular momentum.

The generalized grand partition function can now be calculated:

$$Z=\int \exp-\left[\sum\frac{Hi}{T}-\mu\sum(I_z+l_z)\right]$$
$$\times dI_x dI_y dI_z dl_z dp_r dp_z\ , \tag{2}$$

where the constraint of the total angular momentum $I_T=\sum(I_z+l_z)$ (remember the choice of the z axis) has been introduced by means of the Lagrange multiplier μ. This guarantees that the total angular momentum will be conserved, at least on the average. More explicitly,

$$Z=\prod\int\exp\left[-\frac{I_x^2}{2\mathscr{I}T}-\frac{I_y^2}{2\mathscr{I}T}\right]\exp\left[-\frac{1}{2mT}(p_r^2+p_z^2)\right]$$
$$\times\exp-\left[\frac{I_z^2}{2\mathscr{I}T}-\mu I_z\right]\exp-\left[\frac{l_z^2}{2mr^2T}-\mu l_z\right]$$
$$\times dI_x dI_y dI_z dl_z dp_r dp_z\ , \tag{3}$$

where the terms in I_z, l_z have been grouped together. Integration yields

$$Z=\prod(\sqrt{2\mathscr{I}T}\,)^2(\sqrt{2mT}\,)^2\sqrt{2\pi\mathscr{I}T}$$
$$\times e^{\mu^2\mathscr{I}T/2}\sqrt{2\pi mr^2T}\,e^{\mu^2mr^2T/2} \tag{4}$$

or

$$\ln Z=\sum\left[\ln 2\mathscr{I}T+\ln 2mT+\tfrac{1}{2}\ln 2\pi\mathscr{I}T+\frac{\mu^2}{2}\mathscr{I}T\right.$$
$$\left.+\tfrac{1}{2}\ln 2\pi mr^2T+\frac{\mu^2}{2}mr^2T\right]. \tag{5}$$

The value of the Lagrange multiplier μ is determined by the equation

$$\frac{\partial \ln Z}{\partial\mu}=I_T=\mu\sum(\mathscr{I}T+mr^2T) \tag{6}$$

or

$$\mu=\frac{I_T}{T\sum(\mathscr{I}+mr^2)}\ . \tag{7}$$

By differentiating once more with respect to μ, one obtains

$$\frac{\partial^2\ln Z}{\partial\mu^2}=\sigma_{I_T}^2=T\sum(\mathscr{I}+mr^2)\ . \tag{8}$$

This represents the "spurious" fluctuations in I_T introduced by the grand-canonical approach and can be used to estimate the reliability of the theory in any given situation. Differentiation of the logarithm of the partition function with respect to $\beta=1/T$ yields the total energy,

$$-\frac{\partial\ln Z}{\partial\beta}=E=\sum\tfrac{3}{2}T+\sum\tfrac{3}{2}T+\frac{\mu^2}{2}T^2\sum(\mathscr{I}+mr^2) \tag{9}$$

or

$$E=\tfrac{3}{2}nT+\tfrac{3}{2}nT+\frac{I_T^2}{2\sum(\mathscr{I}+mr^2)} \tag{10}$$

where n is the number of fragments, the first term refers to the intrinsic rotational energy, the second to the translational energy, and the third to the rigid rotation of the system at the critical shape. Again, the first two terms arise from the classical energy equipartition theorem, while the third should be interpreted as the energy of a rigidly rotating body whose moment of inertia is defined by the mass distribution associated with the critical shape. The latter is a distinctly interesting but not altogether unexpected result. It may be worth noticing for the last time how convenient the expression of the translational motion in cylindrical coordinates has turned out to be. The intrinsic spin of each fragment can also be obtained by differentiation:

$$-\frac{\partial \ln Z}{\partial \frac{1}{2}\mathscr{I}T}=\bar{I}^2=2\mathscr{I}T+\mathscr{I}T+\frac{\mu^2}{4}4\mathscr{I}^2T^2 \tag{11}$$

or

$$\bar{I}^2=3\mathscr{I}T+\left[\frac{\mathscr{I}}{\sum(\mathscr{I}+mr^2)}\right]^2 I_T^2 \ . \tag{12}$$

This equation says that the fragment angular momentum arises from two contributions: the first is purely statistical and would exist also for zero total angular momentum; the second is the share of the total angular momentum going to the fragment under study, dictated by the rigid rotation condition. The two contributions are added in quadrature. From the structure of Eq. (12), one would also infer that $\sigma_x^2=\sigma_y^2=\sigma_z^2=\mathscr{I}T$, the average for I_x and I_y being zero and for I_z being

$$\bar{I}_z=\frac{\mathscr{I}}{\sum(\mathscr{I}+mr^2)}I_T \ .$$

The latter inference can be verified directly. By isolation of the factor containing I_z in the partition function, one has

$$Z_{I_z}=\prod \exp-\left[\frac{I_z^2}{2\mathscr{I}T}-\mu I_z\right] . \tag{13}$$

Thus,

$$\bar{I}_z=\frac{\partial \ln Z_{I_z}}{\partial \mu}=\mu\mathscr{I}T$$

$$=\frac{\mathscr{I}}{\sum(\mathscr{I}+mr^2)}I_T \tag{14}$$

as expected. Consequently, $\sigma_x^2=\sigma_y^2=\sigma_z^2=\mathscr{I}T$.

The results obtained so far allow us to describe the fragment spin alignment through the relevant components of the polarization tensor:

$$P_{xy}\propto\sigma_x^2-\sigma_y^2=0 \ , \tag{15}$$

$$P_{zz}=\frac{1}{1+3\frac{\sigma^2}{\bar{I}_z^2}}=\frac{1}{1+3\mathscr{I}T\left[\frac{\sum(\mathscr{I}+mr^2)}{\mathscr{I}I_T}\right]^2} \ . \tag{16}$$

For small fluctuations, one has

$$P_{zz}\cong 1-3\mathscr{I}T\left[\frac{\sum(\mathscr{I}+mr^2)}{\mathscr{I}I_T}\right]^2 . \tag{17}$$

For large fluctuations, one has

$$P_{zz}\cong\frac{1}{3\mathscr{I}T}\left[\frac{\mathscr{I}I_T}{\sum(\mathscr{I}+mr^2)}\right]^2 . \tag{18}$$

DISCUSSION

The great simplicity and transparency of the above treatment is marred by the difficulty that one encounters when trying to produce some predictions. The first difficulty is associated with the evaluation of the total moment of inertia $\sum(\mathscr{I}+mr^2)$. This is defined for the critical shape and mass distribution when the decoupling occurs. In the case of the deep inelastic process, it was not too difficult to guess such shape as that of two touching fragments as either spherical or somewhat deformed. In the case of three or more fragments, the problem is much less defined; in fact, the critical shape, even for the same number of fragments, may vary dramatically in going from moderately low-energy collisions to nearly relativistic collisions. Perhaps, with some optimism, one could turn the problem around and, after having looked for good signs of thermalization [see Eq. (10) for inspiration], one might try to infer the critical shape from the observed angular momenta and polarization.

Another difficulty, which is now associated with the entrance channel, is the definition of the angular momentum window to be considered in analyzing data within the framework of this theory. Some idea may be obtained from the elaborate analyses done for other variables in relativistic collisions, but at lower energies, it is still an unknown.

A comforting last observation arises from Eq. (12). Sizable angular momenta can still be expected even for a "central collision" for which $I_T=0$. In fact, one might venture to guess that in many instances this will be the case, especially at the lower energy end. The angular momentum may then be directly related to the temperature which can perhaps be inferred from other observables such as the internal and translational energy of the fragments. If this were fortunately to be the case, the picture should be reasonably easy to unscramble.

But, in the final analysis, what should really justify a statistical treatment in regimes where prompt processes ought to dominate? Two answers can be given. The skeptical answer is that this is the only regime for which it is easy to develop a theory. The following optimistic answer can be given: Try and chase away phase space with a pitchfork, it will still keep coming back.[16]

CONCLUSIONS

A statistical theory predicting the fate of angular momentum in multifragmentation has been developed. This theory allows one to evaluate the mean energies and angular momenta of each fragment as well as their variances. A generalized limit of rigid rotation at a critical shape describes the equilibrium distribution. The fragment spin polarization has been derived from the first and second moments of the fragment angular momenta. General considerations have been given for the applicability of the theory to various energy and impact parameter ranges.

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