Dynamical correlations in inelastic electron scattering sum rules

F. Dellagiacoma, R. Ferrari, G. Orlandini, and M. Traini
Dipartimento di Fisica, Università di Trento, I-38050 Povo, Trento, Italy
(Received 10 May 1983)

A sum rule approach is presented to study in detail the role of dynamical correlations in electron scattering reactions. Both the induced nucleon-nucleon repulsive short range and tensor correlations are taken into account in the explicit calculations of energy-weighted and non-energy-weighted sum rules in ^{16}O and ^{12}C. The former are largely affected by tensor correlations in a wide region of momentum transfer, while the latter are mostly model independent. The theoretical results for the longitudinal and transverse integrated response functions of ^{12}C are compared with available experimental data.

NUCLEAR REACTIONS Longitudinal and transverse quasielastic electron scattering sum rules; tensor correlations in finite nuclei; comparison with experimental data in ^{12}C.

I. INTRODUCTION

In the last few years one of the most interesting experimental results in inelastic electron scattering reactions has been the separation of the longitudinal and transverse response functions in the region of relatively large momentum transfer.[1,2] Consequently, a new theoretical interest has been stimulated in attempting to explain the recent data,[3–6] since one expects on one hand that fundamental aspects of nuclear structure and dynamics, like for example, meson exchange currents, show up differently in the two channels at such values of momentum transferred,[7] and on the other hand that correlation effects beyond the mean field approximation modify the response functions considerably.[8] In particular, tensor correlations are known to be an important ingredient in many reaction mechanisms. For example, their presence becomes crucial in describing processes like photonuclear reactions[9] or other transitions where spin and spin-isospin operators are involved, sometimes cooperating in producing effects commonly ascribed to isobar configurations.[10,11]

A useful tool for the investigation of such effects is represented by sum rule techniques which allow one to focus on basic features of nuclear dynamics, being that the complexity of the nuclear spectrum is eliminated by the closure property. (For reviews see Refs. 12—15.)

Many years ago Drell and Schwartz[16] and McVoy and Van Hove[17] indicated the possibility of extending sum rule techniques to electron scattering processes. The dependence of such sum rules on the momentum transfer allows one to investigate both long and short range correlations, and in this vein many authors have studied the connections between the integrated electronuclear response functions and non-energy-weighted sums.[5,17–23]

As to energy-weighted sums, their utility has been established in the photon absorption reactions,[14] and many attempts have been made in order to extend them for electron scattering processes[16,24–32] since they are a much more sensitive test for exchange forces and two-body correlation functions.

In this work we study the correlation effects in the deep inelastic electron scattering structure functions by means of non-energy-weighted and energy-weighted sum rules, focusing especially on the dynamical correlations induced by the tensor component of the nucleon-nucleon interaction. The inclusion of dynamical effects in their ground state is obtained by making use of a phenomenological method[33] which allows, even in rather heavy nuclei, one to control the different roles of short range and tensor components in affecting the quantities under study.

A comparison between the longitudinal sum rule and the corresponding experimental one in ^{12}C (Ref. 2) is shown, and for the first time an attempt is made to compare theoretical results with the measured transverse integrated response function in that nucleus.

Stimulated by the recent new data[34] on the photoneutron cross section for ^{16}O, a new calculation for the Thomas-Reiche-Kuhn (TRK) sum rule is also performed in order to study its sensitivity to tensor correlation effects.

In Sec. II A we define the objects of our study and give their explicit expressions. Section II B is devoted to a brief review of the method used to insert dynamical correlations in the nuclear ground state. In Sec. III we present and comment on the numerical results for longitudinal and transverse non-energy-weighted and energy-weighted response functions in ^{12}C and ^{16}O. Some conclusive remarks are then summarized in Sec. IV.

II. GENERAL FORMALISM AND MODEL

A. Structure functions and sum rules

Following the notations of Ref. 31 and referring to it for further general considerations, we construct energy-weighted sum rules at constant momentum transfer, by weighting the nuclear response functions with an appropriate power of the energy transfer and summing over all excited states:

$$S_N^{C,T}(q)=\sum_n \omega_{n0}^N |\langle n | F^{C,T}(\vec{q}) | 0\rangle|^2 , \quad (2.1)$$

where ω_{n0} is the intrinsic nuclear excitation energy

$$\omega=\omega_{n0}+\frac{q^2}{2mA}$$

and

$$F^C(\vec{q})\equiv\rho(\vec{q})=\sum_{k=1}^{A} e_k(q)e^{i\vec{q}\cdot\vec{r}'_k} \quad (2.2)$$

$$F^T(q)\equiv\vec{j}_1^{\,\rm spin}(\vec{q})=i\vec{q}\times\sum_{k=1}^{A}\frac{\mu_k(q)}{2m}\vec{\sigma}_k e^{i\vec{q}\cdot\vec{r}'_k} , \quad (2.3)$$

are the (longitudinal) charge and (transverse) magnetization current operators.

In Eqs. (2.2) and (2.3) we have introduced the charge and magnetic nucleonic form factors $e_k(q)$ and $\mu_k(q)$, respectively, and have neglected the contribution of the convection current to the total transverse structure function since the spin-magnetization current is dominant in the transverse electron-nucleus sum rules in the region of momentum transfer $q \gtrsim 1\ {\rm fm}^{-1}$.[31]

The sum (2.1) can be expressed in terms of simple ground state expectation values by applying closure. For $N=0,1$ one has the following:

$$S_0(q)=\tfrac{1}{2}\langle 0 | \{O^\dagger,O\} | 0\rangle , \quad (2.4)$$

$$S_1(q)=\tfrac{1}{2}\langle 0 | [O^\dagger,[H,O]] | 0\rangle , \quad (2.5)$$

where O represents the intrinsic charge or current operator of Eqs. (2.2) or (2.3) and H is the Hamiltonian of the intrinsic nuclear motion.

In the following, we discuss the role of dynamical correlations on $S_0^C(q)$, $S_0^T(q)$, $S_1^C(q)$, and $S_1^T(q)$. The $S_0^C(q)$ sum is related to the Fourier transform of the two-body proton-proton density[17]

$$S_0^C(q)=Ze^2(q)+Z(Z-1)\times\int\rho_{\rm pp}(\vec{r},\vec{r}')e^{i\vec{q}\cdot(\vec{r}-\vec{r}')}d\vec{r}\,d\vec{r}' , \quad (2.6)$$

where

$$\rho_{\rm pp}(\vec{r},\vec{r}')=\frac{1}{Z(Z-1)}\langle 0 | \sum_{i\neq k=1}^{A} e_i(q)e_k(q)\delta(\vec{r}-\vec{r}_i)\delta(\vec{r}'-\vec{r}_k) | 0\rangle \quad (2.7)$$

gives the probability of finding one proton at $\vec{r}$ and a second proton at $\vec{r}'$.

The expression of $S_0^T(q)$ is

$$S_0^T(q)=f_1^T(q)+f_2^T(q) , \quad (2.8)$$

where

$$f_1^T(q)=\frac{e^2(q)}{2m^2}q^2(Z\mu_{\rm p}^2+N\mu_{\rm n}^2) , \quad (2.9a)$$

$$f_2^T(q)=\frac{e^2(q)}{16m^2}\langle 0 | \sum_{k\neq e}(\mu_+^2+\mu_-^2\tau_k^3\tau_e^3)(\vec{q}\times\vec{\sigma}_k)(\vec{q}\times\vec{\sigma}_e)e^{i\vec{q}\cdot\vec{r}'_{ke}} | 0\rangle . \quad (2.9b)$$

The last term is related to the two body spin-spin density and is expected to be influenced by tensor and short range correlations, as is the case for the proton-proton density in (2.6) ($\mu_+=\mu_{\rm p}+\mu_{\rm n}$; $\mu_-=\mu_{\rm p}-\mu_{\rm n}$).

As far as the non-energy-weighted sums (2.4) are considered, the nuclear dynamics is restricted to the nuclear model for the ground state. The further advantage in considering the energy-weighted sums stems from the fact that the nuclear Hamiltonian comes into play, giving information on the role and the nature of the exchange forces.

In particular, the energy-weighted Coulomb sum rule takes the expression[29,31]

$$S_1^C(q)=\frac{NZ}{2mA}e^2(q)q^2[1+\Delta_{\rm corr}(q)+\Delta_{\rm exc}(q)] , \quad (2.10)$$

where

$$\Delta_{\rm exc}(q)=\frac{1}{2}\frac{2mA}{NZe^2(q)}\frac{1}{q^2}\langle 0 | [\rho^\dagger(\vec{q}),[V,\rho(\vec{q})]] | 0\rangle \quad (2.11)$$

and

$$\Delta_{\rm corr}(q)=\frac{1}{NZ}[Z^2-S_0^C(q)/e^2(q)] . \quad (2.12)$$

$\Delta_{\rm corr}(q)$ embodies the center of mass corrections to $S_1^C(q)$, while $\Delta_{\rm exc}(q)$ gives the contribution to the total sum coming from the nucleon-nucleon potential. $\Delta_{\rm exc}=0$ only if meson exchange terms in the nuclear force are neglected. As has been already stressed in Ref. 31, we expect a large contribution to $\Delta_{\rm exc}(q)$ when the presence of tensor correlations is taken into account explicitly in the nuclear ground state.

An energy-weighted sum rule can be analogously defined for the transverse structure function

$$S_1^T(q)=S_1^T(q)_{\rm kin}+S_1^T(q)_{\rm pot}$$
$$=\frac{q^4}{4m^3}(Z\mu_p^2+N\mu_n^2)$$
$$+\tfrac{1}{2}\langle 0\,|\,[F^T(\vec q\,)^\dagger,[V,F^T(\vec q\,)]]\,|\,0\rangle$$
$$-\frac{q^2}{2mA}S_0^T(q)\ . \tag{2.13}$$

The second term gives the enhancement to the sum rule owing to the nuclear potential and is directly connected with the strength of the nuclear forces in the spin-spin and spin-isospin channels. Furthermore, for the spin-isospin character of the transverse magnetization current operator $F^T(\vec q\,)$, we expect a contribution also from the tensor part of the nucleon-nucleon interaction. The last term in (2.13) is the nuclear recoil correction.[29]

B. Model for the correlated wave function

In a recent work[33] we developed a phenomenological method to study dynamical correlation effects on the nuclear structure properties, especially for the investigation of tensor and short range repulsive correlations. We refer to that paper for details of the model and here summarize only a few salient aspects and present a simple recipe to evaluate the mean value of a two body operator.

If the nuclear ground state is described by a shell model single Slater determinant, the mean value of any two-body operator can be written

$$\langle 0\,|\sum_{i\neq k}O_{ik}\,|\,0\rangle=\sum_{a,b}\langle a(1)b(2)\,|\,O_{12}\,|\,a(1)b(2)\rangle$$
$$-\sum_{a,b}\langle a(1)b(2)\,|\,O_{12}\,|\,a(2)b(1)\rangle\ , \tag{2.14}$$

where $|\,ab\,\rangle$ is a product of single particle wave functions of occupied shell model states. By choosing a harmonic oscillator basis the product $|\,ab\,\rangle$ can be expanded on the basis of the wave functions for the relative motion of the pair via a Moshinsky transformation[35]:

$$|\,a(1)b(2)\rangle=\sum_{\substack{m_aS_a\\ m_bS_b\\ \lambda\mu\\ mM}}\ \sum_{\substack{nl\\ NL\\ SS_Z\\ TT_Z}}C_{ab}(\lambda\mu)\,|\,nlm\,\rangle\,|\,NLM\,\rangle\,|\,SS_Z\,\rangle\,|\,TT_Z\,\rangle\ , \tag{2.15}$$

where C_{ab} contains a product of Moshinsky brackets and geometrical coefficients [cf. formula (2.4) or Ref. 33], while N,L,M and n,l,m represent the harmonic oscillator radial and angular quantum numbers of the center of mass and relative motions of the pair. S,T are the spin and isospin of the pair (with third components S_Z and T_Z).

The inclusion of short range correlations is obtained by modifying the short range behavior of the wave function for the relative motion

$$|\,nlm\,\rangle\rightarrow\frac{g(r)}{\sqrt{N_{nl}}}\,|\,nlm\,\rangle\ , \tag{2.16}$$

where the correlation function

$$g(r)\underset{r\to 0}{\longrightarrow}0$$

and

$$g(r)\underset{r\to\infty}{\longrightarrow}1\ ,$$

and the normalization factor N_{nl} is defined by

$$N_{nl}=\langle\,nlm\,|\,g^2(r)\,|\,nlm\,\rangle\ .$$

In addition, tensor effects can be included by adding a small percentage of D states to all the deuteronlike pairs in the nucleus

$$|\,n,{}^3S_1,J_Z,T=0\rangle\rightarrow\sqrt{1-\eta^2}\,|\,n,{}^3S_1,J_Z,T=0\rangle$$
$$+\eta\,|\,n,{}^3D_1,J_Z,T=0\rangle\ , \tag{2.17}$$

where spectroscopic notations ${}^{2S+1}L_J$ have been used and J_Z is the third component of the total angular momentum of the pair. The parameter η^2 embodies the strength of tensor correlations and gives the D-state probability in analogy with the deuteron case.

The method briefly sketched here improves the independent particle description of the ground state and has the merit of being simple, even for rather heavy nuclei, and of allowing insight into the role of both tensor and short range correlations affecting the mean value of the operators we are going to study.

In the following, we will choose the correlation function $g(r)$ [cf. (2.16)] to have the form

$$g(r)=1-e^{-\gamma\beta^2r^2/4}\ ,$$

where $\beta^2=m\omega_0$ is the harmonic oscillator constant and γ the correlation parameter. We select β and γ fixing the (correlated) root mean square radius of the nucleus at the experimental value and choosing a "healing distance" of 1.2 fm.[33] The parameter η is fixed to give a D-state probability $\eta^2=6\%$.

III. RESULTS AND DISCUSSION

A. Non-energy-weighted sums for the longitudinal and transverse channels.

In Fig. 1 we show results for $S_0^C(q)/e^2(q)$ in ^{16}O and, once more[17–22] we note that short range correlation ef-

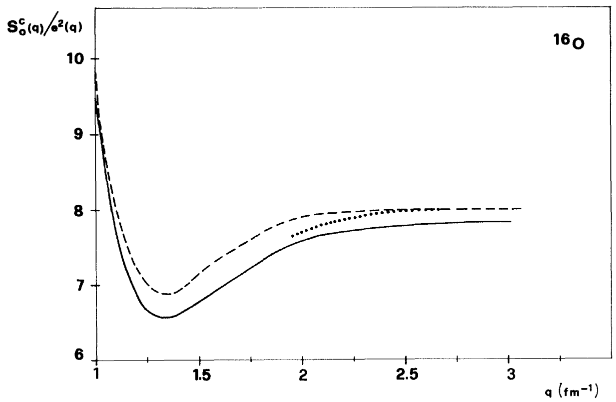

FIG. 1. $S_0^C(q)/e^2(q)$ in the uncorrelated harmonic oscillator model, dashed line, and in the correlated model ($\gamma=20.75$; $\beta^2=0.307$ fm^{-2}), continuous line. A nonrelativistic Fermi gas prediction is also shown for comparison ($K_F=1.36$ fm^{-1}), dotted line.

fects are just a ripple over the main contribution to the sum, i.e., the single nucleonic excitation (incoherent scattering by Z protons). The introduction of induced tensor correlations would further lower the sum a small amount (10—15 % of the short range effects).

In the following, the calculations will contain the proton form factor in the usual dipole form:

$$e^2(q)=\alpha\left[1+\frac{q^2}{a^2}\right]^{-4},$$

where $a^2=18.43$ fm^{-2} and α is the fine structure constant.

In Fig. 2 an explicit comparison between calculated and measured longitudinal integrated response functions in ^{12}C shows a quenching of the data which goes beyond that induced by dynamical correlations (short range and tensor). According to other authors,[5,23,36] the discrepancy could be attributed to meson exchange contributions or higher order relativistic effects which have not been taken into account in our calculations.

Considering on one hand the actual ability of experiments to measure the Coulomb sum, and on the other hand, the several effects concurring to its theoretical determination, it seems premature to draw conclusive statements about dynamical contribution effects to $S_0^C(q)$ with respect to an independent particle description, at least in the region of relatively high momentum transfer.

Specific studies of the transverse non-energy-weighted sum rule for nuclei like ^{16}O and ^{12}C exist only in relation to short range correlations.[17,18,20] Tensor effects have been neglected, and we have found that their contribution has the opposite sign to the short range one.[37]

In Fig. 3 we present the results for the model dependent term $f_2^T(q)$ of $S_0^T(q)$ [see formula (2.9b)] in ^{16}O. We notice that the effects of short range and tensor correlations show up in the region between 1 and 3 fm^{-1}. The former tend to enhance the sum rule, the latter to quench it, and their net result is a small reduction between 1 and 2 fm^{-1} and an increase between 2 and 3 fm^{-1}.

As was the case for the same order Coulomb sum rule, its model-dependent part is overcome by the model independent term which is fixed unambiguously for every nucleus by the gyromagnetic factors of the nucleons. The transverse sum rule and the relative magnitude of the two terms $f_1^T(q)$ and $f_2^T(q)$ is shown in Fig. 4 for ^{12}C. In the same figure, an attempt is made to compare the total sum with the experimental values extracted from the data of Ref. 2, integrating the transverse response function which has been cut off smoothly in the region where pion production and barionic effects begin to become important. The error bars embody both the experimental errors and the uncertainty in the high energy falloff.

The analysis of non-energy-weighted sums and the comparison with their experimental determinations show striking features. In the Coulomb case it seems impossible

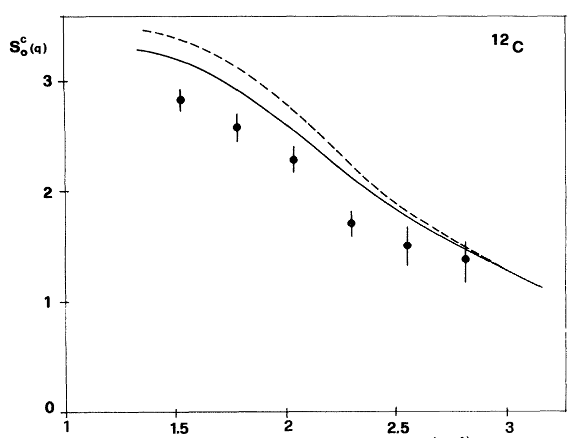

FIG. 2. $S_0^C(q)$ for the uncorrelated harmonic oscillator model, dashed line, and with the short range and tensor correlations ($\gamma=17.90$; $\beta^2=0.358$ fm^{-2}; $\eta^2=0.06$), continuous line. The experimental data are from Ref. 2.

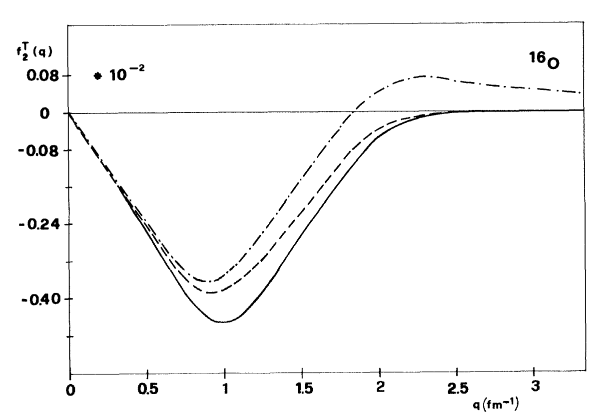

FIG. 3. The term $f_2^T(q)$ of $S_0^T(q)$: the uncorrelated harmonic oscillator model, dashed line; with the inclusion of short range correlations only, dotted-dashed line; with the tensor correlations only, continuous line. Parameters as in Fig. 1; $\eta^2=0.06$.

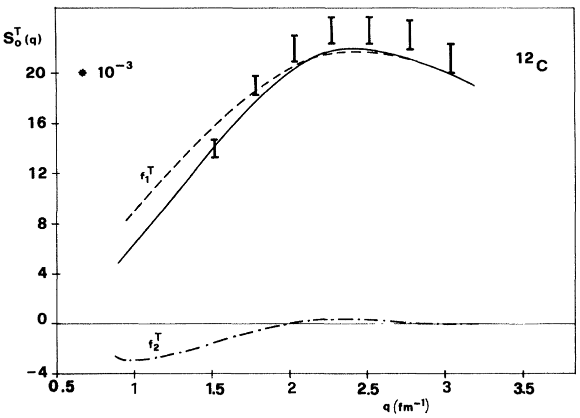

FIG. 4. $S_0^T(q)$, continuous line; its model independent term $f_1^T(q)$, dashed line; and the model dependent term $f_2^T(q)$, dotted-dashed line. $f_2^T(q)$ is evaluated taking into account both the short range and tensor correlations. Parameters as in Fig. 2.

to reproduce the experimental values without including relativistic corrections explicitly, while the short range effects do not improve the calculations crucially. On the contrary, the transverse sum is satisfactorily reproduced for $q=1-3\ \text{fm}^{-1}$ by a simple independent particle model description with the inclusion of Pauli correlations. The well-known quenching of $M1$ and $M2$ transitions at very low momentum transfer[38] is already washed out at $q\approx 1$ fm^{-1}.

From the previous discussion we are led to conclude that the non-energy-weighted sums are not a useful tool to investigate the complexity of nuclear dynamics like tensor effects and short range correlations.

B. Energy weighted sum rules

The longitudinal energy-weighted sum rule $S_1^C(q)$ of formula 2.10 contains the term $\Delta_{\text{exc}}(q)$, which is the essential new ingredient with respect to the analogous non-energy-weighted sum $S_0^C(q)$.

The calculations are performed in a one-boson-exchange model for the nucleon-nucleon potential

$$V(r)=\sum_{\nu}[V^{\nu}(r,m_\nu)-V^{\nu}(r,m_\Lambda)]\left[1-\left[\frac{m_\nu}{m_\Lambda}\right]^2\right]^{-1}, \tag{3.1}$$

retaining only the π and ρ component ($\nu\equiv\pi,\rho$)

$$V^{\pi}(r,m_\pi)=\frac{1}{3}\frac{f_\pi^2}{4\pi}m_\pi\vec{\tau}_1\cdot\vec{\tau}_2\left\{\vec{\sigma}_1\cdot\vec{\sigma}_2+S_{12}\left[1+\frac{3}{m_\pi r}+\frac{3}{(m_\pi r)^2}\right]\right\}\frac{e^{-m_\pi r}}{m_\pi r}, \tag{3.2a}$$

$$V^{\rho}(r,m_\rho)=\frac{1}{3}\frac{f_\rho^2}{4\pi}m_\rho\vec{\tau}_1\cdot\vec{\tau}_2\left\{2\vec{\sigma}_1\cdot\vec{\sigma}_2-S_{12}\left[1+\frac{3}{m_\rho r}+\frac{3}{(m_\rho r)^2}\right]\right\}\frac{e^{-m_\rho r}}{m_\rho r}, \tag{3.2b}$$

where $m_\rho=770$ MeV, $m_\pi=139$ MeV, $f_\pi^2/4\pi=0.08$, $f_\rho^2/4\pi=4.86$, and the regularization mass $m_\Lambda=1051$ MeV. In this framework $\Delta_{\text{exc}}(q)$ can be considered the sum of a $\Delta_{\text{exc}}^{\pi}(q)$ and $\Delta_{\text{exc}}^{\rho}(q)$.

The π component $\Delta_{\text{exc}}^{\pi}(q)$ is practically unaffected by short range correlations; on the contrary, as is shown in Fig. 5, it is strongly enhanced by tensor correlations in a large q range. This result in ^{16}O is consistent with previous calculations[31] performed in ^{4}He by using the Reid soft-core potential and a perturbative method to modify the oscillator wave function.

Of course, owing to the heavy mass of the ρ meson, $\Delta_{\text{exc}}(q)$ is strongly quenched ($\sim$40%) by the short range behavior of the wave function, while tensor correlations further lower it by 10%. The total sum $\Delta_{\text{exc}}(q)$ is dominated by its π component for low q and, consequently, strongly enhanced (100%) by tensor correlations. In the higher q region the ρ component is dominant and the tensor correlation effects amount to 25%.

The expression of the energy-weighted transverse sum rule $S_1^T(q)$ [cf. (2.13)] contains the enhancement term coming from the nuclear potential

$$S_1^T(q)_{\text{pot}}=\tfrac{1}{2}\langle 0\,|\,[F^{T\dagger}(\vec{q}\,),[V,F^T(\vec{q}\,)]]\,|\,0\rangle \tag{3.3}$$

besides the kinetic (model independent) contribution. (In

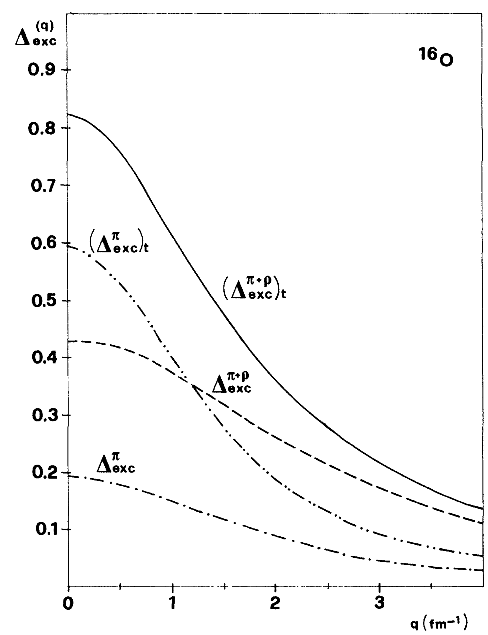

FIG. 5. Potential term, $\Delta_{\text{exc}}(q)$, in $S_1^C(q)$. $\Delta_{\text{exc}}^{\pi}$ and $\Delta_{\text{exc}}^{\pi+\rho}$ are evaluated retaining only short range correlations in the nuclear wave function. $(\Delta_{\text{exc}}^{\pi})_t$ and $(\Delta_{\text{exc}}^{\pi+\rho})_t$ also include tensor correlations. Parameters as in Fig. 1.

the following we will neglect the center of mass corrections, which are of order $1/A$.)

In Fig. 6 a plot of $S_1^T(q)_{\rm pot}$ in ^{16}O is presented and the π component of the sum is shown explicitly. Since the ρ-meson exchange potential originates from a vector coupling ($\vec{\sigma}\times\vec{k}$), contrary to the π exchange which comes from a scalar coupling ($\vec{\sigma}\cdot\vec{k}$), the large contribution of the ρ meson is to be expected because of the transverse nature of the current operator.

The low-q behavior of $S_1^T(q)_{\rm pot}$ is modified by tensor correlations which introduce a q^2 dependence in contrast to the q^4 form of the uncorrelated result. In the higher-q region the correlations are responsible for a relevant enhancement. In contrast to the same order longitudinal sum rule, the net effect of dynamical correlations to the total transverse sum is small owing to the dominance of the aforementioned model independent term (see Fig. 7).

C. Photonuclear enhancement factor

The limiting case of $S_1^C(q)$ for low momentum transfer is particularly interesting[14,29,31]:

$$\lim_{q\to 0}\frac{1}{q^2}S_1^C(q)=\frac{NZ}{2mA}e^2[1+\Delta_{\rm exc}(q=0)]\ . \tag{3.4}$$

Expression (3.4) is equivalent to the famous TRK sum rule for electric dipole transitions, being that $\Delta_{\rm exc}(q=0)$ is just the enhancement factor K defined as

$$K=\frac{mA}{NZe^2}\langle 0\,|\,[D,[V,D]]\,|\,0\rangle\ , \tag{3.5}$$

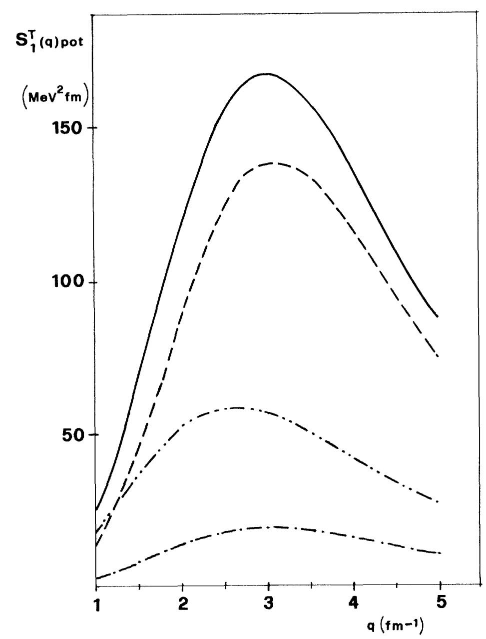

FIG. 6. Potential term of $S_1^T(q)$ in ^{16}O. The dotted-dashed and two-dotted-dashed lines represent only the π contribution without and with inclusion of the tensor correlations, respectively. Dashed and continuous lines represent the sum of π and ρ contributions again without and with the tensor correlations, respectively. Parameters as in Fig. 1.

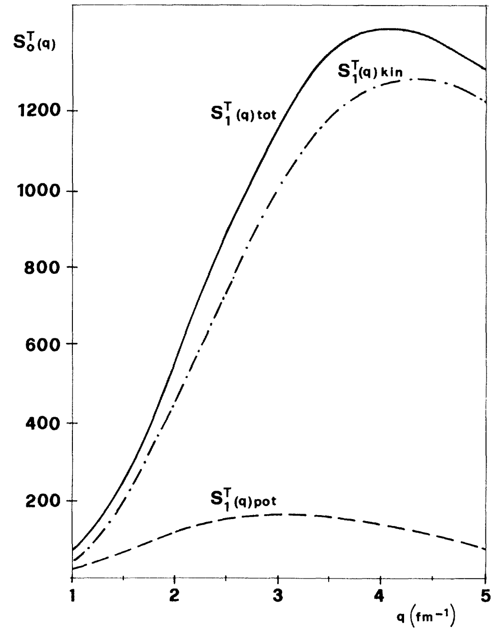

FIG. 7. The transverse sum rule $S_1^T(q)$ together with its potential, $S_1^T(q)_{\rm pot}$, and kinetic, $S_1^T(q)_{\rm kin}$, terms, in ^{16}O.

where

$$D=\sum_{k=1}^{Z} ez_k$$

is the electric dipole operator.

The central role of tensor correlations in the evaluation of K has been established and has stimulated many studies and experimental observations (cf. Ref. 14 and references therein). With the calculations in our simple model we just find a confirmation of the strong tensor influence on this quantity. In fact, the value of K, which one can read on the $q=0$ axis of Fig. 5 and which is summarized in Table I, passes from 0.44 to 0.83 when the induced tensor correlations are explicitly taken into account.

Recently, Berman *et al.*[34] reconstructed the total photonuclear cross section from the experimental information on the partial contributions. The integration up to 140 MeV has given the value $K\simeq 0.40\pm 0.15$; in contrast, the experimental data[39] obtained by subtracting the calculated atomic absorption cross section from the measured total photon absorption cross section, give $K\simeq 1.12$. From the comparison between the present estimate of K and the recent analysis by Berman *et al.*, one could guess that tensor contributions are effective even above the pion threshold.

TABLE I. The values of the enhancement factor K for the electric dipole sum rule in ^{16}O. See further comments in the text.

K^{π}	$K^{\pi+\rho}$	
0.19	0.44	without tensor correlations
0.60	0.83	with tensor correlations

Anyhow, for ^{16}O, there are still too many open questions regarding the measurements themselves, the relation between the experimental values, and the theoretical TRK evaluation,[14] and too few explicit estimates[40] of the photonuclear cross section above the giant resonance region to draw definite conclusions.

D. Scaling law in quasifree scattering region

For high momentum transfer (quasifree scattering) the structure functions may be considered as functions of only one kinematic variable. This phenomenon, known as "scaling," generally reveals the existence of pointlike constituents in the target,[41] while deviations from scaling give information on the structure of the constituents and the basic properties of their interaction. The electronuclear sum rules previously discussed are a useful tool for studying the scaling hypothesis in nuclear physics.[42,43]

Defining a scaling variable as

$$y=\frac{\omega}{q}-\frac{q}{2m} \tag{3.6}$$

for a system of pointlike noninteracting particles, the structure functions are symmetrical with respect to $y=0$. On the contrary, the deviation of the mean value $\bar{y}$ from zero reveals scaling violation and contains information on exchange effects, nuclear correlations, and high momentum components of the wave functions. The mean value $\bar{y}$ for the longitudinal and transverse channel is related to the electronuclear sum rules (2.6), (2.8), (2.10), and (2.13),

$$\bar{y}^{C,T}_{(q)}=\frac{S_1^{C,T}(q)}{qS_0^{C,T}(q)}-\frac{q}{2m}\left[1-\frac{1}{A}\right]. \tag{3.7}$$

Therefore, an explicit evaluation of the sums $S_N^{C,T}(q)$ permits a detailed study of the approach to the scaling limit as the momentum transfer increases.

In Figs. 8(a) and (b) the results of the present calculation for $\bar{y}^{C}(q)$ and $\bar{y}^{T}(q)$ are shown. The oscillations of $\bar{y}^{C}(q)$ are owing to the presence of proton-proton correlations, and its high q behavior is directly related to $\Delta_{\text{exc}}(q)$.

As to $\bar{y}^{T}(q)$, its behavior is dominated by the exchange potential contribution through $S_1^T(q)_{\text{pot}}$ (cf. Fig. 6) for $q \gtrsim 2$ fm^{-1}, and at lower momentum transfer it is much more regular because it is driven by the spin-spin density embodied in $S_0^T(q)$ (cf. Fig. 4).

Therefore, the inclusion of tensor correlations in the simple independent particle ground state gives rise to a considerable enhancement in $\bar{y}^{C,T}(q)$ and a much slower asymptotic approach to zero.

IV. CONCLUSIVE REMARKS

From the analysis of the non-energy-weighted and linear-energy-weighted electronuclear sum rules some conclusive remarks about the dynamical correlation effects are in order here:

(i) Induced tensor correlations do not affect the Coulomb sum $S_0^C(q)$ appreciably. A comparison with experimental data in ^{12}C is unable to discriminate between the correlated and the independent particle description of the nuclear wave function, at least in the region of relatively high momentum transfer.

(ii) The transverse sum rule $S_0^T(q)$ is also scarcely influenced by tensor correlations because of the dominance of its model independent term. The tentative comparison with the existing experimental data turns out to be rather satisfactory, even when including Pauli correlations only.

(iii) The energy-weighted Coulomb sum rule $S_1^C(q)$ is enhanced by the meson exchange potential and is largely affected by tensor correlations in a wide region of momentum transfer.

(iv) Exchange effects due to the ρ meson dominate the potential term of the transverse energy-weighted sum. The introduction of tensor correlations changes its low-q behavior significantly.

(v) A revisitation of the TRK sum rule has been possible by study of the $q \to 0$ limit of $S_1^C(q)$. The tensor correlations are responsible for an enhancement of K^{TRK} from 0.44 to 0.83 in ^{16}O.

(vi) The strong effects of tensor correlations in $S_1^C(q)$ and $S_1^T(q)$ are reflected in the approach to the scaling regime of the corresponding structure functions.

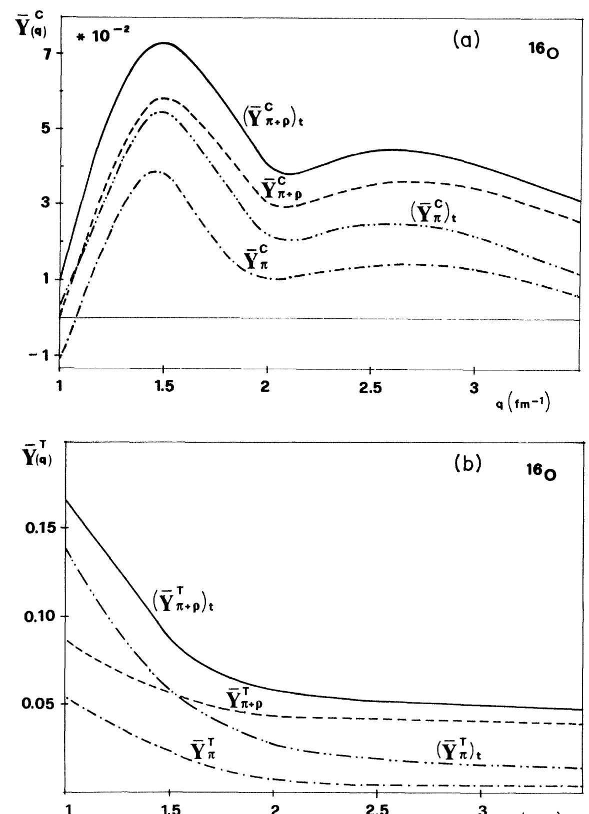

FIG. 8. Mean values of scaling variables for the longitudinal response, (a), and for the transverse one, (b). Notation as in Fig. 5. Same parameters as in Fig. 1.

ACKNOWLEDGMENTS

We are indebted to S. Frullani and J. Morgenstern for clarifying information on the experimental situation and to R. Leonardi, E. Lipparini, S. Stringari, and W. Leidemann for useful suggestions. One of us (G.O.) kindly acknowledges D. Drechsel and J. S. O'Connell for fruitful discussions, and the theoretical group of the Institut für Kernphysik of Mainz University for their warm hospitality.

---

[1]R. Altemus *et al.,* Phys. Rev. Lett. 44, 965 (1980).

[2]P. Barreau *et al.,* Nucl. Phys. A358, 287 (1981); P. Barreau *et al., ibid.* A402, 515 (1983).

[3]J. V. Noble, Phys. Rev. Lett. 46, 412 (1981); P. D. Zimmermann, Phys. Rev. C 26, 265 (1982).

[4]L. S. Celenza, W. S. Pong, M. M. Rahman, and C. M. Shakin, Phys. Rev. C 26, 320 (1982).

[5]J. S. O'Connell *et al.,* Phys. Rev. C 27, 2692 (1983).

[6]M. Cavinato, D. Drechsel, E. Fein, M. Marangoni, and A. M. Saruis, Phys. Lett. 127B, 295 (1983).

[7]T. W. Donnelly, J. W. Van Orden, T. de Forest, Jr., and W. C. Hermans, Phys. Lett. 76B, 393 (1978); M. Kohno and N. Ohtsuka, *ibid.* 98B, 335 (1981).

[8]See, for example, C. Ciofi degli Atti, in *Progress in Particle and Nuclear Physics,* edited by D. H. Wilkinson (Pergamon, New York, 1980), Vol. 3, p. 163.

[9]A. Arima, G. E. Brown, H. Hyuga, and M. Ichimura, Nucl. Phys. A205, 27 (1973).

[10]C. Gaarde, J. S. Larsen, and J. Rapaport, in International Conference on Spin Excitations in Nuclei, Telluride, Colorado, 1982, and references therein.

[11]G. F. Bertsch and I. Hamamoto, Phys. Rev. C 26, 1323 (1982).

[12]J. S. O'Connell, in *Proceedings of the International Conference on Photonuclear Reactions and Applications, Asilomar, 1973,* edited by B. L. Berman (Lawrence Livermore Laboratory, University of California, 1973); J. V. Noble, Phys. Rep. C40, 241 (1978).

[13]D. Drechsel, in *International School on Electro and Photonuclear Reactions, Erice, 1976,* in Lecture Notes in Physics (Springer, Berlin, 1977), Vol. 62, p. 92; D. Drechsel, in *Proceedings on the Fourth Seminar on Electromagnetic Interaction of Nuclei at Low and Medium Energies, Moscow, 1977* (Academy of Science USSR, Moscow, 1979), p. 293.

[14]H. Arenhövel, in *Proceedings of the International Conference on Nuclear Physics with Electromagnetic Interactions, Mainz, 1979,* edited by H. Arenhövel and D. Drechsel, in Lecture Notes in Physics (Springer, Berlin, 1979), Vol. 108, p. 159.

[15]G. Orlandini, *From Collective States to Quarks in Nuclei, Proceedings of the Workshop on Nuclear Physics with Real and Virtual Photons, Bologna, 1980,* edited by H. Arenhövel and A. M. Saruis, in Lecture Notes in Physics (Springer, Berlin, 1981), Vol. 137, p. 72.

[16]S. D. Drell and C. L. Schwartz, Phys. Rev. 112, 568 (1958).

[17]K. W. McVoy and L. Van Hove, Phys. Rev. 125, 1034 (1962).

[18]W. Czyz, L. Lesniak, and A. Malecki, Ann. Phys. (N.Y.) 42, 119 (1967).

[19]J. S. O'Connell, Phys. Lett. 32B, 323 (1970).

[20]J. W. Lightbody, Phys. Lett. 33B, 129 (1970).

[21]A. Malecki and P. Picchi, Phys. Lett. 43B, 351 (1973).

[22]V. Tornow, Y. E. Kim, and Yoon Suk Koh, Nucl. Phys. A369, 281 (1981).

[23]J. W. Van Orden and T. W. Donnelly, Ann. Phys. (N.Y.) 131, 451 (1981).

[24]E. V. Inopin and S. N. Roshchupkin, Yad. Fiz. 17, 1008 (1973) [Sov. J. Nucl. Phys. 17, 526 (1973)].

[25]V. D. Efros, Yad. Fiz. 18, 1184 (1973) [Sov. J. Nucl. Phys. 18, 607 (1974)].

[26]A. Yu. Buki, N. G. Shevchenko, and A. V. Mitrofanova, Yad. Fiz. 24, 457 (1976) [Sov. J. Nucl. Phys. 24, 237 (1976)].

[27]E. L. Kuplennikov, V. A. Goldstein, N. G. Afanasev, V. G. Vlasenko, and V. I. Startsev, Yad. Fiz. 24, 22 (1976) [Sov. J. Nucl. Phys. 24, 11 (1976)].

[28]E. Lipparini, G. Orlandini, and R. Leonardi, Phys. Rev. C 16, 812 (1977).

[29]A. Dellafiore and M. Traini, Nucl. Phys. A344, 509 (1980).

[30]H. Arenhövel, D. Drechsel, and H. J. Weber, Nucl. Phys. A305, 485 (1978).

[31]V. Tornow, G. Orlandini, M. Traini, D. Drechsel, and H. Arenhövel, Nucl. Phys. A348, 157 (1980).

[32]M. Kohno, Nucl. Phys. A366, 245 (1981).

[33]F. Dellagiacoma, G. Orlandini, and M. Traini, Nucl. Phys. A393, 95 (1983).

[34]B. L. Berman, R. Bergère, and P. Carlos, Phys. Rev. C 26, 304 (1982); P. Carlos *et al.,* Nucl. Phys. A378, 317 (1982).

[35]M. Moshinsky, Nucl. Phys. 13, 104 (1959); M. Moshinsky and T. Brody, *Tables of Transformation Brackets* (Monografia de Istituto de Fisica, Mexico City, 1960).

[36]J. M. Laget, Phys. Rep. 69, 1 (1981); Nucl. Phys. A358, 275c (1981).

[37]G. Orlandini, M. Traini, and F. Dellagiacoma, Nuovo Cimento 76A, 246 (1983).

[38]A. Richter, in *Proceedings of the Ninth International Conference on High Energy Physics and Nuclear Structure, Versailles, 1981,* edited by P. Catillon, P. Radvanyi, and M. Porneuf (North-Holland, Amsterdam, 1982), in Nucl. Phys. A374, 177c (1982).

[39]J. Ahrens *et al.,* Nucl. Phys. A251, 479 (1975).

[40]M. Gari and H. Hebach, Phys. Rep. 72, 1 (1981); H. Hebach, Nuovo Cimento 76A, 231 (1983).

[41]G. B. West, Phys. Rep. 18C, 263 (1975), and references therein.

[42]T. Suzuki, Phys. Lett. 101B, 298 (1981).

[43]V. Tornow, D. Drechsel, G. Orlandini, and M. Traini, Phys. Lett. 107B, 259 (1981).